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MEMS Capacitive Micro Thermometer Based on Tip Deflection of Bimetallic Cantilever Beam

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Abstract: Thermometry is an interested field in physics and metrology. In this paper, a capacitive micro thermometer based on the tip deflection of bimetallic cantilever beam was designed. The governing thermo mechanical equations were derived and solved analytically. The temperature rising was expressed with respect to capacitance change of a comb drive. The results of beam deflection were compared well with the existing results.

Keywords: Micro Thermometer, Bimetal, Cantilever, Capacitance

1. Introduction

Recent Micro Electro Mechanical System (MEMS) development has demonstrated that technology can influence all of scientific applications and products. Since producers need to increase accuracy and reliability and decrease price and dimensions of their products, MEMS issue is interested by different researcher because of the benefits of MEMS include small size, low weight, and low cost it can provide all of them. One application of MEMS technology is fabrication of tiny sensors and actuators. Micro sensors gather local information including, for example, thermal, biological, chemical, and optical input.

One kind of the micro sensors is thermometer, which is used to measure the temperature of the desire environment. Existing thermometers don't have satisfy consumers because they are not enough fast and they are too expensive or do not have enough accuracy. They can also be complicated to use and often only work over a limited temperature range. In addition, it is not easy to calibrate these thermometers at low temperatures. Moreover, there is a necessary in experimental physics to fabricate measurements of temperature with high spatial resolution. This requires the development of specific micro sensors having very small dimensions and fast thermal response times.

Primary thermometers demonstrated absolute temperature and they operate very difficult however they are useful in fabricating secondary thermometers, the latter ones are often much easier to operate and thereby more common in research laboratories and industry [1-4]. Modern micro and NANO lithography allows for new thermometer concepts and realizations where sensors can be very small, thermal relaxation times are typically short. Generally, any quantity, which changes as a function of temperature, can be used in measuring temperature. But there is a list of further requirements and merits, which determine the usefulness of particular thermometric quantity in each application. To measure temperatures, one may use sensors based on various physical effects and fabricated from different materials [1].

Several different types of thermometer have been developed, produced and investigated in many applications. The sensors based on MEMS and their fabrication technology has great potential. One kind of thermometers can be mentioned as tunneling thermometers. Typically these thermometers tell about the electron temperature, which means, in general, that a channel for electron thermalisation to the temperature of the thermal bath has to be sufficiently good. Jukka Pekola in his paper [1] demonstrated to Thermometer based on tunneling. Tunneling characteristics through a barrier separating two conductors with none-equal densities of states (DOSs) are usually temperature dependent. And he pointed the Coulomb blockade thermometer (CBT) too. This is a primary thermometer, whose operation is based on competition between thermal energy and electrostatic energy at bias voltage V, and charging energy due to extra or missing individual electrons with unit of charging energy [1]. Lafe Spietz et al have developed a new thermometer that exploits the shot noise in a tunnel junction to measure very low temperatures. In this thermometer, temperature is related to the voltage across the junction. By a relative noise measurement with only the use of the electron charge, Boltzmann's constant, and assumption that electrons in a metal obey Fermi-Dirac statistics [5]. Resistance thermometers are widely used for measurements at low temperatures too. These sensors have been developed based on germanium films on gallium arsenide [6].

Many of the previous thermometers are based on the elaborating fabrication and simulation. Here, a capacitive linear micro thermometer based on MEMS structural is considered which is easy to fabricate with acceptable sensitivity. In the proposed micro thermometer the temperature rising is measured based on capacitance changes due to the tip deflection of the bimetallic cantilever beam. The system of differential equations for bimetallic microbeam based on Euler-Bernoulli beam theory was derived and solved analytically and the temperature rising was expressed with respect to capacitance change of a comb drive.

2. Model Description

As it is shown in Figure 1, this model is a Micro Electro Mechanical System that consists of a bimetallic cantilever beam and one comb drive that is jointed at the tip of the cantilever beam. This system acts as a micro thermal sensor.

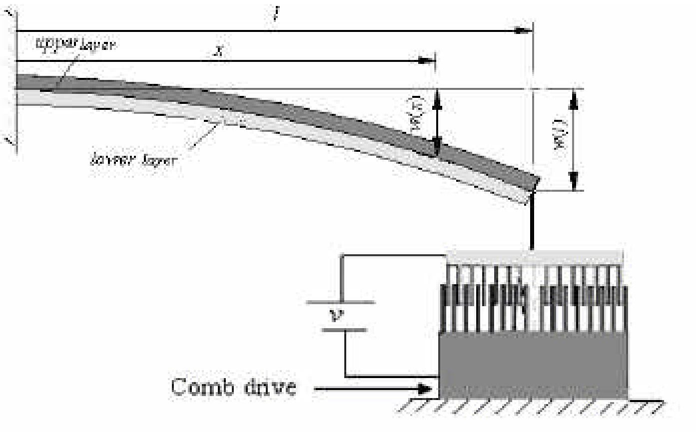

Fig. 1. Schematic of bimetallic cantilever thermometer.

Because of different temperature expansion coefficient of selected materials thereby when the operating temperature changed in system, the microbeam deflected. Thus due to its tip deflection, the effective surface of comb drive plates change which means the influence of temperature on the capacity of the system can be easily measured.

3. Mathematical Modeling

Figure 2 shows an element of bimetallic cantilever beam. Assume a beam with length$l$, thickness$h$, width$b$, expansion coefficient$\boldsymbol{a}$, cross sectional area A and isotropic with Young's modulus$E$, whereas for lower micro beam sub indicate 1 is used and for the other one 2 is used. Suppose that x is the coordinate along the length of the beam with its origin at the left end, and z is the coordinate along the cross section with its origin at the neutral axis of cross section. $w(x)$ is the deflection of the beam, and u is the displacement along x-axis. In this figure dash lines is whereabouts neutral axis.

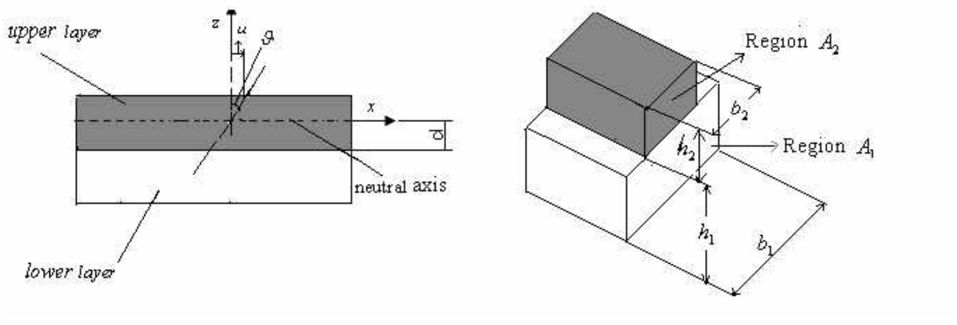

Fig. 2. An element of bimetallic cantilever beam.

For selected microbeams, l/h is usually large enough to neglect the shear deformation and using of strain definition [7], the total strain at the given cross section can be written as:

$$e_{tot} = z\frac{d^2 w}{dx^2} \tag{1}$$

Total strain at x direction at a given cross section of the beam, is the sum of thermal and mechanical strains, thus:

$$\varepsilon_{tot} = \varepsilon_m + \varepsilon_T \tag{2}$$

Where:

$$\varepsilon_T = \alpha \Delta T \tag{3}$$

With

$$\Delta T = T - T_0 \tag{4}$$

Where ε_T and ε_m are the thermal and the mechanical strains, respectively, ΔT is the temperature rising which is to be measured respect to the initial temperature T_0. Substituting Eqs. (1) and (3) into Eq. (2), the following equation can be obtained:

$$\varepsilon_m = z\frac{d^2 w}{dx^2} - \alpha\Delta T \tag{5}$$

Using of Hook's low and Eq. (5), the relationship between the stress and the strain based on Euler-Bernoulli beam theory can be expressed as below:

$$\sigma = Ez\frac{d^2 w}{dx^2} - E\alpha\Delta T \tag{6}$$

The axial force respect to the equilibrium condition along the x-axis is given as:

$$\int_{A_1} \sigma \, dA + \int_{A_2} \sigma \, dA = 0 \tag{7}$$

Substituting the Eq. (6) into Eq. (7):

$$\int_{-h_1-d}^{-d} b_1 (E_1 z\frac{d^2 w}{dx^2} - E_1\alpha_1\Delta T)dz + \int_{-d}^{h_2-d} b_2 (E_2 z\frac{d^2 w}{dx^2} - E_2\alpha_2\Delta T)dz = 0 \tag{8}$$

where d is distance of neutral axis from contact surface of two materials. By integrating, the Eq.(8) can be reduced to:

$$\frac{d^2 w}{dx^2}\left(d(-h_1 b_1 - n h_2 b_2) + \frac{n b_2 h_2^2 - b_1 h_1^2}{2} \right) - \Delta T(\alpha_1 b_1 h_1 + n\alpha_2 b_2 h_2) = 0 \tag{9}$$

where $n = \frac{E_2}{E_1}$. The bending moment *M*(*x*) at a given section is:

$$\int \sigma \, z dA = M(x) \tag{10}$$

Substituting Eq. (6) into Eq. (10):

$$\int_{-h_1-d}^{-d} b_1 (E_1 z^2 \frac{d^2 w}{dx^2} - E_1\alpha_1 z\Delta T)dz + \int_{-d}^{h_2-d} b_2 (nE_1 z^2 \frac{d^2 w}{dx^2} - nE_1\alpha_2 z\Delta T)dz = M(x) \tag{11}$$

The electrostatic forces on this system are in equilibrium state together in x direction (see Figure.3) and these forces don't influence on the tip deflection of the bimetallic cantilever beam. Thus:

$$M(x) = 0 \tag{12}$$

By integrating of the Eq. (11), we have:

$$\frac{d^2w}{dx^2}\left[d^2(b_1h_1 + nb_2h_2) + d(b_1h_1^{\,2} - nb_2h_2^{\,2}) + \frac{(b_1h_1^{\,3} + nb_2h_2^{\,3})}{3}\right] + \Delta T\left[d(\alpha_1b_1h_1 + n\alpha_2b_2h_2) + \frac{\alpha_1b_1h_1^{\,2} - n\alpha_2b_2h_2^{\,2}}{2}\right] = 0 \tag{13}$$

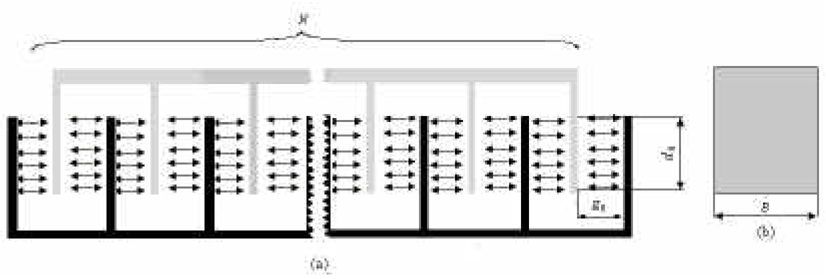

Fig. 3. (a) Equilibrium of electrostatic forces; (b) Width of plate.

Substituting value of d from Eq. (9) into Eq. (13) and simplify it, the final expression which indicates the relationship between deflections of bimetallic cantilever at a given temperature rising can be written as follow:

$$w(x) = \frac{3nb_1b_2h_1h_2(h_1 + h_2)(\alpha_1 - \alpha_2)\Delta T}{b_1^{\,2}h_1^{\,4} + n^2b_2^{\,2}h_2^{\,4} + nb_1b_2h_1h_2(6h_1h_2 + 4h_1^{\,2} + 4h_2^{\,2})}x^2 \tag{14}$$

To calculate the capacity of the comb drive due to changing of comb drive surfaces, the following equation can be used:

$$C = 2N\frac{\varepsilon_0BD}{g_0} \tag{15}$$

Where N is the number of combs, g_0 is the gap, B is the width of plates and D is the variable height of comb drive plates. Due to the considered structure for the capacitive bimetallic thermometer, which is shown in Fig.3, the variable height of comb drive plates can be computed by following equation:

$$D = d_0 + w(l) \tag{16}$$

Where d_0 is the initial effective height of comb drive plates and $w(l)$ is the tip deflection of the bimetallic cantilever beam. Finally, using of Eqs. (14), (15) and (16), the expression to calculate the capacitance of a comb drive with respect to temperature rising is derived as follow:

$$C = 2N\frac{\varepsilon_0 B}{g_0}(d_0 - \frac{3nb_1b_2h_1h_2(h_1+h_2)(\alpha_1-\alpha_2)\Delta T}{b_1{}^2h_1{}^4+n^2b_2{}^2h_2{}^4+nb_1b_2h_1h_2(6h_1h_2+4h_1{}^2+4h_2{}^2)}l^2) \tag{17}$$

4. Calculated Results

To have more sensitivity and large deflection, two materials with high difference in their thermal expansion coefficients are considered (Gold and Silicon), which also Wen-Hwa Chut et al used in their work [8]. The geometrical and material properties are listed in Table 1 as:

Table 1. *Geometrical and material properties of the bimetallic cantilever thermometer.*

Design variable	value
α_1 (Si)	$2.6\times10^{-6}\ k^{-1}$
α_2 (Au)	$14.3\times10^{-6}\ k^{-1}$
E_1	$122\times10^9\,Nm^{-2}$
E_2	$80\times10^9\,Nm^{-2}$
b_1	$100\mu m$
b_2	$80\mu m$
h_1	$4\mu m$
h_2	$1.8\mu m$
d_0	$10\mu m$
g_0	$2.5\mu m$
l	$500\mu m$
B	$15\mu m$
N	14
ε_0	$8.854\times10^{-12}\,\frac{C^2}{N.m^2}$

To demonstrate the feasibility of the proposed model and obtained results, it is tried to compare the calculated results of microbeam deflection with the results predicted of Ref. [8].

Table 2. *Comparison of calculated tip deflection of proposed model with results of Ref. [8] for a bimetallic cantilever microbeam.*

Temperature(c^0)	Tip deflection in this work(μm)	Tip deflection value [8] (μm)	$\Delta\%$
10	2.6970	2.6971	0.004
30	8.0911	8.0913	0.003
50	13.4852	13.4855	0.002
70	18.8793	18.8797	0.002
100	26.9704	26.9710	0.002

where:

$$\Delta(\%) = \frac{ABS\left(Obtained \ \mathrm{Re}\, sults - results \ of \ \mathrm{Re}\, f \ .[8]\right)}{results \ of \ \mathrm{Re}\, f \ .[8]} \times 100 \qquad (18)$$

Due to the Eq. (14), as the temperature increases, the tip deflection of the micro beam increases too. Thus, the variable height of comb drive plates changes, which causes the capacitance of the system changes. The changed capacitance value can be used to evaluate the temperature rising of the desire environment. Figure 4 shows the value of the tip deflection of bimetallic cantilever at the different temperature rising.

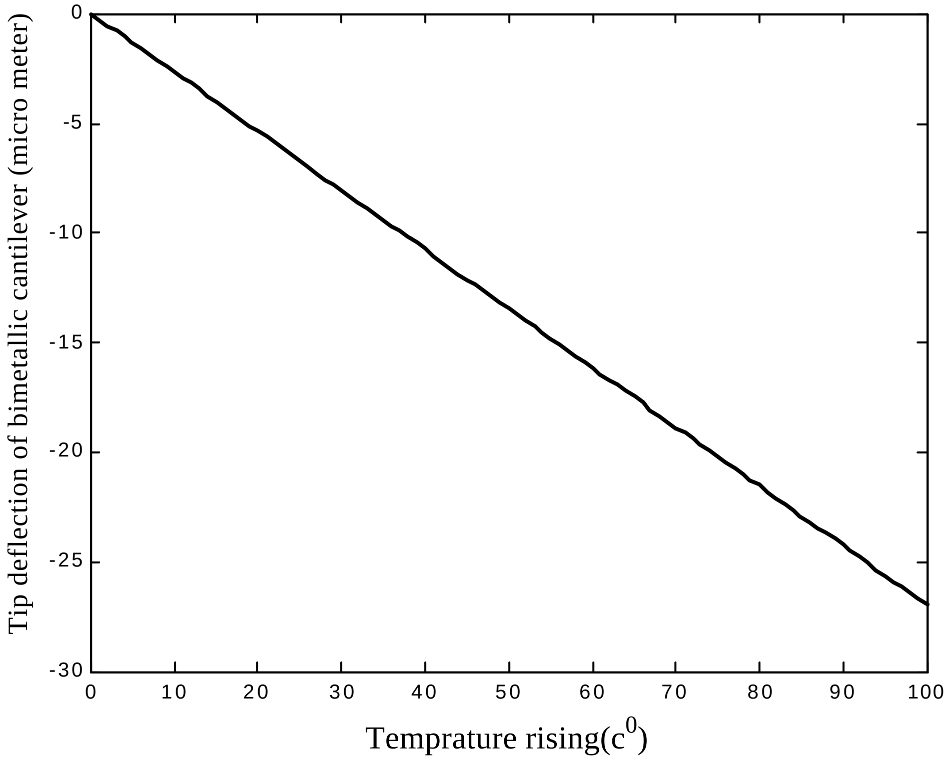

Fig. 4. Tip deflection versus the temperature raising.

Using of Eq. (17), the change of capacitance in the proposed model due to the variation in the temperature can be easily found. Fig. 5 shows the value of the capacitance due to the increasing of the temperature.

As it is shown in Fig.5 and Eq. (17), the value of the capacitance changes linearly due to the increasing of the temperature. Thus a linear formula can be used to express the capacitance value with respect to the value of the temperature rising.

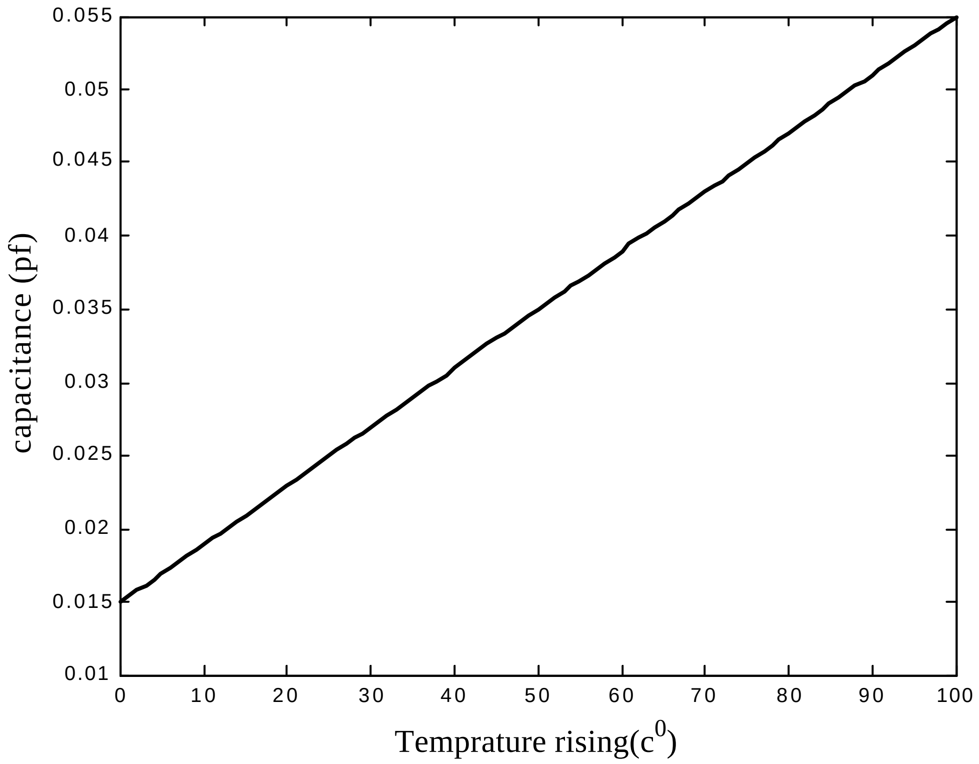

Fig. 5. Capacitance versus the temperature rising.

5. Conclusions

In this work, a new model of micro thermometer used to measure the temperature rising due to the variation of a comb drive capacitance. The governing thermo-mechanical equation of the model was derived and solved analytically. The obtained results were in good agreement with the other existing results. The calculated results showed that by increasing the temperature, the capacitance of the comb drive increasing linearly. Bimetallic microbeams can provide sensitive structure for the fabrication of temperature sensors that cover the temperature in wide range based on these structures.

References

[1]. J. Pekola, Trend in thermometry, Journal of low temperature physics, Vol.135, pp. 723-744, 2004.
[2]. O.V. Lounasmaa, Experimental principles and methods below 1k, *Academic Press*, London, 1974.
[3]. F. Pobell, Mater and method at low temperatures, 2nd editions, Springer, Berlin, 1996.
[4]. T.j. Quinn, Temperature, *Academic Press*, London, 1983.
[5]. L. Spietz, K.W. Lehnert, I. Siddiqi, R.J. Schoelkopf, Primary electronic thermometry using the shot noise of a tunnel junction, *J. Science*, Vol. 300, pp.1929-32, 2003.
[6]. N.S. Boltovets, V. K. Dugaev, V. V. Kholevchuk, P. C. McDonald, V. F. Mitin, I.Yu. Nemish, F. Pavese, I. Peroni, P.V. Sorokin, E. A. Soloviev, E. F. Venger, New generation of resistance thermometers based on Ge films on GaAs substrates, *J. Measurement and Control in Science and Industry*, Vol. 7, pp. 399 – 404, 2003.
[7]. S. Timoshenko, Strengthofmaterial, Part1, New York: Van Nostrand, 1930.
[8]. Wh. Chut, M. Mehregany, R.L. Mullen, Analysis of tip deflection and force of a bimetallic cantilever microactuator, J. *Micromech. Microeng*, Vol.3, pp. 4-7, 1993.