Performance Analysis of Localization Techniques with Generalized Prior Distributions

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Abstract: Low error of indoor localization of wireless transmitters is difficult to achieve with only the received signal strength. This work includes an investigation of the probabilistic method for estimating the position of an unknown node in an indoor environment, Improved Prior Measurement Comparison (PMC+). PMC+ relies on a prior distribution for the unknown node position that emphasizes the probability of a node being in a specific room using a comparison to prior measurements made in that room. We expand the method of calculating this distribution to include arbitrary room shape and sensor placement. This method is then applied to a simulation and measurement campaign in an actual building at the University of New Hampshire, and the results of PMC+ are compared to other localization techniques. PMC+ significantly increases the likelihood of estimating the correct room of the unknown node. Copyright © 2015 IFSA Publishing, S. L.

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1. Introduction

Indoor localization is an important technology that can be used for tasks, such as tracking people in hospitals, reducing building energy usage, and search and rescue missions [1]. While Global Positioning System (GPS) is commonly used for outdoor localization, it is highly inaccurate for indoor applications; indoor environments pose a challenge to localization due to the complex wireless environment. Current indoor localization technologies employ Time of Arrival (TOA), angle of arrival (AOA), received signal strength (RSS), and pattern recognition [2-3]. TOA based systems require high precision timing and synchronization, and AOA based systems require antenna array and complex hardware. Therefore, we consider RSS methods for indoor environments. Indoor tracking has similar aspects as node localization in Wireless Sensor Networks (WSN).

The sensors used for localization in WSNs may be deployed in a random or a systematic manner in an environment [4-6]. The approximate location of unknown nodes is estimated with algorithms based on the known position of pre-deployed sensors [7]. These localization methods better known as Relative Location Estimation (RLE), estimate the distance based on RSS [8]. The term “anchor” refers to a pre-deployed sensor, and both terms are used synonymously throughout this paper. The density of anchors should be such that the unknown node is reachable by at least three anchors at any time. In this paper, we consider a systematic and planned deployment of a sensor network.

http://www.sensorsportal.com/HTML/DIGEST/P_2646.htm
This is similar to the Microsoft RADAR project that uses sensors to determine the position of users [6], whereas the anchor and unknown nodes have different functionality. However, our work considers general wireless signals and not just WiFi. Similar to Fingerprinting (FP), our method uses prior measurements to reduce the error of the estimated position [9]. This work is different in that the prior information is used to select a subset of sensors and also improve the prior probability distribution.

There are a plethora of RLE methods including Minimum Mean Squared Error (MMSE), Probability-based Maximum Likelihood (PML) [10], and enhanced PML (EPML) [11]. The MMSE method estimates the position of a node by minimizing the squared error over all the measured or calculated differences between the sensor and the unknown node. PML minimizes the error based on an assumed probability distribution for the nodes over an area. This assumed probability distribution, or a priori information, is used to determine the most probable estimated position. EMPL (an improved version of PML) uses a subset of anchors that result in the strongest signal to the unknown node. In related works, we presented a technique called Prior Measurement Comparison (PMC) that determines the subset of sensors to use for localization based on reference measurements using FP [9].

In this paper, we describe a technique to intelligently use reference measurements to adjust the a priori knowledge. Sections 1, 2 and 4.1 of this paper are included from our work in [12], and it expands upon that work. In Section 2, we describe the method to calculate RSS and the different localization techniques including our reference-based prior distribution method originally presented in [12]. In Section 3 we present a generalized method of determining prior distributions for each sensor. In Section 4 we compare the performance of localization techniques in two simulation scenarios and a measurement campaign. We provide conclusions and future work in Section 5. Our contributions include a new method for determining the prior distributions for use with the PML/PMC/PMC+ techniques.

2. Localization Methods

In this section, we describe the path loss model used to determine the RSS and the localization methods that are used to compare to our new technique.

2.1. Received Signal Strength

Sensors deployed throughout an area of interest can measure the signal strength from a transmitter (or node). From this received signal strength (RSS) [13], and using path loss, the distance between the unknown node and the sensor can be estimated [14]. The path loss for the transmitter-receiver separation is expressed as a function of distance, $d$, the path loss exponent, $n$, and $\gamma$ the shadowing parameter representing a zero-mean Gaussian distributed random variable with standard deviation $\sigma$ (dB). The RSS is calculated as,

$$RSS = PL_0 - 10n\log_{10}\left(\frac{d}{d_0}\right) + \gamma,$$

where $PL_0$ is the received signal strength at $d_0$. With multiple temporal samples of a transmitted signal, an estimate value for the shadowing standard deviation can be calculated.

Planned deployments render better results than when the sensors are randomly deployed [7]. For planned deployment of sensors, some information is known a priori. This information can include the dimensions of a room, the arrangement and number of sensors. Along with the RSS, this prior information can be used to increase the performance of localization techniques.

2.2. Localization Techniques

There are several methods for localization that will be used in our analysis to compare the efficacy of our proposed technique.

The most widely used technique for indoor localization is MMSE [10]. With MMSE, the error is obtained taking the difference between all the measured distances and the estimated distances. The unknown point or node is then estimated by minimizing the square of all the error from all the anchor points $(\hat{x}, \hat{y})$.

PML uses the prior probability distribution of the unknown nodes to decrease the error associated with localization. The position of the unknown node (detected by $n$ sensors) is estimated by maximizing the posterior probability [7]. $(\hat{x}, \hat{y})$ represents the estimated node location that is calculated by,

$$(\hat{x}, \hat{y}) = \text{arg min}\{PML(x, y)\}$$

$$P(\hat{d}_i|d_i) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{10\log_{10}\left(\frac{d_i}{d_0}\right)^2}{2\sigma^2}}\cdot \frac{10n}{\ln 10} \quad (2)$$

$$PML(x, y) = -\prod_{i=1}^{n} P(\hat{d}_i|d_i)P(d_i)$$

PML can be improved by taking into account the unreachable or the inactive anchor points called Extended Probability-Based Maximum Likelihood (EPML). By selecting only the sensors over a certain RSS power, EPML reduces the uncertainty from far away sensors [7]. However, this approach is still affected by the shadowing-influenced distance metric.
2.3. Prior Measurement Comparison (PMC)

In related work, we show that subsets can reduce the error in an estimating node’s location [9]. The PMC method first compares the RSS from the unknown node to every sensor to the RSS of every reference point; we call this comparison Fingerprinting (FP). Then a subset of anchors is chosen for estimating the position using PML.

To perform the comparison between the detected signal and the references, a matrix of power ratios is calculated at the time of deployment. This matrix is determined by measuring the RSS values at the sensors from each reference point \( x \) for \( X \) total reference points. This matrix is \( R_x = RSS_i/RSS_j \) where \( i \) and \( j \) denote the indexes of \( n \) total sensors. A similar matrix is obtained from the initial transmission of the arbitrary unknown node given by \( \bar{R} = RSS_i/RSS_j \). The two matrices are compared to determine the minimum distance based upon the Frobenius norm which is the likely subset region (or subregion) where the unknown node is located.

To execute PMC, the area is divided into subsets where each subset is bounded by four sensors and a reference point is placed at the center. When a subset is chosen, only the respective four sensors are used for the determination of the location. The selection of the subset for PMC is determined by \( \min_i \| R_x - \bar{R} \| \).

The sensor subset calculated with PMC is then used with PML (Sec. 2.2) to determine the location of an unknown node. In the absence of shadowing, the four sensors bounding the sector would have resulted in the highest RSS and would have been naturally used for the unknown node. PMC improves upon previous methods but does not take full advantage of the prior information gleaned from the reference point measurement comparison.

2.4. Improved Prior Measurement Comparison (PMC+)

The number of anchor nodes deployed in some space is limited by cost and infrastructure demands. With a need to decrease the localization error further, we propose to utilize reference measurements in an intelligent manner. PMC, as explained earlier, compares the received power ratio matrix at the unknown node with that of the reference point matrices and chooses the most similar to the reference point and subsequent subset of anchors. We propose to use multiple references at the room level such that multiple rooms would be inside of the subset area. The motivation for this is to improve localization at the room level given the unique per room wireless channel characteristics.

Based on an analysis of the FP technique, we found that when the shadowing power is low, the correct subset, if not the correct reference point, is selected. When the shadowing power is high, the FP technique may determine that one of several reference points or subsets is likely to be selected. Thus, the correctness of the FP result strongly influences the outcome of the PMC technique. This motivates us to reconsider the prior distribution \( P(d_i) \) [7, 9].

Our new approach is to consider the shadowing power when determining the prior distribution for PMC. We introduce a probability variable, \( P \), which represents the confidence that FP method is correct. When the shadowing power is low, we give more preference (or higher probability) to the room selected, by setting \( P \) close to 1, and we set the value \( P \) to low values when the shadowing power is high. The exact value for \( P \) will depend upon the shadowing in a particular environment. With this new term, we can modify the prior distribution \( P(d_i) \) to include the confidence in the FP technique to increase or decrease the probability that the unknown node is in a particular room.

Consider the layout in Fig. 1, the small squares represent rooms or small regions and the circles represent the anchors or pre-deployed sensors. In PMC, a reference measurement is made at the center of each room and denoted by a dot. The area of one of the subsets of sensors is shown in grey; previous work considered only one reference measurement at the center of the subset area [9]. Because the FP confidence will change the prior distribution, the new prior is a function of the room in reference to a particular anchor. For example, the rooms in Fig. 1 are numbered with respect to the dark anchor.

The new prior distribution is a function of the possible distance to an anchor, \( d_i \), the room, \( r \), in reference to one of the anchors, and \( P \). This probability is expressed as,

\[
P(d_i) = \frac{m(d_i,r,P)}{\int m(x,r,P)dx},
\]

where \( m(x,r,P) \) is defined in Eqs. 4 – 6.
3. Generalization of Prior Distribution

In several reviewed prior works utilizing a form of PML ([7, 9, 11-12]), the distribution functions used in each localization technique are dependent on the use of square or rectangular rooms. These techniques require a uniform distribution of transmitter locations in the region covered by sensors (and by extension, uniform distribution of transmitter locations in each room). The use of square or rectangular rooms and subregions with uniform distribution is likely due to the fact that arc lengths within each subregion simplified to expressions involving inverse trigonometric functions, π/2 radians, subregion length and width, and distance from a sensor, and therefore closed-form distributions could be calculated. While these closed-form expressions are useful for simulations and analysis, they are either extremely difficult or impossible to obtain for rooms and subregions of arbitrary polygonal shape. Additionally, actual building and room size and restrictions on sensor locations may result in such arbitrary polygonal regions. As such, a numerical method of approximating the distribution of transmitters vs. distance from each sensor is required.

Assuming a uniform distribution in a region, the probability that a transmitter is in a subregion is directly proportional to the area of the subregion. Therefore, if a region is divided into small slices of area at each given distance from a sensor, (i.e. the arc length contained within an arbitrary region (such as the sensor region) times an arbitrarily small Δr), a function is derived that is proportional to the distribution of transmitters from a sensor as a function of distance. Finally, due to the law of total probability, the distribution is normalized to obtain a PDF for the distribution of distance from each sensor.

A numerical method can be derived using the same principles from which the closed-form distributions are derived. Specifically, the arc length contained within a subregion or room at a given distance can be approximated by determining a large number of points equally spaced in a circle of a radius equal to the distance from the circle being considered, with that circle centered on the sensor location. If there are N points on this circle, the arc angle between points is therefore 2π/N. Each point in this circle can be analyzed to determine if it is within a subregion or room. The arc length contained within the subregion or room can then be approximated by multiplying 2π/N by the number of points on this circle that are within the subregion or room. The arc length can then be approximated by multiplying the distance from the sensor to the circle (i.e. radius) by the arc length. As N approaches ∞, this approximation approaches the actual arc length contained within the subregion or room. Repeating this process for a large number of uniformly distributed distances from 0 to the maximum distance contained in the subregion of interest from the

\[
m(x, r = 1, P) = \begin{cases} 
\frac{\pi}{2} x^P & 0 < x \leq l \\
\left(\frac{\pi}{2} - 2 \cos^{-1}\left(\frac{1}{x}\right)\right) x^P + 2 \cos^{-1}\left(\frac{1}{x}\right) x(1 - P) & l < x \leq \sqrt{2}l \\
\frac{\pi}{2} x(1 - P) & \sqrt{2}l < x \leq 2l \\
\left(\frac{\pi}{2} - 2 \cos^{-1}\left(\frac{1}{x}\right)\right) x(1 - P) & 2l < x \leq \sqrt{8}l \\
0 & \sqrt{8}l < x 
\end{cases}
\]

\[
m(x, r = 2, P) = \begin{cases} 
\frac{\pi}{2} x(1 - P) & 0 < x \leq l \\
\left(\frac{\pi}{2} - 2 \cos^{-1}\left(\frac{1}{x}\right)\right) x(1 - P) + \cos^{-1}\left(\frac{1}{x}\right) x^P & l < x \leq \sqrt{2}l \\
\sin^{-1}\left(\frac{1}{x}\right) x^P + \left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{x}\right)\right) x(1 - P) & \sqrt{2}l < x \leq 2l \\
\left(\sin^{-1}\left(\frac{1}{x}\right) - \cos^{-1}\left(\frac{2}{x}\right)\right) x^P + \left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{x}\right) - \cos^{-1}\left(\frac{2}{x}\right)\right) x(1 - P) & 2l < x \leq \sqrt{5}l \\
\left(\frac{\pi}{2} - 2 \cos^{-1}\left(\frac{2}{x}\right)\right) x(1 - P) & \sqrt{5}l < x \leq \sqrt{8}l \\
0 & \sqrt{8}l < x 
\end{cases}
\]

\[
m(x, r = 3, P) = \begin{cases} 
\frac{\pi}{2} x^P & 0 < x \leq \sqrt{2}l \\
\left(\frac{\pi}{2} - 2 \sin^{-1}\left(\frac{1}{x}\right)\right) x^P + 2 \sin^{-1}\left(\frac{1}{x}\right) x(1 - P) & \sqrt{2}l < x \leq 2l \\
\left(\frac{\pi}{2} - 2 \sin^{-1}\left(\frac{1}{x}\right) - 2 \cos^{-1}\left(\frac{2}{x}\right)\right) x^P + 2 \sin^{-1}\left(\frac{1}{x}\right) - 2 \cos^{-1}\left(\frac{2}{x}\right) x(1 - P) & 2l < x \leq \sqrt{5}l \\
\left(\frac{\pi}{2} - 2 \cos^{-1}\left(\frac{2}{x}\right)\right) x^P & \sqrt{5}l < x \leq \sqrt{8}l \\
0 & \sqrt{8}l < x 
\end{cases}
\]
sensor, the distribution of transmitters vs. distance from the sensor is generated. This distribution can then be normalized by its numerically-computed integral over all distances to obtain an approximate PDF function. Interpolation can then be used to determine a value for any arbitrary distance.

This method can be easily expanded to PMC+. In PMC+, at a selected distance, the arc length contained in a room of interest is multiplied by the constant \( P \) (the probability that the FP is correct and the transmitter is in the room), the arc length in the sensor subgroup region but not in the room of interest is multiplied by \((1-P)\), and the two values are summed. The same method of approximating the arc length in each room or region as previously described can be used to easily determine a PDF distribution for each sensor, for each subgroup region and each room within that subgroup region. If the value of \( P \) is changed, the entire distribution must be recalculated.

This numerical method is complex and may take a significant amount of time to compute. For each distribution, \(N^*M\) locations must be considered, where \( N\) is the number of points considered on each circle, and \( M\) is the number of distances considered. Each location must be determined to be within a polygon formed by the region (and in the case of PMC+, determined to be within a polygon formed by the room of interest as well), which is computationally complex. Assuming \(4^*S\) sensors form a subgroup, for \( S\) subgroups, the number of distributions that must be pre-calculated is equal to \(4^*S\). In PMC+, assuming \( R\) rooms per subgroups, and assuming \( Q\) values of \( P\), \(4^*R^*S^*Q\) prior distributions must be calculated. However, these calculations can be pre-computed for real-time sensing applications.

For simulations in subsequent sections where the numerical method was utilized, 1800 points were evaluated on each circle, and 500 distances were evaluated in each distribution. The numerical method utilizing these values was verified by comparing the calculated sensor PDF functions to the closed-form distribution expressions obtained in previous work. Using several randomly-selected distributions from prior papers for PMC and PMC+ [9], [12] our numerical distributions were accurate to within 0.1% at all distances, sufficiently accurate to be used in subsequent simulations.

### 4. Simulations & Measurements

In this section, we compare the performance of PMC+ with other localization techniques. First, a simple scenario with 16 sensors arranged with square sensor subgroups and square rooms was considered. Next, a more complex scenario modeling part of Kingsbury Hall at the University of New Hampshire was considered with a non-rectangular sensor subgroup and non-uniform, polygonal room sizes. Finally, a measurement campaign was conducted in Kingsbury Hall based on the simulation scenario.

#### 4.1. Simulation of Square Area

In this subsection, we evaluate the performance of PMC+ in comparison with other localization techniques for sensors and subgroups arranged in square subgroups. For this analysis, we simulated a 100 by 100 sq. meter area separated into nine subsets as shown in Fig. 1. Each subset contains four rooms of size \( l = 16.5\) m.

For the first analysis, a random point is selected in the area and the RSS values are then simulated and used to perform each localization method. The simulation determines the sample mean of errors over the entire area with PMC+, PMC, and PML. We used the following parameters for this analysis, \(n = 3.8\), \(\sigma = \{3, 7\}\), and \(PL_0 = -55\) dB. For this scenario, we selected a prior parameter of \( P = 0.75\). The Monte Carlo simulations are repeated until the confidence interval reaches 95\% of 0.2 m.

The results of this simulation are presented in Fig. 2 and show that the PMC+ method has the least amount of error for both levels of shadowing standard deviation. PMC+ reduces the effect of shadowing on the results because the subset sensors are closer to the unknown node and the prior distribution adds weight to a region (or room) of the subset. These results confirm that PMC+ performs better than PML and PMC at low and high values of \(\sigma\).

![Fig. 2. Estimation error comparison of PMC+, PMC and PML, Square Region Simulation.](image)

We also consider the mean error versus the shadowing power. Fig. 3 shows the comparison between the PMC and PMC+ for standard deviations ranging from 3 to 9 dB. Due to the randomness in the environment, shadowing, and path loss exponent, it is possible that the FP function of PMC will select the wrong subset, i.e. a subset of sensors whose region does not contain the unknown node. When PMC chooses the wrong subset, the error in localization increases. The plot shows that PMC+ consistently performs better than PMC at low and high levels of shadowing because PMC+ adds weight to the more likely region with the parameter \(P\). Also, while
PMC+ may choose the wrong room at high shadowing power, it is likely that the correct subset of anchors will be used for the localization.

Finally, we compared room level accuracy between PMC+ and PMC. From the Monte Carlo simulations, we calculate the percentage of simulations where the estimated location was in the correct room. Over all levels of $\sigma$, PMC+ was found to be 7.5% higher than PMC. These results show that the added complexity of the prior distribution decreases the error of probabilistic indoor localization techniques.

4.2. Building Model Simulation

A new simulation based on the second floor of the south wing of Kingsbury Hall at the University of New Hampshire was developed to test the performance of the PMC+ algorithm compared to PMC, PML, and MMSE. This scenario was chosen for its ability to be replicated in a measurement campaign to test the algorithms in a real-world environment. In this scenario, four sensors were placed in locations as close to exterior walls of the building and corners of the room as possible. The sensors were placed on the ends of ceiling-mounted overhead service carriers in three laboratories, and on the top of a cabinet in a fourth laboratory.

Due to unique and varying room geometries, multiple fingerprint points were chosen per room in most rooms to ensure that any measurement taken within the room was closer to a fingerprint point in the same room as a measurement than a fingerprint point in an adjacent room. Fig. 4 shows a map of the simulation region illustrating sensor positions, subgroup and room boundaries, and fingerprint points. Measurement points illustrated in Fig. 4 were used in a measurement campaign described in the subsequent section and should be ignored for this simulation. Instead, measurement points were selected in the polygonal region formed by the four sensors using a uniform distribution.

In this simulation, a path loss exponent $n = 4.9$, reference distance $d_0 = 11.17$ m, and path loss at reference distance $d_0 (PL_0) = -55.55$ dB were used in a log-distance path loss model. These values were determined from a small measurement campaign performed in Kingsbury hall at 915 MHz using a signal generator producing a 200 kHz bandwidth AWGN signal with 10 dBm of power and a spectrum analyzer. Log-normal shadowing was assumed, with standard deviation $\sigma = \{3, 5, 7, 9, 11\}$ dB considered. A static value of $P = 0.75$ was used in PMC+ calculations. A Monte Carlo simulation was performed for each standard deviation value for MMSE, PMC, and PMC+, with iterations occurring in each case until 95% confidence of the mean error being within 0.2 m was reached.

The results of our simulation show that PML, EPML and PMC, collapse to the same result. This situation occurs because only one subset is used. PML uses $RSS$ data (and distributions) from all sensors in its equations, which in this simulation is all 4 available sensors. EPML only uses data from all sensors with $RSS$ power above a certain threshold; due to the relatively close proximity of all 4 sensors, the $RSS$ at all 4 sensors will remain above a threshold with a reasonable value. Therefore EPML will also use the $RSS$ measurements (and corresponding distributions) from each of the 4 sensors. Finally, PMC uses fingerprinting to select a sensor subgroup of 4 sensors; since there is only one sensor subgroup in this simulation, the $RSS$ measurements from these 4 sensors will also be used. Beyond sensor selection, all three methods estimate a location by maximize
the posterior probability using RSS and distributions from each sensor in exactly the same way.

The mean error versus shadowing standard deviation for MMSE, PML/PMC and PMC+ is illustrated in Fig. 5. Overall, the mean error for this simulation is lower than that of the previous simulation. This is likely due to the closer proximity of the four sensors that comprise the subgroup compared to the previous simulation. MMSE performs poorly at high values of $\sigma$, with error in excess of 16 m at a value of $\sigma = 11$. PML/PMC and PMC+ perform almost identically, with mean error values being well within the 0.2 m confidence interval of each other. One possible reason for the similar values of mean error for this simulation in comparison with the last is that some sources of error such as using measurements from farther away sensors with more uncertainty (as in the PML method) or having fingerprinting identify the wrong sensor subgroup (as in the PMC method) are absent from this simulation, thus keeping the error lower than would be the case with a greater number of sensors, particularly at higher values of $\sigma$. PMC+ similarly benefits from not being able to select an incorrect subgroup in this simulation (as with PMC). However, unlike PMC, PMC+ may select an incorrect room within the subgroup (particularly at higher values of $\sigma$), which will increase the error in the estimate.

![Fig. 5. Mean error of PMC+ and PMC for different values of shadowing power, $\sigma$ (dB), Kingsbury Hall Simulation.](image)

Further investigation of the CDF of error for the three methods at various values of $\sigma$ (Fig. 6 and Fig. 7) shows that PMC and PMC+ perform almost identically in terms of error, as both have almost identical distributions. At least in the context of our building, it would appear that adding weight to a room in the prior distribution neither helps nor hinders the accuracy of PMC+.

![Fig. 6. Estimation error comparison of PMC+, PMC and PML, Kingsbury Hall Simulation, $\sigma = 3$.](image)

![Fig. 7. Estimation error comparison of PMC+, PMC and PML, Kingsbury Hall Simulation, $\sigma = 7$.](image)

Finally, the probability of estimating a position in the same room as the actual position is evaluated for each of the 3 localization methods for various values of $\sigma$, as illustrated in Fig. 8. In this metric, PMC+ shows its strength. Despite having comparable levels of error to PMC, PMC+ has a 4 to 10% increase in probability of estimating the correct room over PMC for all levels of $\sigma$, with the biggest increases occurring at $\sigma = 5$ and 7 dB. Both methods show an improvement of more than 30% over MMSE in all cases.

This simulation shows that PMC+ performs at least as well as PMC in terms of error in position estimation, but shows a significant improvement in
correctly estimating the room that a transmitter is in. Therefore PMC+ would be particularly useful for an application such as room occupancy estimation.

Fig. 8. Probability of Correct Room Estimation for PMC+, PML/PMC and MMSE for different values of $\sigma$ (dB).

4.3. Building Measurement Campaign

A measurement campaign mirroring the simulation performed in the previous subsection was performed. Sensors consisting of Ettus Research USRP B200 boards equipped with Ettus VERT900 antennas were placed at each of the 4 sensor locations indicated in Fig. 4. Each USRP board was connected to an Intel NUC PC running Ubuntu Linux with GNU Radio. RSS power measurements were taken simultaneously by all four sensors for each measurement and processed by a central controlling PC which performed the three location techniques on the collected data. A Hewlett Packard E4432B Signal Generator with an Ettus VERT900 antenna was configured to transmit a 10 dBm 200 kHz bandwidth AWGN signal at 915 MHz. The antenna connected to the signal generator was placed at each measurement point, approximately 2 m off the ground. For each fingerprint point, the transmitter was positioned at the corners of a 1m square centered at the fingerprint location, with RSS measurements being taken at each of the 4 sensors. At each sensor, the 4 RSS measurements were averaged, and the set of average RSS measurements from the 4 sensors was used for the fingerprint at the fingerprint location.

After fingerprint measurements were conducted, RSS measurements occurred at the sensors with the transmitter antenna located at each of the 38 measurement points indicated in Fig. 4. These measurement points were chosen to be relatively uniform over the area sensor subgroup region. These measurements along with the fingerprint measurements were fed into the same script that performed the simulation in the previous section for processing by each of the 3 location techniques. Because the value of $\sigma$ is inherent in the environment, it is constant, and thus data from this measurement campaign cannot be plotted versus $\sigma$. A value of $\sigma$ for our building was not determined in our limited measurement campaign, so only generalized comparison to simulation results can be made rather than comparing the results to simulations for a specific value of $\sigma$.

Table 1 shows the results of the measurement campaign for each of the three location techniques. MMSE produces the greatest mean error, though it is significantly closer to the error in PML/PMC and PMC+ that would be expected based on simulation results. PMC+ has a mean error that is 32 cm smaller than PML/PMC. This could be due to a slight bias of measurement points away from the walls of rooms (due to a significant number of desks and bookshelves against walls, preventing measurement points against walls in many rooms).

Table 1. Mean Error and Probability of Correct Room Estimation for MMSE, PML/PMC, and PMC+, Kingsbury Measurement Campaign.

<table>
<thead>
<tr>
<th></th>
<th>Mean Error (m)</th>
<th>Pr (Estimate correct room)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSE</td>
<td>4.9755</td>
<td>0.3158</td>
</tr>
<tr>
<td>PML/PMC</td>
<td>3.7767</td>
<td>0.5000</td>
</tr>
<tr>
<td>PMC+</td>
<td>3.4500</td>
<td>0.7105</td>
</tr>
</tbody>
</table>

Because measurement points away from walls are not as likely to be closer to a fingerprint point in an incorrect room (even with high levels of shadowing), the prior probabilities have an emphasis on the correct room a higher percent of the time, resulting in more accurate estimation points. Also, as observed in the simulation, the PMC+ location method has a significantly higher probability of selecting the correct room than PML/PMC and MMSE. PMC+, however, had a much higher increase in this probability over PML/PMC (over 20 % greater) as when compared to the difference in probabilities in simulation (4 to 10 % greater).

Fig. 9 shows a CDF of the error of 38 measurement points from the Kingsbury measurement campaign. As noted from the analysis of the mean error above, MMSE performs better than expected, with its error CDF only slightly below the PML/PMC and PMC+ until just under 90 % of measurement points, when its error increases greatly. PMC+ appears to perform better than PML/PMC a majority of the time, but in cases when PMC+ selects an incorrect room (about 29 % of the time), the error will be greater than PML/PMC. This is reflected in the fact that at above an error of about 5.7 m, the probability that the error is less than this value is
greater with PML/PMC than with PMC+. Therefore, it would be advantageous to use a distribution more like PML/PMC than PMC+ for cases where there is less certainty that an unknown transmitter is in a room, which can be achieved by decreasing the value of $P$ to close to 0.5. Future work will investigate the effect of different $P$ values on the performance of PMC+. We propose comparing the minimum Frobenius norm of the fingerprint locations in each room, and selecting a high value of $P$ when the minimum norm of one room is significantly lower than all other rooms, and using a lower value when the two smallest minimum norms of two rooms are close together (indicating less certainty of which of the two rooms the unknown transmitter is in).

**Fig. 9.** Estimation error comparison of PMC+, PMC and PML, Kingsbury Hall Measurement Campaign.

## 5. Conclusions and Future Work

In this paper, the performance of the PMC+ localization technique was compared to other localization techniques in both simulated and real-world measurement scenarios. A numerical method of approximating transmitter distribution from each sensor using arc length estimates was used to calculate distributions for arbitrary room shapes and sensor configurations, allowing for the simulation and measurement campaign in a building with non-rectangular room configurations. In both simulation and measurement campaign, PMC+ outperformed other localization methods in estimating the room of an unknown transmitter. Future work will investigate the use of different values of $P$ tied to the Frobenius norm of measurements compared to fingerprint location to decrease error in cases when room selection is incorrect.