Sensors & Transducers

Node Load Balance Multi-flow Opportunistic Routing in Wireless Mesh Networks

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Abstract: Opportunistic routing (OR) has been proposed to improve the performance of wireless networks by exploiting the multi-user diversity and broadcast nature of the wireless medium. It involves multiple candidate forwarders to relay packets every hop. The existing OR doesn't take account of the traffic load and load balance, therefore some nodes may be overloaded while the others may not, leading to network performance decline. In this paper, we focus on opportunities routing selection with node load balance which is described as a convex optimization problem. To solve the problem, by combining primal-dual and sub-gradient methods, a fully distributed Node load balance Multi-flow Opportunistic Routing algorithm (NMOR) is proposed. With node load balance constraint, NMOR allocates the flow rate iteratively and the rate allocation decides the candidate forwarder selection of opportunities routing. The simulation results show that NMOR algorithm improves 100 %, 62 % of the aggregative throughput than ETX and EAX, respectively. *Copyright © 2014 IFSA Publishing, S. L.*

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1. Introduction

Traditional wireless routing (TR) often follows the design methodology for wired networks by abstracting the wireless links as wired links to look for the shortest delay or least cost path(s) for a user between a pair of source and destination nodes. However for unreliable wireless networks, the forwarding capacity of intermediate nodes, which overhear packet transmissions, can be explored to improve the performance of WMNs. This observation motivates the emergence of a novel technique known as *opportunistic routing*, which allows any node overhearing a packet to participate in forwarding it instead of deterministically choosing the next hop before transmitting a packet. Then opportunistic routing can effectively combine multiple weak links into a strong link and take advantage of transmissions reaching unexpectedly near or unexpectedly far. It exploits the broadcast nature and multi-user diversity of the wireless channel. And recent researches [1-5] have validated that compared with Traditional Routing, OR can evidently increase the reliability of packet transmissions and promote the end to end throughput of multi-hop wireless network, especially Wireless Mesh Networks (WMNs) [6].

The existing ORs use different route metrics to select candidate forwarders, such as the transmission

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number [3, 7, 8, 9], transmission time [10, 11], cost [12] or utilities [13] of all possible paths from source to destination. But these route metrics decide the route and candidate forwarders without taking into account the distribution of flows in multi-flow wireless mesh networks. Although the distributions of flows and traffic load are different, the results of route are still same. Due to the temporal and spatial locality of flow dates, flows may tend to concentrate on a small area stemming from ignoring the space distribution while routing. Some node on some popular paths may be overloaded, while the others may not.

The load imbalance among candidate forwarders would prevent the network from providing good and fair services to its users. The heavy load can exhaust prematurely a candidate forwarder's resources, such as bandwidth, processing power, and memory storage. The overload nodes may be congested and become the bottleneck of end-to-end transmission, consequently resulting in longer end-to-end delay and significant throughput reduction stemming from packet loss and buffer overflow. The use of underutilized paths and nodes can enhance the overall system throughput. In addition, a node can serve for multiple flows concurrently. And how much resources the node should be assigned to different flows in order to obtain higher throughput are still open problems. These two problems can be resolved by introducing a joint candidate forwarder selection and rate allocation opportunistic routing scheme.

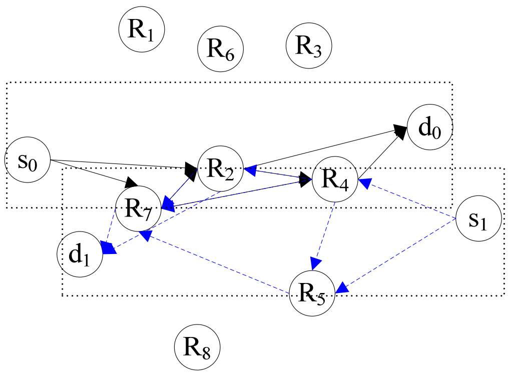
With multiple flows, some works maximize the network throughput or utility via allocating the sending rate, channel resource or gateway [14-19]. But they pre-select route before allocating resource which isn't included the node resource. The route playing an important role in network performance, we need to allocate node and select route in reason with multi-flow. Compared with traditional routing, opportunistic routing has multiple candidate forwarders resulting in the nodes of flows crossing more frequently. According to the distribution of flow, how to allocate node resources in order to maximize the network throughput and keep fairness is an urgent need to be resolved.

In this paper, we address the problem of choosing OR route and allocating rate for multi-flow with load balance of nodes in lossy WMNs. Opportunities routing selection with node load balance is described as a convex optimization problem. In order to solve the problem, by combining primal-dual and sub-gradient methods, a fully distributed joint candidate forwarders selection and rate allocation Node load balance Multi-flow Opportunistic Routing algorithm (NMOR) is proposed. NMOR allocates the flow rate iteratively and the rate allocation decides the candidate forwarder selection of opportunities routing. The simulation results show that NMOR algorithm improves 100 %, 62 % of the aggregative throughput than expected number of transmissions (ETX) [9] and expected any path number of transmissions (EAX [7]), respectively.

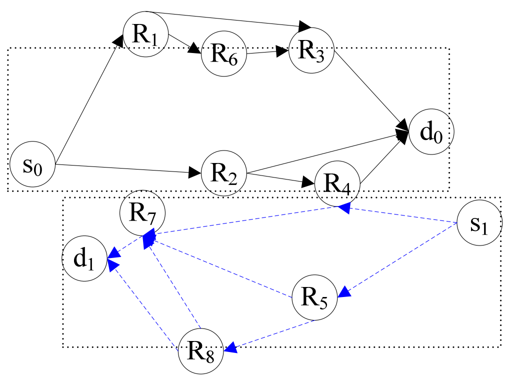
The rest of this paper is organized as follows. The motivation is introduced in Section 2. After analyzing the constraint of the problem, we give the problem formulation in Section 3. Section 4 proposes the distributed algorithm. Details on the simulation to evaluate the proposed algorithm are provided in Section 5. Section 6 summarizes and concludes the paper.

2. Motivation

In this section, an example is used to present the motivation as shown in Fig. 1. A wireless mesh network consists of two concurrent flows from s_0 to d_0 and from s_1 to d_1. s and d present the sources and destinations of flows, R present the other nodes which can be selected as candidate forwarders. The sources, destinations and candidate forwarders of each flow are marked by a dashed rectangle. Because the source and destination of the two flows are nearby, they choose all most same nodes as candidate forwarders by ETX or EAX route metrics. For example, R_2, R_4, R_7 are the candidate forwarder of flow (s_0,d_0) and flow (s_1,d_1); R_5 is the candidate forwarder of (s_1,d_1); R_1, R_3, R_6 and R_8 are candidate forwarders of none flow. Some node may be overloaded while the others may not.



(a) Opportunistic routing selection ignoring the flow distribution.



(b) Opportunistic routing selection with the flow distribution.

Fig. 1. Opportunistic routing selection.

Fig. 1(b) is the opportunistic routing selection result with the flow distribution. The two flows make full use of nodes as candidate forwarders. Additionally, nodes serving for multiple flows (such as R_4) suitably allocate rate for each flow can produce better network throughputs. How to assign candidate forwarders and nodes rates to multiple flows in order to maximize the throughput is our problem.

3. System Model

We use a undirected graph $G(V,E)$ to model the wireless mesh network in which V is the set of n nodes, and E is the set of m links. $p(u,v)$ is packet delivery ratio (PDR) of link $l(u,v)\in E$ according to the propagation model. If u can correctly decode the packets from v at least with the possibility P_0 (P_0 <<1), that is $p(u,v)$ is greater than P_0, node u is a neighbor of node v. The neighbor set of node v is denoted by $R(u)$. In this WMNs, there are K concurrent flows whose sources and destinations are $\{(s_k, d_k), k=1..K\}$. We should allocate reasonable candidate forwarders and forwarders' rate for these flows in order to maximize the total network throughput at the same time keeping the fairness.

In this section, we firstly analyze the constraint of problem. Then the object of network performance is proposed, and we transform the problem in order to solve it conveniently according to the dependence of variables.

3.1. Variables and Constrains on OR

In order to describe the opportunistic routing, firstly several variables are defined. Then we mainly use the model introduced in [17] to formulate the basic properties of opportunistic routing, which has been shown to be sufficiently effective and tractable for network analysis.

We define a binary variable β_u^k denoted whether or not node u is the candidate forwarder of session k, 1 yes and 0 no, defined as follows:

$$\beta_u^k = \begin{cases} 1, \text{ node } u \text{ is the candidate forwarder} \\ \quad \text{of session } k \\ 0, \text{ otherwise} \end{cases} \tag{1}$$

We define a binary variable α_{uv}^k, which has value 1 if the link from u to v is active in the session k in the routing solution, and value 0 otherwise, defined as follows:

$$\alpha_{uv}^k = \begin{cases} 1, \text{ the link from } u \text{ to } v \text{ is active in the session } k \\ 0, \text{ otherwise} \end{cases} \tag{2}$$

Note that only if both node u and v are the candidate forwarder of session k and node v is the neighbor of node u, the link from u to v is active in session k.

$$\alpha_{uv}^k = \beta_u^k * \beta_v^k * BH_{uv}, \forall k \in [1,K], \forall (u,v) \in E\ , \tag{3}$$

where BH_{uv} presents whether or not node v is the neighbor of node u, 1 yes and 0 no.

1) Multipath flow conservation constraint.

For a unicast session k where the source s_k wants to send data with rate λ_k to d_k, by the flow conservation condition, we have:

$$\begin{aligned} &\sum_v \alpha_{uv}^k r_k(u,v) - \sum_w \alpha_{wu}^k r_k(w,u) = h_k(u), \\ &\forall k \in [1,K], \forall u \in V \end{aligned} \ , \tag{4}$$

where

$$h_k(u) = \begin{cases} \lambda_k, \text{if } u = s_k \\ -\lambda_k, \text{if } u = d_k \\ 0, otherwise \end{cases}$$

and $r_k(u,v)$ is the information flow rate of session k from node u to v, which is the average injection rate of innovative packets on link (u,v). For session k, the equation represents the flow conservation constraint that the source node's *net* transmission rate is λ_k, the destination node's net transmission rate is $-\lambda_k$, and any intermediate node's *net* transmission rate is 0.

Moreover, only if the link from u to v is active in session k, the information flow rate of session k on link (u,v) has value non zero and value 0 otherwise. The constraint can be expressed as follow:

$$\alpha_{uv}^k r_k(u,v) = r_k(u,v), \forall k \in [1,K], \forall (u,v) \in E \tag{5}$$

2) MAC broadcasting rate constraint.

We use the broadcast MAC model of Zhang and Li [17], which extends the unicast MAC model to obtain a necessary condition for feasible broadcast schedules. In this model, the transmission range and the interference range are considered to be the same, and the reception probability beyond this range can be ignored. Specifically, the wireless network is modeled as an ideal time-slotted broadcast MAC where competing transmitters can optimally multiplex the channel without any collisions.

Since the transmission range is defined as the distance where the reception probability falls below a small threshold, it is fair to assume that the interference range equals to the transmission range [17]. The neighbor set $R(u)$ is equal to the set of nodes whose transmission range (interference range) node u is located in. Let $B_k^t(u)$ denote a binary decision variable indicating whether node u is transmitting session k's data in slot t. Thus, according to the above definition of collision, a schedule is collision free:

$$\sum_{k\in[1,K]} \beta_u^k B_k^t(u) + \sum_{k\in[1,K]} \sum_{v\in R(u)} \beta_v^k B_k^t(v) \le 1, \forall u \ne s_k \quad (6)$$

This equation indicates that any receiver u allows the broadcast transmission from at most one transmitter within its range at each time slot. Note that for session k the source node s_k is excluded, since s_k does not need to receive information from other nodes for this user. $B_k^t(u)$ indicates whether node u is transmitting session k's data in slot t. If node u isn't the candidate forwarder of session k which means that β_u^k is zero, the value of $B_k^t(u)$ must be zero, because the node u don't send any packet for session k.

Assuming that the schedule length is T, according to (6) we have

$$\frac{W}{T}\sum_{t\in[1,T]}\sum_{k\in[1,K]} \beta_u^k B_k^t(u) + \frac{W}{T}\sum_{t\in[1,T]}\sum_{k\in[1,K]}\sum_{v\in R(u)} \beta_v^k B_k^t(v) \le W,\ \forall u \ne s_k, \quad (7)$$

where W is the MAC layer capacity, which is the maximum broadcast rate of a node when no interferer presents. The *average broadcast rate* of node u for session k can be computed by $b_k(u) = \lim_{T\to\infty} \frac{W}{T} \sum_{t\in[1,T]} B_k^t(u)$. Apply to (7), we must have:

$$\sum_{k\in[1,K]} \beta_u^k b_k(u) + \sum_{k\in[1,K]}\sum_{v\in R(u)} \beta_v^k b_k(v) \le W, \forall u \ne s_k \quad (8)$$

We transformed an integer variable $B_k^t(u)$ into a continuous one $b_k(u)$ by averaging. Moreover, according the relationship between $B_k^t(u)$ and β_u^k, we can obtain the relationship between $b_k(u)$ and β_u^k. Only if the node u is the candidate forwarder of session k, the *broadcast rate* of node u for session k has value non zero and value 0 otherwise. The constraint can be expressed as follow:

$$\beta_u^k b_k(u) = b_k(u), \forall k \in [1,K], \forall u \in V \quad (9)$$

3) Coding constraint.

In our paper opportunistic routing with network code (e.g. MORE[3]) is considered, where the forward rate of node is unaffected by the order of forwarding, only affected by the quality of links. Therefore the information flow rate of session k on link (u,v) must not exceed the corresponding unicast transmission rate, which is the following straightforward network coding model.

$$b_k(u) * p(u,v) \ge r_k(u,v), \forall k \in [1,K], \forall (u,v) \in E, \quad (10)$$

where $p(u,v)$ is the packet delivery ratio of link (u,v). As discussed in [17, 18], although it is not a tight bound, it involves approximations to the behavior of an actual WMN, and includes all the tractable information that users can use to induce a better payoff. Other coding models which make an exact characterization with an exponential number of constraints make the problem intractable.

4) Node load balance constraint.

We define a *load balance area* (i.e. a set of nodes) for each node. Let $v \in A(u)$ present that v is in u's load balance area. That is to say,

$$\begin{aligned} &|b(u) - b(v)| \le \theta(u,v), \theta(u,v) > 0, \forall v \in A(u) \\ &b(u) = \sum_{k\in[1,K]} b_k(u), \forall u \in V \end{aligned}, \quad (11)$$

where $b(u)$ is *Node max load* of u, $\theta(u,v)$ is a parameter set by node u. We introduce $A(u)$ and $\theta(u,v)$ to help u have a controllable balanced load with other nodes. In this paper, we consider a symmetric definition (i.e. $v \in A(u) \Leftrightarrow u \in A(v)$, and $\theta(u,v) = \theta(v,u)$). In practice, the required load balance constraint could be asymmetric.

3.2. Problem Formulation

The solving problem is selecting opportunistic route for the K flows in order to maximize the total network throughput at the same time keeping the fairness. Therefore the objective of this problem can be designed to maximize the product of all flows rate via optimal multi-hop flow routing. It is defined as maximize $\prod_{k\in[1,K]} \lambda_k$ equaled to maximize $\sum_{k\in[1,K]} \ln(\lambda_k)$. With the above constrains, we can formulate the problem as the following system:

$$\begin{aligned} &\text{maximize} \sum_{k\in[1,K]} \ln(\lambda_k) \\ &\text{subject to} \\ &\begin{cases} \alpha_{uv}^k = \beta_u^k * \beta_v^k * BH_{uv}, \forall k \in [1,K], \forall (u,v) \in E \\ \sum_v \alpha_{uv}^k r_k(u,v) - \sum_w \alpha_{wu}^k r_k(w,u) = h_k(u), \forall k \in [1,K], \forall u \in V \\ \alpha_{uv}^k r_k(u,v) = r_k(u,v), \forall k \in [1,K], \forall (u,v) \in E \\ \sum_{k\in[1,K]} \beta_u^k b_k(u) + \sum_{k\in[1,K]}\sum_{v\in R(u)} \beta_v^k b_k(v) \le W, \forall u \ne s_k \\ \beta_u^k b_k(u) = b_k(u), \forall k \in [1,K], \forall u \in V \\ b_k(u) * p(u,v) \ge r_k(u,v), \forall k \in [1,K], \forall (u,v) \in E \\ |\sum_{k\in[1,K]} b_k(u) - \sum_{k\in[1,K]} b_k(v)| \le \theta(u,v), \forall u \ne v, \forall v \in A(u) \\ \text{over: } \alpha_{uv}^k = 0 or 1, \forall k \in [1,K], \forall (u,v) \in E \\ \beta_u^k = 0 or 1, \forall k \in [1,K], \forall u \in V \\ 0 \le r_k(u,v) \le W, \forall k \in [1,K], \forall (u,v) \in E \\ 0 \le b_k(u) \le W, \forall k \in [1,K], \forall u \in V \end{cases} \end{aligned} \quad (12)$$

where the variables are α, β, r, b, and

$$h_k(u) = \begin{cases} \lambda_k, \text{if } u = s_k \\ -\lambda_k, \text{if } u = d_k \\ 0, otherwise \end{cases}.$$

The nonlinear constrains make the problem hard to be solved, therefore we transform it according to the relationship of variables.

The constraints in (4) (5) contain the product of two variables α_{uv}^k and $r_k(u,v)$ which is in non-linear form. According to property in (5), we can rewrite the constraints (4) (5) as follows. If the information flow rate of session *k* on link *(u,v)* has value non zero, the link from *u* to *v* must be active in session *k*.

$$\sum_v r_k(u,v) - \sum_w r_k(w,u) = h_k(u), \forall k \in [1,K], \forall u \in V, \quad (13)$$

$$\alpha_{uv}^k = \begin{cases} 1, & r_k(u,v) \neq 0 \\ 0, & r_k(u,v) = 0 \end{cases}, \forall k \in [1,K], \forall (u,v) \in E, \quad (14)$$

Similarly we can reformulate (8), (9) into linear constraints as follows. If the *broadcast rate* of node *u* for session *k* has value non zero, the node *u* must be the candidate forwarder of session *k*.

$$\sum_{k\in[1,K]} b_k(u) + \sum_{k\in[1,K]} \sum_{v\in R(u)} b_k(v) \le W, \forall u \neq s_k, \quad (15)$$

$$\beta_u^k = \begin{cases} 1, & b_k(u) \neq 0 \\ 0, & \text{otherwise} \end{cases}, \forall k \in [1,K], \quad (16)$$

After converting the nonlinear constrains, the value of α, β are only dependent on *r* and *b* and don't need to be included in our final formulation. Now all of our constraints are linear. Using the above definition of objective function, we represent the problem formulation as follows.

$$\text{maximize} \sum_{k\in[1,K]} \ln(\lambda_k)$$

subject to

$$\begin{cases} b_k(u) * p(u,v) \ge r_k(u,v), \forall k \in [1,K], \forall (u,v) \in E \\ \sum_v r_k(u,v) - \sum_w r_k(w,u) = h_k(u), \forall k \in [1,K], \forall u \in V \\ \sum_{k\in[1,K]} b_k(u) + \sum_{k\in[1,K]} \sum_{v\in R(u)} b_k(v) \le W, \forall u \neq s_k \\ |\sum_{k\in[1,K]} b_k(u) - \sum_{k\in[1,K]} b_k(v)| \le \theta(u,v), \forall u \neq v, \forall v \in A(u) \\ 0 \le r_k(u,v) \le W, \forall k \in [1,K], \forall (u,v) \in E \\ 0 \le b_k(u) \le W, \forall k \in [1,K], \forall u \in V, \end{cases} \quad (17)$$

where r,b are the optimization variables. A node decides whether or not to forward packet for a flow according to the flow rate, unlike [17, 18] which perform a node selection procedure beforehand, then allocate the flow rate.

Since the objective function is strictly convex and the constraints are linear, the problem (17) represents a strictly convex optimization problem. We convert (17) into a standard form of convex optimization problem (18). Though the problem (18) can be readily solved by standard convex programming algorithm, it is desirable to provide a decentralized solution in WMNs. We develop a distributed algorithm to solve the problem (18) in section 5.

$$\text{minimize} - \sum_{k\in[1,K]} \ln(\lambda_k)$$

subject to

$$\begin{cases} r_k(u,v) - b_k(u) * p(u,v) \le 0, \forall k \in [1,K], \forall (u,v) \in E \\ \sum_v r_k(u,v) - \sum_w r_k(w,u) = h_k(u), \forall k \in [1,K], \forall u \in V \\ \sum_{k\in[1,K]} b_k(u) + \sum_{k\in[1,K]} \sum_{v\in R(u)} b_k(v) - W \le 0, \forall u \neq s_k \\ |\sum_{k\in[1,K]} b_k(u) - \sum_{k\in[1,K]} b_k(v)| - \theta(u,v) \le 0, \forall u \neq v, \forall v \in A(u) \\ 0 \le r_k(u,v) \le W, \forall k \in [1,K], \forall (u,v) \in E \\ 0 \le b_k(u) \le W, \forall k \in [1,K], \forall u \in V \end{cases} \quad (18)$$

4. Distributed Algorithm

In this section, we propose a decentralized algorithm for the problem (18) based on decomposition techniques [20]. Specifically, we decompose the original problem into two separate sub-problems with decoupled variables based on the dual decomposition. Then we solve the sub-problems independently, and finally solve the master dual problem by updating dual variables.

4.1. The Dual Decomposition Method Solution

By introducing dual variables $x^{(i)}(u), y_k^{(i)}(u,v)$, $z^{(i)}(u,v)$ to relax the three sets of constraints in (18) respectively, we have the Lagrangian function of the primal problem (18) subject to constraint (13) as (19).

$$\begin{aligned} L(r,b,x,y,z) &= -\sum_{k\in[1,K]} \ln(\lambda_k) \\ &+ \sum_{u\in V} x(u)\Big(\sum_{k\in[1,K],u\neq sk} b_k(u) + \sum_{k\in[1,K]} \sum_{v\in R(u),u\neq s_k} b_k(v) - W\Big) \\ &+ \sum_{k\in[1,K]} \sum_{(u,v)\in E} y_k(u,v)(r_k(u,v) - b_k(u)p(u,v)) \\ &+ \sum_{u,v\in V,\forall v\in A(u)} z(u,v)\Big(\sum_{k\in[1,K]} b_k(u) - \sum_{k\in[1,K]} b_k(v) - \theta(u,v)\Big) \\ &= -\sum_{k\in[1,K]} \ln(\lambda_k) + \sum_{k\in[1,K]} \sum_{(u,v)\in E} y_k(u,v) r_k(u,v) \\ &+ \sum_{k\in[1,K]} \sum_{u\in V,u\neq s_k} x(u) b_k(u) + \sum_{k\in[1,K]} \sum_{u\in V} \sum_{v\in R(u),v\neq s_k} x(v) b_k(u) \\ &+ \sum_{u,v\in V,\forall v\in A(u)} z(u,v)\Big(\sum_{k\in[1,K]} b_k(u) - b_k(v)\Big) \\ &- \sum_{k\in[1,K]} \sum_{(u,v)\in E} y_k(u,v) b_k(u) p(u,v) \\ &- \sum_{u\in V} x(u) W - \theta(u,v) \sum_{u,v\in V,\forall v\in A(u)} z(u,v) \end{aligned} \quad (19)$$

Thus, the minimization operation of *L*(*r,b,x,y,z*) over (*r,b*) can be decomposed as two sub-problems as follows:

SUB1: the information flow rate problem of session *k*

$$\begin{aligned}&\text{minimize } -\ln(\lambda_k)+\sum_{(u,v)\in E} y_k(u,v) r_k(u,v)\\&\text{subject to}\\&\begin{cases}\sum_v r_k(u,v)-\sum_w r_k(w,u)=h_k(u), \forall u\in V\\ 0\le r_k(u,v)\le W, \forall (u,v)\in E\end{cases}\end{aligned} \quad (20)$$

SUB2: the broadcast rate problem of node u

$$\begin{aligned}&\text{minimize } b_k(u)(x(u)_{u\neq s_k}+\sum_{v\in R(u),v\neq s_k} x(v)-\\&\sum_{(u,v)\in E} y_k(u,v)p(u,v)+\sum_{v\in A(u)} z(u,v)-\sum_{v\in A(u)} z(v,u))\\&\text{subject to } \quad 0\le b_k(u)\le W, \forall k\in[1,K]\end{aligned} \quad (21)$$

The Lagrange dual function is the minimum value of the Lagrangian function as following.

$$G(x,y)=\inf_{r,b}\{L(r,b,x,y,z)\} \quad (22)$$

The dual problem of problem (18) is formulated as maximize $G(x,y,z)$. We use the subgradient method to solve the Lagrange dual problem. In the i^{th} iteration, the dual variables are updated as (23) where the subgradients of dual variables are M, H and C.

$$\begin{aligned}&\forall u\in V, x^{(i)}(u)=\max(0,x^{(i-1)}(u)+\eta M_u^{(i-1)})\\&\forall (u,v)\in E, y_k^{(i)}(u,v)=\max(0,y_k^{(i-1)}(u,v)+\eta H_{(k,u,v)}^{(i-1)})\\&\forall u\in V \textit{ and } v\in A(u), z^{(i)}(u,v)=\max(0,z^{(i-1)}(u,v)+\eta C_{(u,v)}^{(i-1)})\\&M_u^{(i-1)}=\sum_{k\in[1,K],u\neq sk} b_k^{(i-1)}(u)+\sum_{k\in[1,K]}\sum_{v\in R(u),u\neq sk} b_k^{(i-1)}(v)-W\\&H_{(k,u,v)}^{(i-1)}=r_k^{(i-1)}(u,v)-b_k^{(i-1)}(u)p(u,v)\\&C_{(u,v)}^{(i-1)}=\sum_{k\in[1,K]} b_k^{(i-1)}(u)-\sum_{k\in[1,K]} b_k^{(i-1)}(v)-\theta(u,v)\end{aligned} \quad (23)$$

We solve the two sub-problems.

1) The information flow rate problem of session k (SUB1).

The objective function is strictly convex. According to the flow-path formulation technique in [17], we consider an equivalent problem of **SUB1**:

$$\begin{aligned}&\text{minimize} -\sum_{\pi\in P}\ln(\gamma_k(\pi))+\sum_{\pi\in P}\varphi_k(\pi)\gamma_k(\pi)\\&\text{subject to } 0\le \sum_{\pi\in P}\gamma_k(\pi)\le W\end{aligned}, \quad (24)$$

where P is the set of single paths of session k from s_k to d_k, $\gamma_k(\pi)$ is the flow rate of session k on path π, and $\varphi_k(\pi)=\sum_{(u,v)\in\pi} y_k(u,v)$. Obviously, solving this equivalent problem will result in that the min-cost path (with respect to $\sum_{(u,v)\in\pi} y_k(u,v)$) is always chose, i.e. a single path flow. We use Γ_k to denote the value of this single path flow. Thus this problem above is transformed to (25).

$$\begin{aligned}&\text{minimize} -\ln(\Gamma_k)+\varphi_k^{\min}\Gamma_k\\&\text{subject to } 0\le\Gamma_k\le W\end{aligned}, \quad (25)$$

where $\varphi_k^{\min}=\min_\pi \sum_{(u,v)\in\pi} y_k(u,v)$ is the cost of the min-cost path. Since the objective function is an increasing, strictly convex and continuously differentiable function, (25) can be easily solved and $\Gamma_k=1/\varphi_k^{\min}$ is the solution. $r_k^{(i)}(u,v)$ is set to the solution to (24), if link (u,v) is on this min-cost path. Otherwise the result of $r_k^{(i)}(u,v)$ in (24) is zero. Therefore, each session k can use a distributed shortest path algorithm to compute its $r_k^{(i)}$.

2)The broadcast rate problem of node u (**SUB2**).

Since the objective of the problem **SUB2** (21) is linear, the Lagrange multiplier method may not necessarily generate the optimal solution. To handle this difficulty, we adopt the proximal method and add a small regularization term $\varepsilon\|b_k^{(i)}(u)-b_k^{(i-1)}(u)\|^2$ to make it strictly convex. ε is positive constant scalar. When ε is sufficiently small, the term $\varepsilon\|b_k^{(i)}(u)-b_k^{(i-1)}(u)\|^2$ is close to 0, therefore the solution for problem (26) is arbitrary close to the solution for problem **SUB2** (21).

$$\begin{aligned}&\text{minimize} b_k^{(i)}(u)(x^{(i)}(u)_{u\neq sk}+\sum_{v\in R(u),v\neq sk} x^{(i)}(v)\\&-\sum_{(u,v)\in E} y_k^{(i)}(u,v)p(u,v)+\sum_{v\in A(u)} z^{(i)}(u,v)-\sum_{v\in A(u)} z^{(i)}(v,u))\\&+\varepsilon\|b_k^{(i)}(u)-b_k^{(i-1)}(u)\|^2\\&\text{subject to } 0\le b_k^{(i)}(u)\le W, \forall k\in[1,K]\end{aligned} \quad (26)$$

In the i^{th} iteration interval, $b_k^{(i)}$ can be computed as follows:

$$\begin{aligned}&b_k^{(i)}(u)=b_k^{(i-1)}(u)+\frac{1}{2\varepsilon}(\sum_{(u,v)\in E} y_k^{(i)}(u,v)p(u,v)-x^{(i)}(u)_{u\neq sk}\\&-\sum_{v\in R(u),v\neq sk} x^{(i)}(v)+\sum_{v\in A(u)} z^{(i)}(u,v)-\sum_{v\in A(u)} z^{(i)}(v,u))\end{aligned} \quad (27)$$

4.2. Distributed Algorithm of NMOR

In each iteration, the link is or isn't active in a session, then the flow rate $r_k^{(i)}(u,v)$ is zero or not which can't present the flow rate situation in a period. We take an equally-weighted average of the flow rate in each iteration i. Similarly, we take an equally-weighted average of the broadcast rate to present the broadcast rate in a period. The average flow rate and broadcast rate till the i_{th} iteration interval can be computed as follows:

$$\tilde{r}_k^{(i)}(u)=\frac{1}{i}\sum_{m=0}^{i} r_k^{(m)}(u), i\ge 1 \quad (28)$$

$$\tilde{b}_k^{(i)}(u) = \frac{1}{i}\sum_{m=0}^{i} b_k^{(m)}(u), i \geq 1 \tag{29}$$

The distributed algorithm of Node load balance Multi-flow Opportunistic Routing algorithm (NMOR) is shown as Table 1. Firstly in initialization which is the zero iteration, the algorithm sets $r_k^{(i)}(u,v), b^{(i)}(u)$, $x^{(i)}(u), y_k^{(i)}(u,v), z^{(i)}(u,v)$ to non-negative values randomly for all $\forall u \in V, \forall (u,v) \in E, \forall k \in K$. Secondly in the i_{th} iteration, every node locally updates its dual variables $x^{(i)}, y^{(i)}, z^{(i)}$ by (30) according to its own and neighbors' broadcast rate and flow rate of its links in the *(i-1)*th iteration. Thirdly every node locally solves the sub-problem **SUB1** and **SUB2** according to its dual variables $x^{(i)}, y^{(i)}, z^{(i)}$ in the i_{th} iteration. For sub-problem **SUB1**, every node solves the equal problem (31) and obtains the flow rate of its links. For sub-problem **SUB2**, every node obtains the broadcast rate by (32). Then the algorithm goes to the $(i+1)_{th}$ iteration.

Table 1. The Distributed Algorithm of Node load balance Multi-flow Opportunistic Routing (NMOR).

Input: Network topology G(*V,E*), Flows *K*, Sources and Destinations $\{(s_k, d_k), k=1..K\}$, Link reception p, Link Capacity W Output: flow rate $r_k^{(i)}(u,v)$ and broadcast rate $b_k^{(i)}(u)$
1. Initialization: set i=0; set $r_k^{(i)}(u,v), b^{(i)}(u)$, $x^{(i)}(u), y_k^{(i)}(u,v), z^{(i)}(u,v)$ to non-negative values randomly for all $\forall u \in V, \forall (u,v) \in E, \forall k \in K$; 2. Every node u updates the dual variables $x^{(i)}, y^{(i)}, z^{(i)}$ by (30). Then it broadcasts $x^{(i)}(u), y_k^{(i)}(u,v), z^{(i)}(u,v)$ to its direct neighbors. $$\begin{aligned} & x^{(i)}(u) = \max(0, x^{(i-1)}(u) + \eta M_u^{(i-1)}) \\ & y_k^{(i)}(u,v) = \max(0, y_k^{(i-1)}(u,v) + \eta H_{(k,u,v)}^{(i-1)}) \\ & z^{(i)}(u,v) = \max(0, z^{(i-1)}(u,v) + \eta C_{(u,v)}^{(i-1)}) \\ & M_u^{(i-1)} = \sum_{k\in[1,K],u\neq sk} b_k^{(i-1)}(u) + \sum_{k\in[1,K]}\sum_{v\in R(u),u\neq sk} b_k^{(i-1)}(v) - W \\ & H_{(k,u,v)}^{(i-1)} = r_k^{(i-1)}(u,v) - b_k^{(i-1)}(u)p(u,v) \\ & C_{(u,v)}^{(i-1)} = \sum_{k\in[1,K]} b_k^{(i-1)}(u) - \sum_{k\in[1,K]} b_k^{(i-1)}(v) - \theta(u,v) \end{aligned} \tag{30}$$ Where $x^{(i-1)}(u)$, $y_k^{(i-1)}(u,v)$ and $z^{(i-1)}(u,v)$ are the dual variables in the *(i-1)*th iteration, η is the step, $b_k^{(i-1)}(u)$ is the broadcast rate in the *(i-1)*th iteration, $r_k^{(i-1)}(u,v)$ the flow rate of link *(u,v)* in session *k* in the *(i-1)*th iteration. 3. Every node u solves the sub-problem **SUB1** and **SUB2** and gets the flow rate $r^{(i)}, b^{(i)}$, Then it broadcasts $r_k^{(i)}(u,v), b^{(i)}(u)$ to its direct neighbors. 3.1 Solving the sub-problem **SUB1** Solve the following problem $$\begin{aligned} & \text{minimize} - \ln(\Gamma_k^{(i)}) + \varphi_k^{\min,i}\Gamma_k^{(i)} \\ & \text{subject to } 0 \leq \Gamma_k^{(i)} \leq W \end{aligned} \tag{31}$$ where $\Gamma_k^{(i)}$ is the flow rate of session k in the i^{th} iteration, $\varphi_k^{\min,i} = \min_{\pi} \sum_{(u,v)\in\pi} y_k^{(i)}(u,v), \pi \in P$, P is a set of all single paths of session k from s_k to d_k, $y_k^{(i)}(u,v)$ is the dual variable (link cost) in the i^{th} iteration, $\varphi_k^{\min,i}$ is the cost of the min-cost path and π is the min-cost path of session k in the i^{th} iteration. If link $(u,v) \in \arg\min_{\pi} \sum_{(u,v)\in\pi} y_k^{(i)}(u,v)$, then $r_k^{(i)}(u,v) = \Gamma_k^{(i)}$; Otherwise $r_k^{(i)}(u,v)$ =0. 3.2 Solving the sub-problem **SUB2** $$\begin{aligned} & b_k^{(i)}(u) = b_k^{(i-1)}(u) + \frac{1}{2\varepsilon}\Big(\sum_{(u,v)\in E} y_k^{(i)}(u,v)p(u,v) - x^{(i)}(u)_{u\neq sk} \\ & - \sum_{v\in R(u),v\neq sk} x^{(i)}(v) + \sum_{v\in A(u)} z^{(i)}(u,v) - \sum_{v\in A(u)} z^{(i)}(v,u)\Big) \end{aligned} \tag{32}$$ 4. set $i \leftarrow i$+1 and go to step 2 (until satisfying termination criterion)

Every node locally keeps its broadcast rate $b(u)$, the flow rate of links inputting or outputting from it $r(u,v)$, the dual variable itself $x(u)$ and dual variable of links inputting or outputting from it $y(u,v)$ and $z(u,v)$. Every node can carry out the algorithm locally and no global iteration information is needed.

1) In the step 2, for $x^{(i)}(u)$ and $z^{(i)}(u,v)$, node u needs its own and its one-hop neighbors' broadcast rates. Its own broadcast rate is kept locally. Its one-hop neighbors' broadcast rates can be obtained by exchange information with its one-hop neighbors. For $y_k^{(i)}(u,v)$, node u needs $r_k^{(i)}(u,v)$ and $b_k^{(i)}(u)$ which are kept locally.

2) In the step 3, for $r_k^{(i)}(u,v)$ node solves the problem (31) via a distributed shortest path algorithm. For $b_k^{(i)}(u)$, node u needs the dual variable x of the neighbor nodes which are obtained by exchange information with its one-hop neighbors.

3) The initialization ($r^{(0)}, b^{(0)}, x^{(0)}, y^{(0)}, z^{(0)}$) can be negotiated in a distributed manner.

After the algorithm converges, we can obtain the values of α, β by (14) and (16). Nodes decide whether or not to forward packet for a flow according to the allocated flow rate, instead of nodes allocate flow rate of candidate forwarders after selecting the candidate forwarders. NMOR allocates the flow rate iteratively and rate allocation decides the candidate forwarder selection of opportunities routing.

After the above dual decomposition is finished, the result can be proved by using standard techniques with the distributed gradient algorithm's convergence analysis.

Theorem 1.By Algorithm NMOR, dual variables $x^{(i)}(u) \in R^N, y_k^{(i)}(u,v) \in R^{K\times N\times N}, z^{(i)}(u,v) \in R^{N\times N}$ converge to the optimal dual solution x^*, y^*, z^* and the corresponding primal variables r^*, b^* are the globally optimal solutions of (18).

Proof. Since strong duality holds for problem (18), its two sub-problems (20) (21) and its Lagrange dual problem (22), we solve the dual problem through the distributed gradient method and recover the prime optimizers from the dual optimizers by Danskin's Theorem.

$$\begin{aligned}
&\frac{\partial(L(r^{(i)}, b^{(i)}, x^{(i)}, y^{(i)}, z^{(i)}))}{\partial x^{(i)}(u)} = \sum_{k\in[1,K], u\neq src(k)} b_k^{(i)}(u) + \\
&\sum_{k\in[1,K]} \sum_{v\in R(u), u\neq sk} b_k^{(i)}(v) - W, \forall u \in V \\
&\frac{\partial(L(r^{(i)}, b^{(i)}, x^{(i)}, y^{(i)}, z^{(i)}))}{\partial y_k^{(i)}(u,v)} = r_k^{(i)}(u,v) - b_k^{(i)}(u) p(u,v), \\
&\forall k \in [1,K], \forall (u,v) \in E \\
&\frac{\partial(L(r^{(i)}, b^{(i)}, x^{(i)}, y^{(i)}, z^{(i)}))}{\partial z^{(i)}(u,v)} = \sum_{k\in[1,K]} b_k^{(i)}(u) - \sum_{k\in[1,K]} b_k^{(i)}(v) \\
&-\theta(u,v), \forall u \in V \text{ and } v \in A(u)
\end{aligned} \tag{33}$$

Hence, the algorithm is a gradient projection algorithm for the dual problem (22). Since the dual objective function is a convex function, there exists a step size η that guarantees $x^{(i)}(u) \in R^N$, $y_k^{(i)}(u,v) \in R^{K\times N\times N}$, $z^{(i)}(u,v) \in R^{N\times N}$ to converge to the optimal dual solution x^*, y^*, z^*. Furthermore, since problem (18) and its three sub-problems are convex optimization problems and (22) has unique solutions, r^*, b^* is the globally optimal primal solution of (18).

5. Simulations

In this section, we present some simulation results to demonstrate the properties of the proposed algorithm. The results validate that our algorithm increases the throughput and keeps load balance effectively, yet in a distributed fashion.

5.1. Methodology

We randomly distribute 16 wireless nodes in a terrain area of 300 meters × 300 meters. P_0 is equal to 0.1, that is only if the PDR of two nodes must be greater than 0.1, there is a link between them. In the optimization, ε is set to 0.05. The step-size at the i_{th} iteration is $\eta^i = 0.3/\sqrt{i}$. The parameters $\theta(u,v)$ are set to 1 Mbps. We have pre-defined the convergence threshold of 10^{-4}. The shadowing model is used in which frame losses are proportional to the distance between wireless nodes. Note that this model assumes that losses between the source and different forwarders are independent. Therefore, intra-path and inter-path collisions occur in a random manner. We list the parameters used to obtain numerical results in Table 2.

Table 2. Parameters.

Parameters	Values
Transmission power Pt	0.28183815 W
TX Antenna gain Gt	1.0
RX Antenna gain Gr	1.0
Frequency freq	2.4e9
System loss sysLoss	1.0
Pathloss Model	shadowing model
Path loss exponent pathlossExp_	2.0
Shadowing deviation std_db_	4.0
Distances dist0_	1.0
Receive Threshold rxThresh_	2.44547e-08
Channel Bit Rate	11 M
Packet Length	1000 bytes

We evaluate the performance of NMOR, comparing with ETX [9] and EAX [7] route metrics by randomly generated wireless networks. We introduce the ETX and EAX metrics. The node chooses its neighbors as candidate forwarders, if the neighbor's ETX or EAX to destination is smaller than itself. ETX is calculated as follow:

$$ETX(s,d) = 1/p(s,c) + ETX(c,d), \tag{34}$$

where s and d are the source and destination, c is the next hop of s. $p(u,v)$ is the packet delivery ratio(PDR) between u and v. ETX(c,d) is the ETX values from c to d.

EAX metric calculates the number of transmissions considering all possible candidate forwarders instead of only one next hop. Thus, the EAX from a source s to a destination d is defined as:

$$EAX(s,d) = \frac{1+\sum_i EAX(i,d) p_i \prod_{j=1}^{i-1}(1-p_j)}{1-\prod_{i\in J}(1-p_i)}, \quad (35)$$

where s and d are the source and destination, J is the candidate forwarder set of s, i represents the candidate forwarder. The larger index of i is, the low priority it is. In addition, p_i is the PDR between s and i. $1/(1-\prod_{i\in J}(1-p_i))$ is the cost from s to candidate forwarder set J, that is, at least one candidate forwarder successfully receive the packet from s. EAX(i,d) is the EAX values from i to d. $\frac{p_i \prod_{j=1}^{i-1}(1-p_j)}{(1-\prod_{i\in J}(1-p_i))}$ is the probability of i to forward packet for s. Expected Summing all EAX of i is the cost from all candidate forwarders to d.

The metrics used to evaluate different route metrics are explained as follows. Total throughput: the sum of all flow's throughputs. Total Delay: the aggregative delays of all flows. Flow balance metric: defined in 3.2 as the log of product of all flow's throughputs. The more balance flows are, the bigger the value of flow balance metric is. The minimum flow throughput: the minimal throughput of all flows. The bigger the minimum flow throughput is, the more balance flows are too. We examine NMOR, ETX and EAX sequentially to obtain flow balance metric and total throughput between the same source-destination pair.

Various factors affect the performance. We perform two set of simulations to analyze the effect of the number of flow and the parameters θ's value. As follow, we show the simulation results respectively.

5.2. The Effect of the Number of Flows

We present the total throughput, total delay, flow balance metric and the minimal throughput with different number of concurrent flows. We randomly increase the number of concurrent flows from two to eight. Each number of flows experiment runs 10 times in four randomly network topologies. Fig. 2 present the total throughput of NMOR, ETX and EAT with different the number of concurrent flows. The result shows that NMOR increases the total throughput by 100 % and 62 %, compared with ETX and EAX. Because the ETX and EAX only select limited nodes as candidate forwarder, it's difficult to satisfy the node load balance constraint.

Fig. 3 shows that the average total delays with different number of flows. The average delay of NMOR decreases by 48 % and 40 % compared with ETX and EAX respectively.

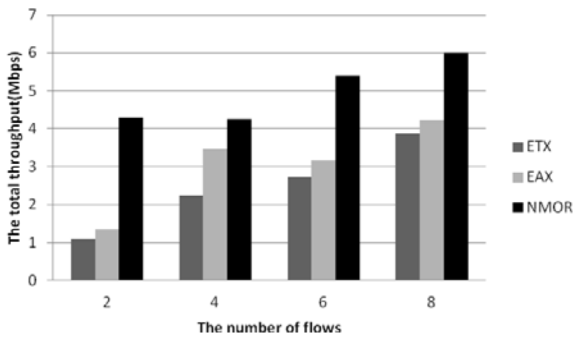


Fig. 2. The total throughput with flows.

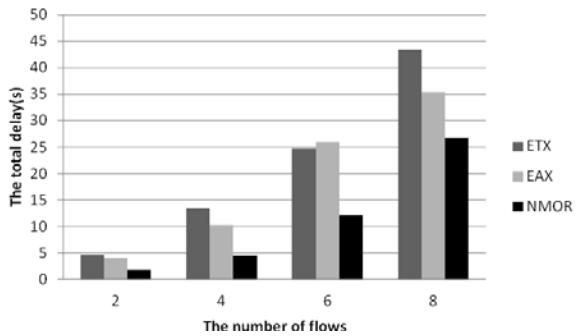


Fig. 3. The total delay with flows.

In Fig. 4, the flow balance metric increase as the increase of the number of concurrent flows and the values of NMOR are always higher than that of ETX and EAX. At average, the flow balance metric of NMOR achieves 76 %, and 47% higher than that ETX and EAX respectively. As the number of flows increases, the minimal throughput decreases in Fig. 5. Because the number of flows increases, the available bandwidth allocated to each flow decrease. And the more the flows are, the lower bandwidth the flow with minimal throughput can compete. But the minimal throughput of NMOR is 2.5 times and 1.8 times that of ETX and EAX respectively.

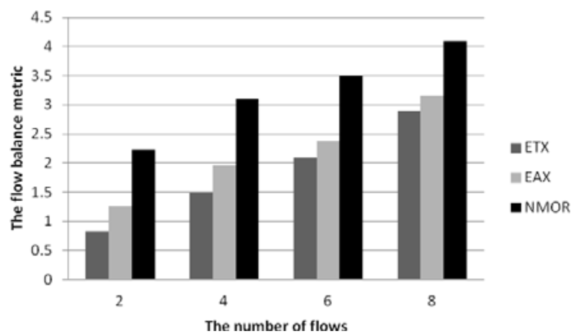


Fig. 4. The flow balance metric with flows.

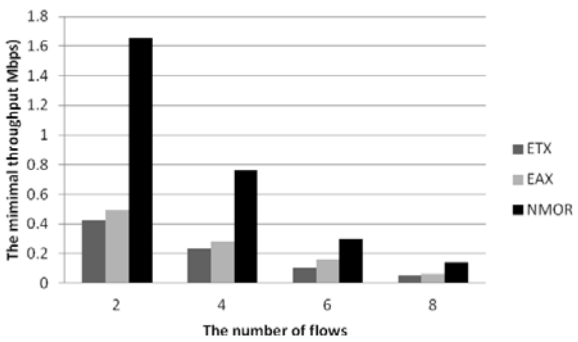


Fig. 5. The minimum flow with flows.

NMOR uses all possible nodes and links while selecting route instead of the nodes and links pre-selected by ETX or EAX. Even if we optimally allocate rate on the nodes and links selected by ETX or EAX, it's still limited. NMOR optimally allocates rates on all possible nodes and links. Therefore, it can achieve better throughput.

5.3. The Effect of θ's Value

To consider load balance, we introduce a parameter θ into our constraint. We use different values of θ to test the performance of NMOR. We vary θ from 1 to 5 Mbps. Fig. 6 and Fig. 7 show the total throughput and the minimal throughput achieved by NMOR with different θ, for comparison, as well as the performances of ETX and EAX. At the same time, we obtain the performance without the node load balance constraint denoted by "no balance".

Generally, the total and minimal throughput of NMOR is higher than that of ETX and EAX. As the increase of the value of θ, the total throughput increase and the minimal throughput decrease. The bigger the value of θ is, the weaker the node load balance constraint is and the more imbalances the fairness of flows is. Therefore the minimal throughput decreases with the increase of the value of θ. According to the fairness demand, we select proper value of θ.

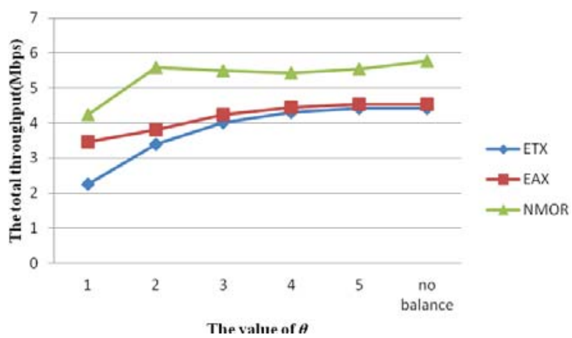


Fig. 6. The total throughput with different θ.

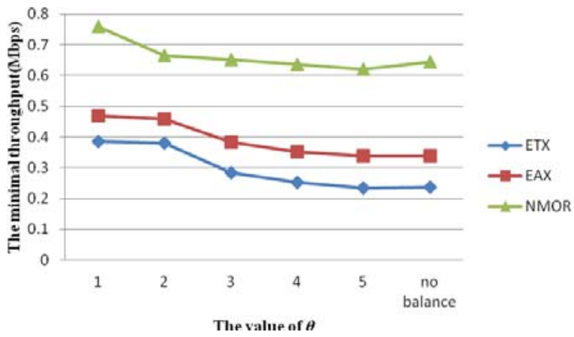


Fig. 7. The minimal throughput with different θ.

7. Conclusions

In this paper, we have studied the route selection problem of opportunistic routing with node load balance. And a distributed algorithm NMOR, which allocates the flow rate iteratively and rate allocation decides the candidate node selection, are proposed. Evaluation results show that NMOR achieves higher throughput and fairness than existing opportunistic routing algorithms

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References

[1]. S. Biswas, R. Morris, ExOR: Opportunistic Multi-Hop Routing for Wireless Networks, in *Proceedings of the International Conference on Applications, Technologies, Architectures, and Protocols for Computer Communications (SIGCOMM'05)*, Philadelphia, Pennsylvania, USA, 22-26 August 2005, pp. 133-144.

[2]. E. Rozner, J. Seshadri, Y. Mehta, et al, Simple opportunistic routing protocol for wireless mesh networks, *IEEE Transactions on Mobile Computing*, Vol. 8, Issue 12, 2009, pp. 1622-1635.

[3]. S. Chachulski, M. Jennings, S. Katti, et al, Trading structure for randomness in wireless opportunistic routing, in *Proceedings of the International Conference on Applications, Technologies, Architectures, and Protocols for Computer Communications (SIGCOMM'07)*, Kyoto, Japan, 27-31 August 2007, pp. 169-180.

[4]. S. Katti, D. Katabi, H. Balakrishnan, et al, Symbol-level network coding for wireless mesh networks, in *Proceedings of the International Conference on Applications, Technologies, Architectures, and Protocols for Computer Communications (SIGCOMM'08)*, Seattle, WA, USA, 17-22 August 2008, pp. 401-412.

[5]. T. Li, D. Leith, L. Qiu, Opportunistic routing for interactive traffic in wireless networks, in

Proceedings of the IEEE 30th International Conference on Distributed Computing Systems (ICDCS'10), Genoa, Italy, 21-25 June 2010, pp. 458-467.

[6]. Yan Zhang, Jijun Luo, Honglin Hu, Wireless mesh networking: architectures, protocols and standards, *Auerbach Pulications*, 2008.

[7]. Zifei Zhong, S. Nelakuditi, On the efficacy of opportunistic routing, in *Proceedings of the 4th Annual IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks*, *SECON'07*, San Diego, California, USA, 18-21 June 2007, pp. 441-450.

[8]. Y. Li, W. Chen, and Z.-L. Zhang, Optimal forwarder list selection in opportunistic routing, in Proceedings of the *IEEE 6th International Conference on Mobile Adhoc and Sensor Systems, MASS'09*, Macau, China, 12-15 October 2009, pp. 670-675.

[9]. D. D. Couto, D. Aguayo, J. Bicket, et al, A high-throughput path metric for multi-hop wireless routing, in *Proceedings of the 9th Annual International Conference on Mobile Computing and Networking (MobiCom'03)*, San Diego, CA, USA, 14-19 September 2003, pp. 134-146.

[10]. R. Laufer, H. Dubois-Ferrière, and L. Kleinrock, Multirate anypath routing in wireless mesh networks, in *Proceedings of the IEEE International Conference INFOCOM*, Rio de Janeiro, Brazil, 19-25 April 2009, pp. 37-45.

[11]. R. Draves, J. Padhye, B. Zill, Routing in multi-radio, multi-hop wireless mesh networks, in Proceedings of the *ACM Annual International Conference on Mobile Computing and Networking (MOBICOM)*, Philadelphia, PA, USA, September 26 – October 1, 2004, pp. 114–128.

[12]. H. Dubois-Ferrière, M. Grossglauer, and M. Vetterli, Valuable detours: least-cost anypath routing, *IEEE/ACM Transactions on Networking*, Vol. 19, Issue 2, 2011, pp. 333-346.

[13]. J. Wu, M. Lu, F. Li, Utility-based opportunistic routing in multi-hop wireless networks, in *Proceedings of the 28th International Conference on Distributed Computing Systems, ICDCS*, Beijing, China, July 17-20, 2008, pp. 470–477.

[14]. M. Lu and J. Wu, Opportunistic routing algebra and its applications, in *Proceedings of the IEEE International Conference INFOCOM*, Rio de Janeiro, Brazil, April 19-25, 2009, pp. 2374-2382.

[15]. X. Zhang and B. Li, Dice: a game theoretic framework for wireless multipath network coding, in *Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, Hong Kong SAR, China, May 26-30, 2008, pp. 293-302.

[16]. Xi Fang, Dejun Yang, Guoliang Xue, Consort: node-constrained opportunistic routing in wireless mesh networks, in *Proceedings of the IEEE International Confernce (INFOCOM'11)*, Shanghai, China, April 10-15, 2011, pp. 1907-1915.

[17]. Fan Wu, Vijay Raman, and Nitin Vaidya, Being opportunistic or being concurrent, in *Proceedings of the 7th Annual IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (IEEE SECON'10)* Boston, MA, USA, June 21-25, 2010, pp. 1-3.

[18]. Kae Won Choi, Wha Sook Jeon, and Dong Geun Jeong, Efficient load-aware routing scheme for wireless mesh networks, *IEEE Transcations on Mobile Computing*, Vol. 9, Issue 9, 2010, pp. 1293-1307.

[19]. E. Ancillotti, R. Bruno, M. Conti, A. Pinizzotto, Load-aware routing in mesh networks: models, algorithms and experimentation, *Computer Communications*, Vol. 34, Issue 8, 2011, pp. 948-961.

[20]. D. P. Palomar and M. Chiang, A tutorial on decomposition methods for network utility maximization, *IEEE Journal on Selected Areas in Communications*, Vol. 24, Issue 8, 2006, pp. 1439-1451.