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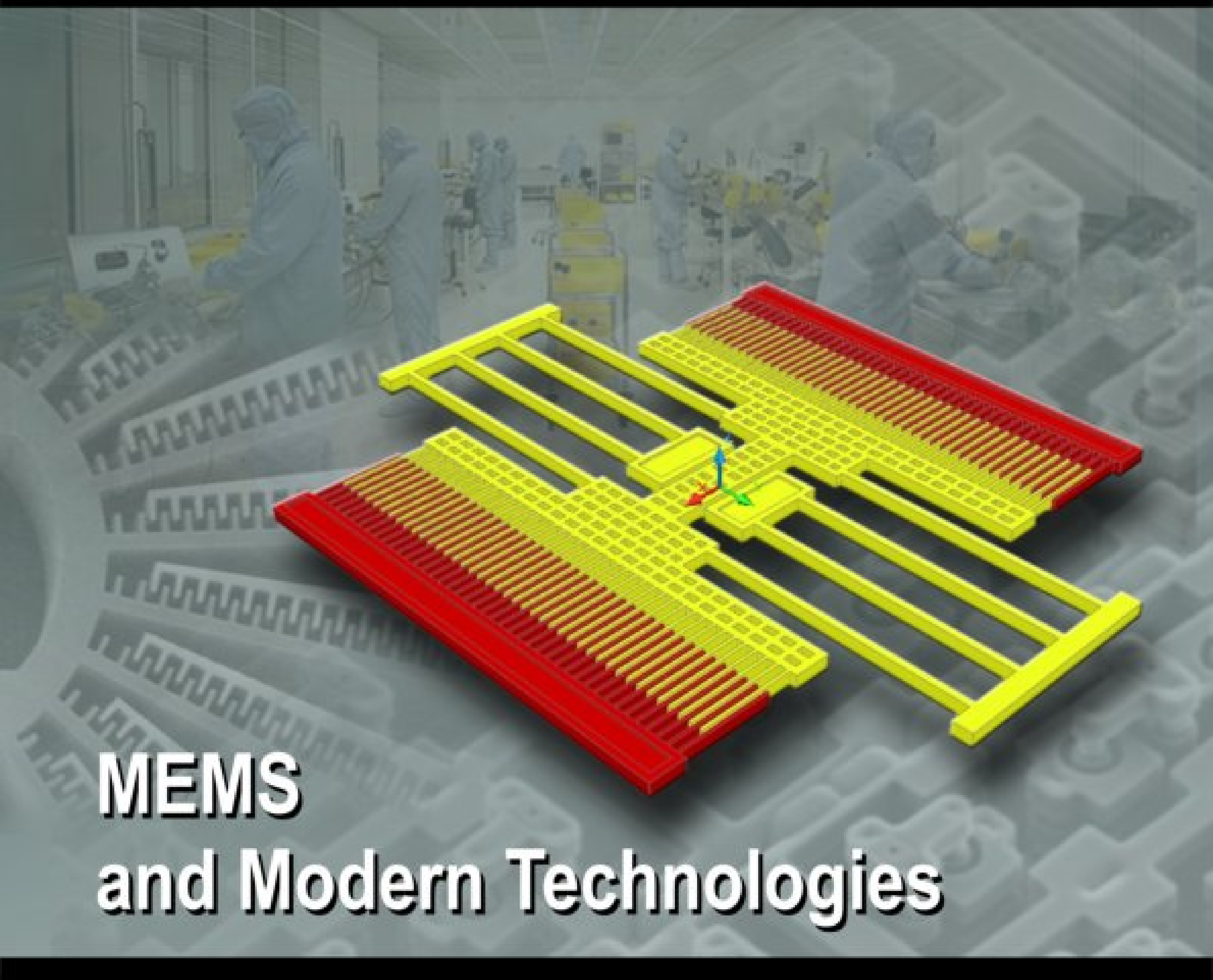

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Thermo-Mechanical Behavior of a Bilayer Microbeam Subjected to Nonlinear Electrostatic Pressure

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Abstract: In the present work mechanical behavior of a bilayer cantilever microbeam subjected to nonlinear electrostatic pressure and temperature variations has been studied. Nonlinear integro-differential thermo-electro mechanical equation based on Euler-Bernoulli beam theory has been derived and solved using Step-by-Step Linearization Method and finite difference method. The effect of temperature variations on static instability of the microbeam due to nonlinearity created by applied electrostatic forces has been investigated. In our model it has been shown that as the temperature increases, the applied voltage decreases, which in turn guides the system to an unstable condition, in other words, decreases the pull-in voltage. On the other hand, the pull-in voltage increases as the temperature decreases. Numerically obtained results for pull-in voltage and beam deflection are in good agreement with existing results in the literature. *Copyright © 2009 IFSA.*

Keywords: MEMS, Thermal, Electrical, Pull-in voltage, Static instability, Bilayer cantilever microbeam

1. Introduction

There are a number of actuation methods that can be used to excite Micro Electro Mechanical Systems (MEMS), such as piezoelectric, electromagnetic, thermal, and electrostatic mechanisms.

Study of electrostatically actuated MEMS is a branch of micromechanics whose principle is a very commonly used one on sensing or actuating devices in MEMS. The foregoing systems have found wide applications in switches [1], micro-mirrors [2], micro-resonators [3], micro-actuators [4], micro thermo-meters [5], and tunable capacitors [6].

Boundary supported suspended microstructures, such as MEMS cantilevers, are currently used in various microengineering sensor/actuator fields. Their relatively simple geometries make them very advantageous both from a design and microfabrication point of view. A cantilever is a beam carrying loads to a strong mounting point with one end of the beam anchored, and having the other end suspended in the air. The beam forms a lever, which carries the load by being held in position by the mount, turning the loads into torque on the mount.

Due to acting of applied force, the axis of the beam deflects from its initial position. Accurate values of beam deflection are sought in many practical cases. To evaluate the value of the deflection of beams, the basic differential equation should be developed and solved. In some cases the loads applied to the beam cause the deflection of the beam to be linear. Therefore linear equation should be derived and solved by common analytical methods. But in some cases due to complexity of the nature of electrostatic forces applied on the beam, the governing equation is nonlinear and should be solved by numerical methods.

In the microscopic scale, high energy densities and large forces are available, and the electrostatic actuation may be more effective than other kinds of actuation methods. An electrostatically actuated microbeam is an elastic beam suspended above a stationary rigid plate, both made of conductive materials, and a dielectric medium filling the gap between them. An applied electric voltage between the two electrodes results in the deflection of the elastic beam and a consequent change in the MEMS capacitance. In case if the applied voltage exceeds a certain limit namely the pull-in voltage, the nonlinear electrostatic force gives rise to the well-known pull-in phenomenon that causes beam to collapse towards the stationary rigid plate. In fact as a result of the nonlinearity, stable force balance and deflection can only be achieved to one third (in lamped modeling) of the initial gap distance. Beyond this value, the stability of the displacement is lost, and the gap is sharply reduces to zero value with no further increase in the applied voltage. Determination of the pull-in voltage plays an important role in the design of MEMS devices.

Several studies have investigated this behavior of microbeams under various loading conditions. The earliest study may be found in the pioneering work of Nathanson et al. [7, 8]. In their study of a resonant gate transistor, they constructed and analyzed the mass-spring model of electrostatic actuation. Recently, some authors have used the nonlinear equation representing the idealized electrostatic structure to analyze the mechanical behavior of the MEMS devices [9, 10]. Other studies analyze the response of a microbeam to a generalized transverse excitation and with axial force using Rayleigh's energy method in order to approximate the fundamental natural frequency of the straight, undeflected beam.

A temperature change is one of the basic actuation parameter that can tune the system directly. Thermal actuation is known for its capability in producing a large and linear displacement with respect to a heating power. This mechanism usually is derived using the known bilayers (multilayers) including different thermal expansion coefficients. In microcapacitive thermal sensors and thermal tunable capacitors, the used bilayer microbeam is not only subjected to the electrostatic force but also to thermal bending moment. In addition, the all of micro actuators and micro sensors exhibit to temperature changes.

By applying simultaneously effect of electrostatic force and thermal moment, the pull-in parameters can change. But there are no enough studies in the effect of temperature changes on the stability of electrostatically actuated microstructures. In the present work, a bilayer microbeam, which is loaded by nonlinear electrostatic force and thermal bending moment, is studied. The static behavior of the microbeam and its static instability (pull-in phenomenon) are investigated. Numerical obtained results for pull-in voltage and cantilever deflection are in good agreement with results existing in the literature.

2. Design and Modeling

Fig. 1 shows a schematic of the proposed model. It consists of a deformable bilayer cantilever microbeam separated from a fixed ground plate as a substrate by an air gap g_0.

A dc voltage source, *V*, between the capacitor electrodes is used to create electrostatic force along the microbeam.

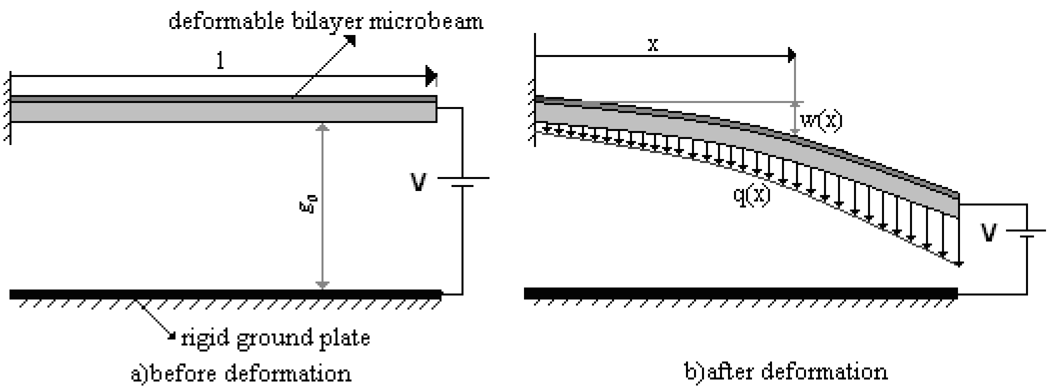

Fig. 1. A Schematic view of a bilayer cantilever microbeam subjected to electrostatic force and thermal moment: a) before deformation; b) after deformation.

In this work the following reasons can cause the deflection of the microbeam:

- Temperature changes (Different expansion coefficients of two selected materials cause the deflection of the microbeam as a result of temperature changes)
- Electrostatic force resulted from applied voltage

The temperature changes and externally electrostatic force applied to the movable microbeam cause the microbeam to move downward which in turn leads the air gap between the movable microbeam and the fixed ground plane to decrease, therefore increase the capacitance between them.

3. Mathematical Modeling

Fig. 2 shows an element of bilayer cantilever microbeam. Assume a microbeam with length *l*, thickness *t*, width *b*, expansion coefficient α, cross sectional area A, and isotropic with Young's modulus *E*, whereas for upper microbeam subscript 1 and for the lower one 2 is used. $w(x)$ is the deflection of the microbeam. Let that *x* be the coordinate along the microbeam with its origin at the left end, and *z* be the coordinate along the cross section with its origin at the mid plane of the microbeam or neutral axis of the cross section when there are no temperature changes.

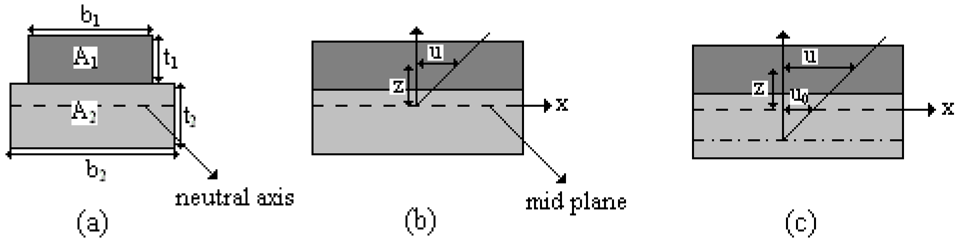

Fig. 2. An element of the bilayer cantilever microbeam: a) section view; b) side view when there are no temperature changes; c) side view in the presence of temperature changes.

As a result of temperature variations in the system, the mid plane is stretched while u_0 is the displacement or extension of the mid plane in *x* direction. *u* is the total extension along *x* axis of a layer located at a given distance *z* due to temperature changes and electrostatic force.

For selected microbeam, t/l is usually small enough to neglect the shear deformation and strain definition is used. Total strain at *x* direction at a given cross section of the microbeam is the sum of thermal strain (ε_T) and mechanical strain (ε_m) based on Euler-Bernoulli beam theory. Thus it can be written as [11]:

$$\varepsilon_{x\,total} = \varepsilon_T + \varepsilon_m = \frac{du}{dx} = \frac{du_0}{dx} - z\frac{d^2w}{dx^2},\ \varepsilon_T = \alpha\theta \quad ;\theta = T - T_0\ , \tag{1}$$

where θ is the temperature change, measured with respect to initial temperature T_0. Using Hook's law and equation 1, the relationship between the stress, strain and temperature changes for Euler-Bernoulli beam in a cross section area of upper and lower microbeam can be expressed as below:

$$\begin{aligned} \sigma_x^{(1)} &= E_1\frac{du_0}{dx} - E_1 z\frac{d^2w}{dx^2} - E_1\alpha_1\theta \\ \sigma_x^{(2)} &= E_2\frac{du_0}{dx} - E_2 z\frac{d^2w}{dx^2} - E_2\alpha_2\theta \end{aligned} \tag{2}$$

Considering that the axial force with respect to free end along the *x* axis is zero, the equation of bilayer microbeam deflection can be expressed as [11]:

$$-(EI)_{eq}\frac{d^2w}{dx^2} = M(x) + M^T, \tag{3}$$

where

$$(EI)_{eq} = \left(E_1\int_{A_1} z^2 dA + E_2\int_{A_2} z^2 dA\right), M^T = \left(E_1\alpha_1\int_{A_1} z dA + E_2\alpha_2\int_{A_2} z dA\right)\theta = \Omega\theta \tag{4}$$

M^T is the thermal moment and $M(x)$ is the external bending moment in a given section resulted from electrostatic pressure acting along the microbeam. With respect to Fig. 3:

$$M(x) = \int_x^l q(\xi)(\xi - x)d\xi\ , \tag{5}$$

where $q(\xi)$ is the electrostatic force per unit length of the microbeam and is defined as below [1]:

$$q(\xi) = \frac{\varepsilon_0 b_2 V^2}{2\left(g_0 - w(\xi)\right)^2}, \quad (6)$$

where ε_0, V and g_0 are the permittivity of air, applied voltage and initial gap between the microbeam and ground plate, respectively.

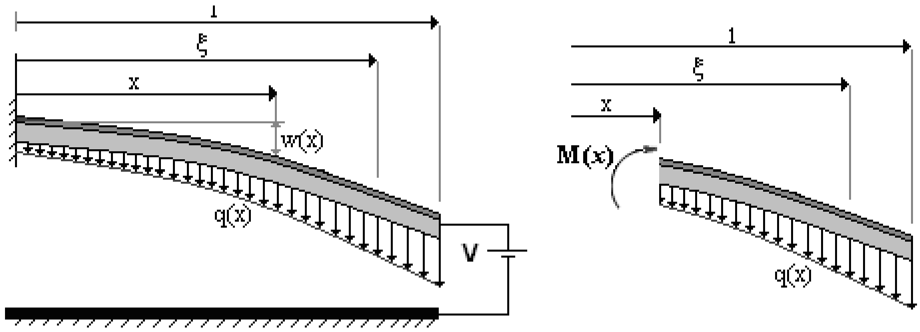

Fig. 3. Moment and electrostatic pressure in the model.

4. Numerical Solution

Solution of nonlinear integro-differential equation 3, due to its nonlinear nature, is more complex and time consuming. In order to overcome the complexity, it is common to linearize it. Since the deflection of the microbeam (*w*) is high enough as compared to initial gap, the linearization of equation 3 with respect to *w* of initial position causes considerable errors in results. Due to step by step applying voltage and temperature a method namely Step-by-Step Linearization Method (SSLM) is proposed for linearization [4], while linearizes the nonlinear equation in each step with respect to the previous one. At last a linear equation in terms of the difference of the deflection of the microbeam between two successive steps ($\psi(x)$) is derived and solved by Finite Difference Method (FDM).

Selection of the small step sizes of applied voltage and temperature causes $\psi(x)$ is small, so the generated errors by this method are surly neglected.

To perceive abovementioned method more, assume T_i and V_i be the temperature and the applied voltage in i^{th} step which cause to the deflection of the microbeam $w_i(x)$.

$$T_{i+1} \to T_i + \delta T \quad \& \quad V_{i+1} = V_i + \delta V \quad \Rightarrow \quad w_{i+1} \to w_i + \delta w = w_i + \psi(x),\ i = 1,2,...,N, \quad (7)$$

where δT and δV are the temperature and the voltage changes between two successive steps, respectively. Rewriting equation 3 for $(i+1)^{th}$ step, the following equation is obtained:

$$-\left(EI\right)_{eq} \frac{d^2(w_i + \psi(x))}{dx^2} - M_{i+1}^T = M_{i+1}(x) \quad (8)$$

whereas:

$$M_{i+1}(x)=\int_x^l \frac{\varepsilon_0 b_2 V_{i+1}^{\ 2}}{2\left(g_0-w_i(\xi)-\psi(\xi)\right)^2}(\xi-x)d\xi\ ,\ M_{i+1}^T=\Omega\theta_{i+1} \tag{9}$$

Based on Calculus of variation theory and using Taylor's series expansion, $M_{i+1}(x)$ can be written as:

$$M_{i+1}(x)=M_i(x)+\left.\frac{\partial M}{\partial w}\right|_{(w_i,V_i)}\delta w+\left.\frac{\partial M}{\partial V}\right|_{(w_i,V_i)}\delta V\ , \tag{10}$$

The higher order of the Taylor's series can be neglected due to small value of δV. Considering that $w_i(x)$ and $w_{i+1}(x)$ satisfy the differential equation of the microbeam deflection, the linear equation for $\psi(x)$ can be expressed as:

$$-\left(EI\right)_{eq}\frac{d^2\psi(x)}{dx^2}-\int_x^l \frac{\varepsilon_0 b_2 V_i^{\ 2}}{\left(g_0-w_i(\xi)\right)^3}\psi(\xi)(\xi-x)d\xi=\int_x^l \frac{\varepsilon_0 b_2 V_i\delta V}{\left(g_0-w_i(\xi)\right)^2}(\xi-x)d\xi+\delta M^T\ , \tag{11}$$

For a cantilever microbeam, the corresponding boundary conditions are zero displacement and zero slope in the fixed end of the microbeam. The unknown function $\psi(x)$ also satisfies the same boundary conditions.

Hence the nonlinear equation 3 is converted to the linear equation 11 in terms of $\psi(x)$, therefore using FDM and imposing the boundary conditions, equation 11 can be solved easily.

5. Numerical Results and Discussion

In order to validate the achieved equation, the pull-in results for single and bilayer cantilever microbeam using SSLM and FDM are now presented.

5.1. Single Layer Microbeam

The physical and geometrical properties of the cantilever single layer microbeam are shown in Table 1.

Table 1. Geometrical and physical properties of the single layer cantilever microbeam.

Design variable	Value [12]
E	$155.8\times10^9\ Nm^{-2}$
b	$5\,mm$
t	$57\ \mu m$
l	$20\,mm$
ε_0	$8.85\ pFm^{-1}$
g_0	$92\ \mu m$

Before getting results to compare, it is necessary to find the best step size for applying voltage and number of grid points for FDM. The convergence of the results is listed in Table 2. As declared in Table 2 the best results can be 0.15 *volt* for step size of applying voltage and 50 grid points for finite-difference method.

Table 2. Obtained pull-in voltages for single layer microbeam with different grid points and step sizes for applying voltages.

Number of grid points	Different steps for bias voltages				
	0.25	**0.22**	**0.20**	**0.17**	**0.15**
35	70.2	70.0	69.9	69.7	69.6
40	70.0	69.9	69.8	69.5	69.4
45	69.9	69.7	69.6	69.4	69.3
50	69.8	69.7	69.5	69.4	69.2
55	69.8	69.6	69.4	69.2	69.2

Table 3 shows the calculated tip gap versus different applied voltages, which are compared with the results of reference [12]. The calculated pull-in voltage for this model using SSLM is 69.2 volt and it agrees very well with the results in [12].

Table 3. Tip gap versus applied voltage.

Applied voltage (volt)	Tip gap in proposed model (μm)	Tip gap (μm) [12]	Experiment (μm) [12]	Δ%
20	90.3	90.2	90.5	0.2
40	84.5	84.3	84.6	0.1
60	70.7	71.5	70.0	0.01
65	63.2	67.2	64.0	0.01

$$\Delta(\%) = \frac{ABS(obtained\ results\ in\ proposed\ \mathrm{mod}\,el - \exp erimental\ results[12])}{\exp erimental\ results[12]} \times 100$$

5.2. Bilayer Microbeam

For proposed bilayer model the geometrical and physical properties, assuming $t_1 = t_2 = t$ and $b_1 = b_2 = b$, are shown in Table 4. To have more sensitivity and large deflection, two materials with high difference in their thermal expansion coefficients are considered (Gold and Silicon). The convergency of the results for a bilayer microbeam is listed in Table 5. Based on results given in the Table 5 the best step size for voltage application is 0.15 V and the best number of the grid points is 50.

The pull-in voltage for cantilever bilayer microbeam is 10.1 *volt*, when there are no temperature changes.

Table 4. Geometrical and physical properties of the proposed bilayer model.

Design variable	Value
(Au) α_1	$14.3\times10^{-6}\ k^{-1}$
(Si) α_2	$2.6\times10^{-6}\ k^{-1}$
E_1	$80\times10^{9}\ Nm^{-2}$
E_2	$122\times10^{9}\ Nm^{-2}$
b	$90\ \mu m$
t	$3\ \mu m$
l	$500\ \mu m$
ε_0	$8.85\ pFm^{-1}$
g_0	$2\ \mu m$

Table 5. Obtained pull-in voltages for proposed bilayer cantilever microbeam with different grid points and step sizes for applying voltages $(\theta = 0)$.

Number of grid points	Different steps for bias voltages				
	0.25	**0.23**	**0.20**	**0.18**	**0.15**
20	10.6	10.6	10.4	10.4	10.4
30	10.5	10.4	10.4	10.3	10.2
40	10.4	10.4	10.3	10.3	10.2
50	10.4	10.4	10.2	10.2	10.1
90	10.3	10.3	10.2	10.1	10.1
120	10.3	10.3	10.2	10.1	10.1

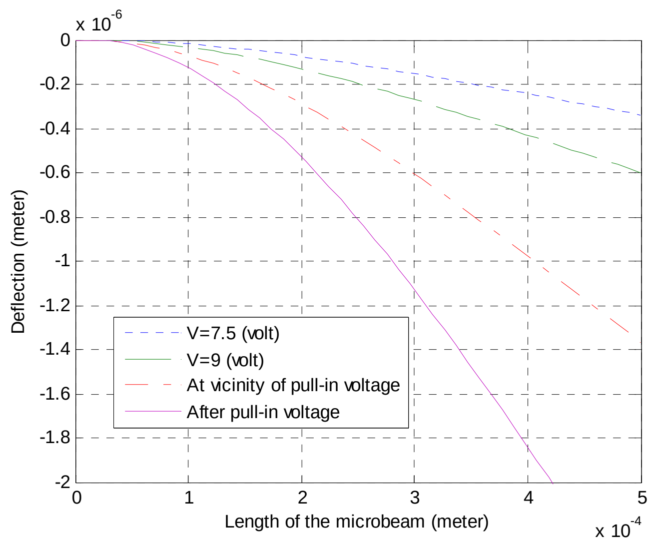

Fig. 4. Deflection of the bilayer cantilever microbeam versus microbeam length for different applied voltages $(\theta = 0)$.

Fig. 4 indicates the deflection of the bilayer cantilever microbeam at vicinity of pull-in voltage, two voltages smaller than the pull-in voltage and either after the pull-in phenomenon happens. As shown in Fig. 4, bending stiffness of the microbeam decreases while its deflection increases, as the voltage increases. As a result of applying δV , the microbeam experiences a greater δw compared to its former state as approaching the pull-in voltage while at the pull-in voltage entirely loses its stiffness and collapses towards the rigid plate, in other words buckling phenomenon occurs.

Here system behavior is investigated applying temperature and voltage simultaneously. It is obvious that the sequence of applying these two actuators in turn is of main importance in determining the microbeam response.

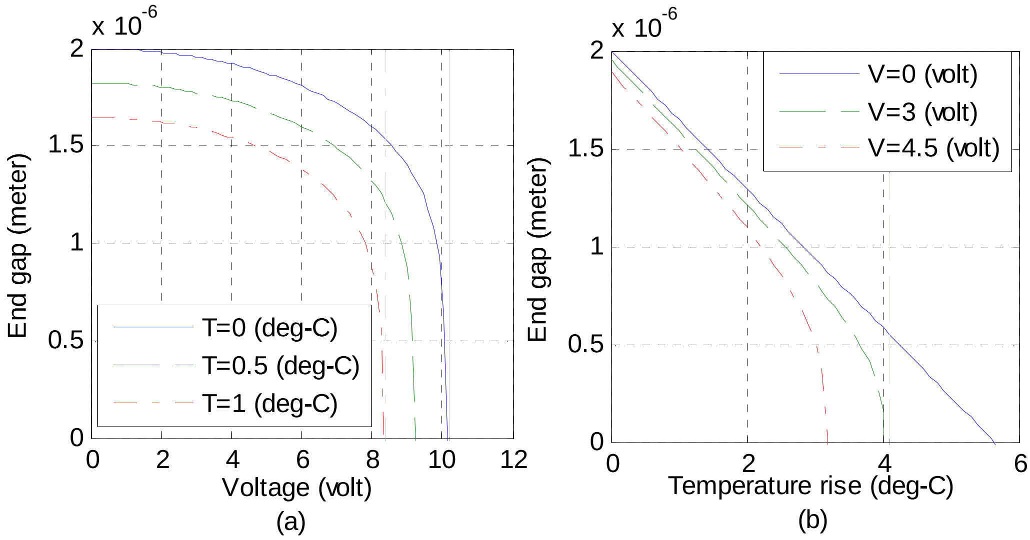

Fig. 5. (a) Tip gap of the microbeam versus voltage;
(b) Tip gap of the microbeam versus temperature changes θ .

Fig. 5(a) shows the tip gap of the microbeam versus applied voltage for different temperature changes. As shown when the microbeam heats up, a pull-in phenomenon happens earlier than when there is no temperature rises. It means that the pull-in voltage will decrease as the temperature rise increases.

Fig. 5(b) shows the tip gap of the microbeam versus the temperature rise when the microbeam deflects due to a given applied voltage and then the temperature rise causes the microbeam to increase deflection up to the pull-in temperature. When there is no applied voltage and microbeam deflects by temperature changes only, there is no nonlinearity and the tip gap varies linearly with the temperature and the system keeps its stability and it solely contacts with the ground plate.

6. Conclusion

In the present work, a bilayer cantilever microbeam subjected to nonlinear electrostatic pressure and temperature changes was studied. Nonlinear integro-differential thermo electro mechanical equation based on Euler-Bernoulli beam theory was derived and linearized using Step-by-Step Linearization Method and solved by applying Finite Difference Method. The static instability of the bilayer microbeam was studied. The results illustrated that due to nonlinear nature and displacement dependency of the electrostatic force, increasing the applied voltage guides the structure to an unstable state. In the lack of the electrostatic forces increasing the temperature variations does not have any influence on the stability, but when the structure is subjected to a bias voltage, due to displacement dependency of the electrostatic force, the temperature changes guides the structure to an instable state.

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Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726-5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually. *Sensors & Transducers Journal* is indexed and abstracted very quickly by Chemical Abstracts, IndexCopernicus Journals Master List, Open J-Gate, Google Scholar, etc.

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Contributions are invited on all aspects of research, development and application of the science and technology of sensors, transducers and sensor instrumentations. Topics include, but are not restricted to:

- Physical, chemical and biosensors;
- Digital, frequency, period, duty-cycle, time interval, PWM, pulse number output sensors and transducers;
- Theory, principles, effects, design, standardization and modeling;
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- Sensor instrumentation;
- Virtual instruments;
- Sensors interfaces, buses and networks;
- Signal processing;
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- Technologies and materials;
- Nanosensors;
- Microsystems;
- Applications.

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