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Pull-In Voltage of Fixed-Fixed End Type MEMS Switches with Variative Electrostatic Area

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Abstract: In this paper we developed a new approach to calculation of pull-in voltage of fixed-fixed end type MEMS switches. In the previous works Pull-in voltage has been calculated on a lumped model, when one of them used a distributed model by assuming the electrostatic area is applied the whole of the surface of the beam. In the present work, we used a distributed model to determinate the pull-in voltage. To do this the nonlinear governing equation of the beam with non-uniform electrostatic pressure that applied in the part of beam has been derived. The derived nonlinear equation has been solved numerically and has been showed that the results of our approach are more accurate than the other previous works.

Keywords: MEMS, distributed model, electrostatic pressure

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1. Introduction

The advent of monolithic integration of micromechanical structure with electronics requires the development of simulation and analysis tools that can allow coupled circuit and micromechanical simulation [1]-[8]. Pull-in phenomenon is a discontinuity related to the interplay of the elastic and electrostatic pressures. The determination of the pull-in voltage and position requires the solution of a coupled electrostatic-elastic system. Previously static analyses have been performed to determine the beam equilibrium position for a given voltage using both lumped parameter and distributed model. The researchers have used three methods to decrease the actuation voltage [1-8]. One of them is variative electrostatic area [2]. Previous models cannot capture precision details, such as variative electrostatic area and varying section. Design and optimization of MEMS switch requires efficient solution techniques. In the current investigation an efficient analyzes is implemented to provide an understanding of MEMS switch behavior.

Previous work addressing Pull-in voltage has taken both electrical and mechanical approaches. Chen and Kang [9] described a method when distributed electrostatic pressure applied at whole of beam and initial gap was not considered in electrostatic pressure. Zhang and Zhao [10] presented four dimensionless numbers but didn't solve governing equation and they apply the electrostatic pressure at whole of beam. Design and optimization of MEMS devices requires efficient solution techniques [11], [12]. Existing models have no sufficient accuracy, so that there has developed an emergent requirement for more accurate model to approach the design of MEMS switch. Compared with the 1D lumped model, the 2D distributed model can attain more accurate solutions. In MEMS Switch model the electrostatic pressure doesn't apply to whole of the beam, and beam divided to 3 sections that electrostatic pressure has been apply in midsection, so it apply the one section of beam, in other words the electrostatic area can be varying. Also, by checking the location and amount of electrostatic area, under the beam, errors have been studied between computer analysis and calculation method.

We considered the electrostatic area to 20%, 30% and 40% of length of beam, and calculated pull-in voltage. Also, the electrostatic pressure is function of deflection. It causes that nonlinear equation produce. Then we considered that the area of beam above substrate as shown in Fig. 1 is difference from other two sections.

2. Model Description

Because of the miniaturization and more accurate it is necessary to take some effects such as variative electrostatic area. In one-dimensional model, beam is electrostatically actuated by an applied DC voltage across rigid parallel plates, where one plate is fixed and the other is attached to a spring. The displacement versus voltage characteristic for this geometry is shown in Fig. 2. In our nonlinear distributed model, electrostatic pressure is applied only to a certain part of a beam and in this part, the section is difference from other sections. In our model, we compare different electrostatic area (20 %, 30 % and 40 %) and their effect in pull-in voltage.

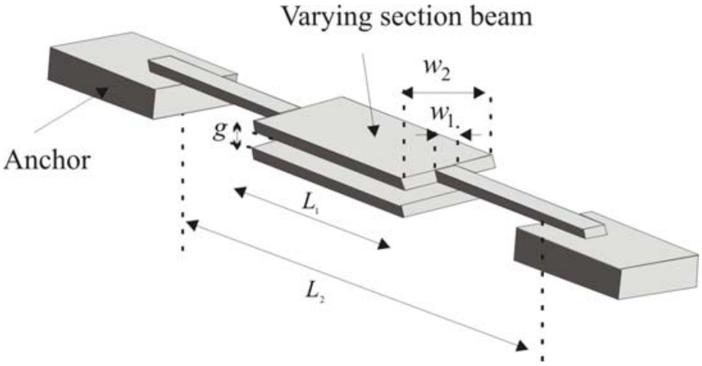

Fig.1. The schematic diagram of the fixed-fixed end type MEMS switches with varying section.

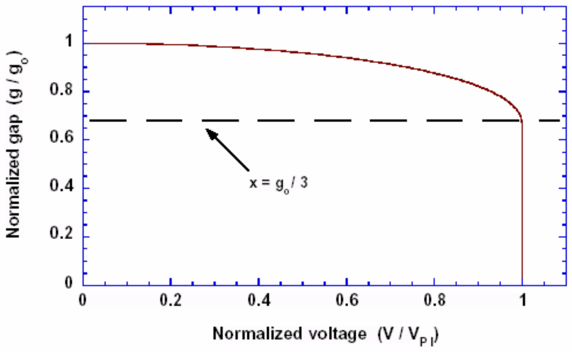

Fig. 2. Gap versus voltage for the one-dimensional model, showing the pull in instability at a displacement of one-third the original gap.

The detailed schematic of fixed-fixed bridge structure is shown in Fig. 3.

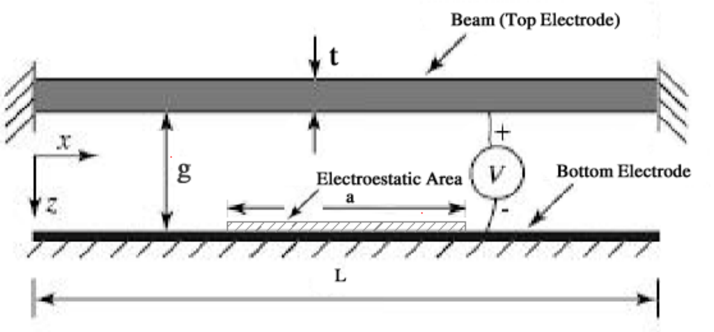

Fig.3. Detailed schematic of fixed-fixed end type MEMS switch.

2.1 Governing Equations

The micro switch configuration is modeled using Euler-Bernoulli beam theory with a constant cross-sectional area along the length of the beam. The coupled electromechanical equation is shown in general in equation (1):

$$\tilde{E}\hat{I}\frac{d^4u}{dx^4} = q(x) \ , \tag{1}$$

here $\tilde{E}$, $\hat{I}$, are the effective modulus, which are wide relative to their thickness and length, the effective moment of inertia of the cross-section and equals $\frac{wt^3}{12}$, where *t* is the film thickness and *w* is width of the beam and *q(x)* is electrostatic pressure. The electrostatic pressure (right-hand side) is derived from a parallel-plate approximation with an applied voltage V, with considering the variative electrostatic area and nonlinear electrostatic pressure because when gap is decreases because of deflection the electrostatic pressure is nonlinear increases, the electrostatic pressure is:

$$q(x) = \frac{\varepsilon_0 V^2 w}{2[g - u(x)]^2} \ , \tag{2}$$

Where *V* is the voltage applied between the movable and the ground plates on the fixed substrate, g is the initial gap between the movable and the fixed plates, ε_0 is the permittivity of air. But should be attend that when the electrostatic pressure is applied at whole of beam, the electrostatic pressure is equation above. We improve this model in next section.

2.2 Mode Improvement

In this section, we correct electrostatic pressure. The electrostatic pressure is the pressure per unit length that acts on the beam in the region directly above the gate. A node is placed at each end of the gate in the mode

$$q(x) = \frac{\varepsilon_0 V^2 B}{2[g - u(x)]^2}[H(x - (\frac{l}{2} - \frac{b}{2})) - H(x - (\frac{l}{2} + \frac{a}{2}))], \tag{3}$$

where *H(x)* is Heaviside function. Respectively, *a* is the length of left and right side of electrostatic area. In this model the Pull-in voltage were calculated for100 Grid points using that program until convergence was achieved. Material and geometrical parameters for the structure are indicated in Table 1.

Table 1. Material and Geometrical parameters of varying section fixed-fixed end type MEMS switch.

Parameter	**Value**
E (Young's modulus)	169 (MPa)
υ(Poisson's ratio)	0.3
ρ (Density)	2331 (Kg/m)^3
ε (Permittivity of air)	8.8541878×10^{-12} (F/m)
L (Length of the beam)	800 µm
Width of the beam(W$_1$)	50 µm
Width of the beam(W$_2$)	100 µm
t (Thickness of the beam)	14.4 µm
g (Initial gap)	1 µm

3. Solving Differential Equation

By algebra equation (1) can be written as equation (4),

$$\frac{\hat{E}t^3}{12}\frac{d^4u}{dx^4} = \frac{\varepsilon_0 V^2}{2[g-u(x)]^2}[H(x-(\frac{l}{2}-\frac{b}{2}))-H(x-(\frac{l}{2}+\frac{a}{2}))]\,, \tag{4}$$

A solution of the governing differential equation will now be determined. Due to the nonlinearity in the electrostatic pressure, an analytical solution is impractical to obtain and a numerical solution is sought. Numerical finite difference method has been used for evaluating the pull-in voltage from equation (4). Increment the applied voltage until pull-in occurs and relies on a discretized nodal array. The four spatial boundary conditions needed to find a solution are determined from the requirement that two end of the beam are clamped (zero displacement and zero slop).These conditions allow (1) to be expressed solely in terms of the nodal deflections (u_i, i = 1, 2,…N), where node *1* is located at the fixed end of the beam and node *N* is located at the beam tips.

The boundary conditions are:

$$u(0)=0\,,\; u(l)=0\,,\; \frac{du}{dx}(0)=0\,,\; \frac{du}{dx}(l)=0\,, \tag{5}$$

$$u_i^{(4)} = \frac{u_{i+2}-4u_{i+1}+6u_i-4u_{i-1}+u_{i-2}}{h^4} \tag{6}$$

The grid points represented by the displacements u_0, u_{N+1} and u_{N+2} are image points needed for the finite difference approximations of the boundary conditions and field equations. At beam center, by symmetry we have zero slop and no shear force at beam center (This is the derivative of the moment, which is proportional to the radius of curvature. In other words, we solved the nonlinear equation for only one second of beam and other section is exactly equal to this section. The boundary conditions are:

$$u(0)=0,\; \frac{du}{dx}(0)=0,\; \frac{du}{dx}(half)=0,\; \frac{d^3u}{dx^3}(half)=0 \tag{7}$$

4. Results

The varying section and variative electrostatic area are two methods to accurately determine pull-in voltage in MEMS switch. By considering the variative electrostatic area in width it will be saw that the exact pull-in voltage is different from last models (Table 2). Fig. 4 shows pull-in voltage versus center gap for varying section model in fixed-fixed end type MEMS switches. Fig. 4 shows that if electrostatic area were small, the pull-in voltage decreases and gap increases. Also the small electrostatic area is closed to lumped model. In lumped model, the electrostatic pressure is applied in mid point of beam and if the electrostatic area grows, errors between last model and exact results increase.

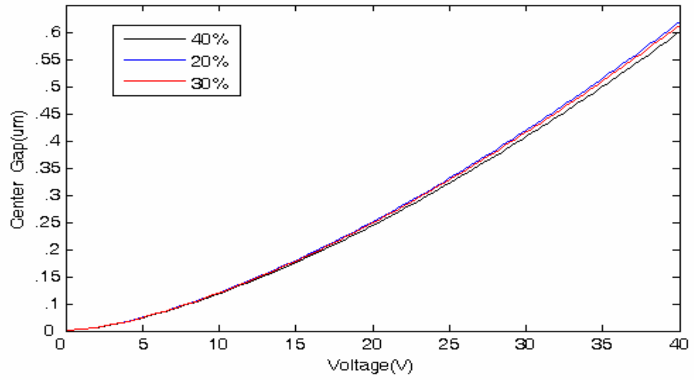

Fig.4. Center gap versus voltage for the varying section model acknowledgements.

Table 2 shows value of pull-in voltage and center gap of lumped model and distributed model with variative electrostatic area. It is seeing when considered the variative electrostatic area pull-in voltage is different from model without this effect and the pull-in voltage doesn’t occur at a displacement of one-third the original gap. The pull-in voltage without considering electrostatic area is 40.5 V [13] and the pull-in voltage of 40 V for the same beam structure obtained by the FEM/BEM technique and when the electrostatic area is applied, the pull-in voltage is different.

Table 2. Pull-in voltage and center gap in different model.

Type Model	Pull-In Voltage	*Center Gap*
Lumped Model	36.3221	*.6667*
20%	40.0508	*.6023*
30%	40.0818	*.6022*
40%	*40.1253*	*.6016*

By solving governing equation, the deflection of beam in pull-in voltage and normal voltage are plotted. Fig. 5 shows the gap versus discretize beam. λ Equals $\frac{V}{V_{pull-in}}$ and are .8, .6 and .4.if the voltage increases to pull-in, the beam will be unstable and a large difference occurs between the deflection of $\lambda = 1, \lambda = .8$.

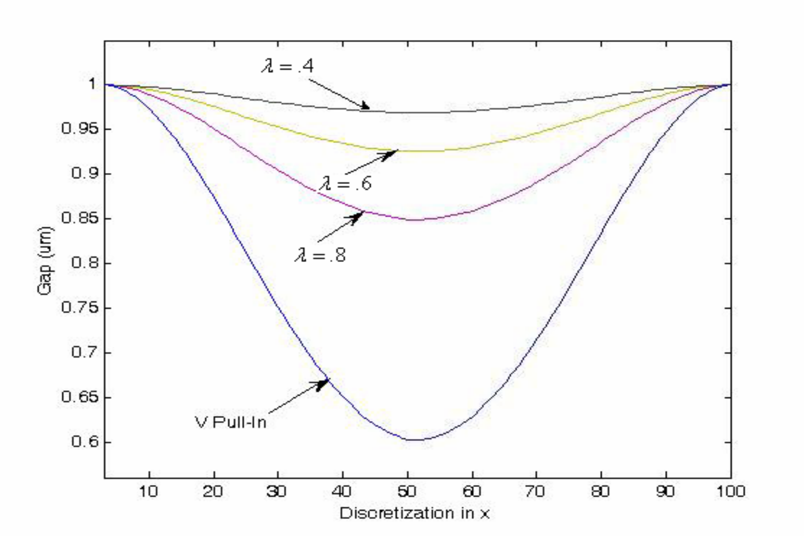

Fig. 5. Modeled deflection of fixed-fixed beam.

5. Conclusion

The Pull-in voltage of MEMS switch including variative electrostatic area and varying section has been modeled using a numerically method. The switch is modeled using Euler-Bernoulli beam theory. The results have showed that when the electrostatic area is difference caused the difference in pull-in voltage. The model and simulation can be used as design tools to improve switch performance in future design.

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