On Kalman Information Fusion for Multiple Wireless Sensors Networks Systems with Multiplicative Noise

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Abstract: Kalman information fusion filter is dealt with for WSNs (Wireless sensor networks), where multiplicative noises occur in the system model. There are $l > 2$ sub-systems, and for each sub-system, there are $m > 2$ observations that are sent through different wireless channel. Packet loss is inevitable for the wireless channel, and the optimal information fusion filter is given by the projection theorem and the scalar weighting criterion. Numerical example shows that the proposed approach is efficient.

Keywords: Information fusion, Wireless sensor networks, Packet loss, Multiplicative noises.

1. Introduction

The standard Kalman filter is usually calculated by Ricatti equations and the used measurements are supplied by one sensor. However, the measurements from one sensor may be error owing to different reasons, while it isn’t detected in time, so Kalman filter may be wrong from one sensor [1-2]. Different from one (or single) sensor, multi sensor information fusion problem [3, 4] can overcome the above wrong case, and has attracted much attention recent years [4–6].

The above optimal multi-sensor information fusion problem has been dealt with well for normal systems [5, 7, 8]. Particularly, [5] proposes information fusion Kalman filter fast algorithms, and the optimal information fusion Kalman filter is given under the LMSE sense in [8]. However, the above references assume that the measurements can be used, which may be impractical in application, for example, for wireless channel, the above standard information fusion result can’t be used any more. Recently, Kalman filter for wireless sensor networks systems has attracted much attention [9, 10, 16]. [9] is an important result where two bounds of the steady error covariance are given for wireless sensor systems with the loss of measurements. In [10], the observation is partitioned into two parts where each part can be lost independently. The above references only consider the case of systems with additive noises, for the case of systems with multiplicative noises, the above methods can’t be used. Moreover, they can’t be used for the information fusion case, for WSNs with both additive noises and multiplicative noises, only [15] has considered the case of $l > 2, m = 2$.

In the paper, the general case for the information fusion for multiple WSNs with multiplicative and additive noises will be given, which extends the result of [15], i.e., there are $l > 2$ sub-systems, and for each sub-system, there are $m > 2$ observations. The information fusion filter also extends our previous works [11, 13, 14] where multiplicative noises are considered. The proposed numerical example shows that the proposed approach is efficient.
2. Problem Statements

Consider the WSN systems

\[ x_i(t+1) = A_i x_i(t) + B_i x_i(t) w_i(t) + u_i(t), \]  
\[ y_i(t) = C_i x_i(t) + D_i y_i(t) w_i(t) + v_i(t), \]  
\[ \bar{y}_i(t) = y_i(t), \]  
\[ \bar{y}_o(t) = \bar{y}_i(t), \]

where \( I, m > 2 \). \( x_i(t) \in R^n \), \( y_i(t) \in R^n \), \( \bar{y}_i(t) \) are state, measurements for origin and via WSNs. \( u_i(t) \in R^p \), \( w_i(t) \in R^q \), \( v_i(t) \in R^i \) are input and noises. \( x_i(t) \rightarrow x_i(t) \), \( \{y_i(t), v_q(s)\} = R_{ij} \delta_{ij} \delta_{l}, \)

\[ E \left( \begin{bmatrix} u_i(t) \\ \vdots \\ u_i(s) \end{bmatrix} \right) = \begin{bmatrix} Q_1 & \cdots & Q_l \\ \vdots & \ddots & \vdots \\ Q_1 & \cdots & Q_l \end{bmatrix}, \]

\[ \{w_i(t), y_i(t)\} = M_{ij} \delta_{ij}, \] and \( \{x_i, x_j\} = \Pi_{ij} \delta_{ij} \). More assumptions can be referred to [15].

The information fusion Kalman filter problem for multiple wireless sensors networks systems with multiplicative and additive noises can be given as follows,

**P:** For the known observations \( \{\bar{y}_0(t), \cdots, \bar{y}_o(t)\} \), and scalars \( \{\gamma_0(0), \cdots, \gamma_o(t)\} \), seek a Kalman filter \( \hat{x}_o(t | t) \) of \( x(t) \) satisfying the indexes below:

i) Unbiasedness, i.e., \( E\hat{x}_o(t | t) = Ex(t) \).

ii) Optimality, i.e., to seek the matrix weights \( c_i(t), \cdots, c_l(t) \), such that

\[ \hat{x}_o(t | t) = \sum_{i=0}^{l} c_i(t) \hat{x}_i(t | t), \]

wherein \( \hat{x}_i(t | t), \cdots, \hat{x}_l(t | t) \) denote \( l \) unbiasedness filter of \( x(t) \), in order to minimize the trace of \( P(t | t) \), i.e., \( tr\{P_o(t | t)\} = min\{trP(t | t)\} \).

3. Main Results

The main results for multiple wireless sensors networks systems with multiplicative and additive noises will be presented in the section.

3.1. Scalar Weighting Criterion

**Lemma 1:** [7, 12, 15] Denote 
\[ e_i(t | t) = x(t) - \hat{x}_i(t | t) \]. The covariance matrix is \( q_i(t | t) \) and the cross-covariance matrix is \( g_{ij}(t | t) (i,j=1,2,\cdots,l) \). Then the information fusion Kalman filter \( \hat{x}_o(t | t) \) is

\[ \hat{x}_o(t | t) = \sum_{i=0}^{l} c_i(t) \hat{x}_i(t | t), \]

where

\[ \begin{bmatrix} c_1(t) \\ c_2(t) \\ \cdots \\ c_l(t) \end{bmatrix} = \left( e^T trP^{-1}(t | t) e \right)^{-1} e^T trP^{-1}(t | t), \]

\[ P(t | t) = \begin{bmatrix} P_1(t | t) \\ \vdots \\ P_l(t | t) \end{bmatrix}, \]

\[ e^T = [1 \cdots 1]. \]

The error covariance matrix is

\[ P_o(t | t) = \sum_{i,j=0}^{l} c_i(t)c_j(t)P_{ij}(t | t), \]

and \( trP(t | t) \leq trP_{ij}(t | t) (i,j=1,2,\cdots,l) \). For the convenience of discussion, \( P_i(t) \) is replaced by \( P_i(t) \).

3.2. Some Important Cases

Some cases of Kalman filter for each fixed-\( i \) subsystem will be given in this part, we can use the similar definitions and denotations as in [15]. For the system (1)-(3), when \( i \) is fixed, there are \( 2^n \) cases since \( \gamma_i(0) \) can be equal to 0 or 1 for every \( j \).

When no packet loss occurs, we have the following.

**Lemma 2:** Consider the state-space description (1)-(3) for fixed-\( i \), if \( \gamma_i(0) = \cdots = \gamma_o(t) = 1 \), and the system will be the standard Kalman filter as,

\[ \hat{x}_i(t + 1) = A_i \hat{x}_i(t) + K_{i,\cdots,\infty}(t)\bar{y}_{i,\cdots,\infty}(t), \]

\[ P_i(t + 1) = \left[ A_i - \gamma_{i,\cdots,\infty}(t)K_{i,\cdots,\infty}(t)C_{i,\cdots,\infty} \right] P_i(t), \]

where

\[ \bar{Y}_{i,\cdots,\infty}(t) \]

\[ \bar{y}_{i,\cdots,\infty}(t) \]

\[ \bar{y}_{1,\cdots,\infty}(t) \]

\[ \bar{y}_{i,\cdots,\infty}(t) \]

\[ \bar{y}_{i,\cdots,\infty}(t) \]

\[ \bar{y}_{i,\cdots,\infty}(t) \]
\[
\gamma_{i_{(m-1)}}(t) \triangleq \begin{bmatrix}
\gamma_i(t) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \gamma_m(t)
\end{bmatrix}, \quad \gamma_{i_{(m-1)}}(t) \triangleq \begin{bmatrix}
\gamma_i(t) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \gamma_{i(m-1)}(t)
\end{bmatrix},
\]

\[
C_{i_{(m-1)}} \triangleq \begin{bmatrix}
C_{i_1} \\
\vdots \\
C_{i_m}
\end{bmatrix}, \quad D_{i_{(m-1)}} \triangleq \begin{bmatrix}
D_{i_1} \\
\vdots \\
D_{i_m}
\end{bmatrix},
\]

\[
v_{i_{(m-1)}}(t) \triangleq \begin{bmatrix}
v_{i_1}(t) \\
\vdots \\
v_{i_m}(t)
\end{bmatrix},
\]

\[
R_{i_{(m-1)}} \triangleq \begin{bmatrix}
R_{i_1}(t) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & R_{i_m}(t)
\end{bmatrix},
\]

with
\[
K_{i_{(m-1)}}(t) \triangleq \left[ A_P(t)C_{i_{(m-1)}} + B_tM_i(t)D_{i_{(m-1)}} \right] \times \left[ C_{i_{(m-1)}}P(t)C_{i_{(m-1)}}^T + D_{i_{(m-1)}}^T \right]^{-1} \times \Pi(t)M_i(t)D_{i_{(m-1)}}^T + R_{i_{(m-1)}}^{-1}.
\]

For the case of \( \gamma_i(t) = \cdots = \gamma_{i_{(m-1)}}(t) = 0, \gamma_{i_{m}}(t) = 1 \), we have the following.

**Lemma 3:** Consider state-space description (1)-(3) for fixed-i, if \( \gamma_i(t) = \cdots = \gamma_{i_{(m-1)}}(t) = 1, \gamma_{i_{m}}(t) = 0 \), i.e., only the \( m \)th measurement is lost at time \( t \) for the \( i \)th sub-system, Kalman filter can be given as,

\[
\hat{x}_i(t+1) = A_i\hat{x}_i(t) + K_{i_{(m-1)}}(t)[\bar{y}_{i_{(m-1)}}(t) - \gamma_i(t)C_{i_{(m-1)}}\hat{x}_i(t)],
\]

\[
\hat{x}_i(t+1) = A_i\hat{x}_i(t) + K_{i_{(m-1)}}(t)[\bar{y}_{i_{(m-1)}}(t) - \gamma_i(t)C_{i_{(m-1)}}\hat{x}_i(t)],
\]

\[
P_i(t+1) = [A_i - \gamma_i(t)K_{i_{(m-1)}}(t)C_{i_{(m-1)}}]P(t)\times [A_i - \gamma_i(t)K_{i_{(m-1)}}(t)C_{i_{(m-1)}}]^T + Q_i + \gamma_i(t)K_{i_{(m-1)}}(t)R_{i_{(m-1)}}K_{i_{(m-1)}}^T,
\]

where

\[
\bar{y}_{i_{(m-1)}}(t) \triangleq \begin{bmatrix}
\bar{y}_{i_1}(t) \\
\vdots \\
\bar{y}_{i_{(m-1)}}(t)
\end{bmatrix},
\]

For the case of \( \gamma_i(t) = \cdots = \gamma_{i_{(m-1)}}(t) = 0, \gamma_{i_{m}}(t) = 1 \), we will give the following lemma

**Lemma 5:** [15] considers the state-space description (1)-(3), if \( \gamma_i(t) = \cdots = \gamma_{i_{m}}(t) = 0 \), the Kalman filter can be given as follows since the open loop,
\[ \hat{x}_i(t + 1) = A \hat{x}_i(t), \]  
\[ P_i(t + 1) = AP_i(t)A^T_i + B_i \Pi_i(t)B_i^T_i + Q, \]  
\[ \Pi_i(t + 1) = A_i \Pi_i(t)A_i^T_i + B_i \Pi_i(t)B_i^T_i + Q, \]

For the general case that some measurement are lost, for example, the second measurement and the \( m \)-th measurement are available and the other measurement (the first, the third to the \( (m-1) \)-th measurement) are lost, we have the following.

**Lemma 6:** Consider the state-space description (1)-(3) for fixed-\( i \), if \( y_{i1}(t) = y_{i2}(t) = \ldots = y_{i(m-1)}(t) = 0 \), i.e., only the second measurement and the \( m \)-th measurement are available at time \( t \) for the \( i \)-th sub-system, and Kalman filter can be given as,

\[ \hat{x}_i(t + 1) = A \hat{x}_i(t) + K_{2,m}(t)\tilde{y}_{2,m}(t) \]
\[ P_i(t + 1) = [A_i - \gamma_{2,m}(t)K_{2,m}(t)C_{2,m}(t)]P_i(t) \]
\[ \times [A_i - \gamma_{2,m}(t)K_{2,m}(t)C_{2,m}(t)]^T + B_i - \gamma_{2,m}(t)K_{2,m}(t)D_{2,m}(t)\Pi_i(t) \]
\[ + \gamma_{2,m}(t)K_{2,m}(t)R_{2,m}(t) + Q_i, \]

where

\[ \tilde{y}_{2,m}(t) = \begin{bmatrix} y_{i2}(t) \\ y_{i,m}(t) \end{bmatrix}, \]
\[ \gamma_{2,m}(t) = \begin{bmatrix} \gamma_{i2}(t) & 0 \\ 0 & \gamma_{i,m}(t) \end{bmatrix}, \]
\[ C_{2,m}(t) = \begin{bmatrix} C_{i2} \\ C_{i,m} \end{bmatrix}, \]
\[ D_{2,m}(t) = \begin{bmatrix} D_{i2} \\ D_{i,m} \end{bmatrix}, \]
\[ \hat{y}_{i,m}(t) = \begin{bmatrix} \hat{y}_{i2}(t) \\ \hat{y}_{i,m}(t) \end{bmatrix}, \]
\[ R_{2,m}(t) = \begin{bmatrix} R_{i2}(t) \\ R_{i,m}(t) \end{bmatrix}, \]

with

\[ K_{2,m}(t) = \begin{bmatrix} AP_i(t)C_{i2}^T + B_iM_i \Pi_i(t)D_{i2}^T \\ C_{i2}P_i(t)C_{i2}^T + D_{i2} \end{bmatrix}, \]

\[ D_{2,m}(t) = \begin{bmatrix} D_{i2} \\ D_{i,m} \end{bmatrix}, \]

3.3. Kalman Filter for Sub-system Equations

Five important cases have been given in the above subsection, and we can give the main theorem for WSN,

**Theorem 1:** Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], one-step predictor for \( i \)-th sub-system is

\[ \hat{x}_i(t + 1) = A \hat{x}_i(t) + \gamma_i(t)\gamma_i(t)\gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i2}(t) - \gamma_i(t)C_{i2}(t)\hat{x}_i(t)) \]
\[ + \gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i,m}(t) - \gamma_i(t)C_{i,m}(t)\hat{x}_i(t)) \]
\[ - \gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i2}(t) - \gamma_i(t)C_{i2}(t)\hat{x}_i(t)) \]
\[ + \ldots \]
\[ + (1 - \gamma_i(t))\gamma_i(t)(1 - \gamma_i(t)) \]
\[ \times (1 - \gamma_i(t))\gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t) \gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i2}(t) - \gamma_i(t)C_{i2}(t)\hat{x}_i(t)) \]
\[ + \gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i,m}(t) - \gamma_i(t)C_{i,m}(t)\hat{x}_i(t)) \]
\[ + \ldots \]
\[ + (1 - \gamma_i(t))\gamma_i(t)(1 - \gamma_i(t)) \]
\[ \times (1 - \gamma_i(t))\gamma_i(t) \]
\[ \times (\hat{y}_{i2}(t) - \gamma_i(t)C_{i2}(t)\hat{x}_i(t)) \]
\[ + \gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)\gamma_i(t)K_{i,i}(t) \]
\[ \times (\hat{y}_{i,m}(t) - \gamma_i(t)C_{i,m}(t)\hat{x}_i(t)) \]

where \( \Pi_i(t + 1) \) is as in (11).
Remark 1: It can be easily seen from (39) and (40) that, there are $2^n$ parts in the structures of $\hat{x}(t+1)$ and $P(t+1)$, which shows the $2^n$ cases of Kalman predictors.

Furthermore, we can give the filter formula as in the following theorem

**Theorem 2:** Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], the Kalman filter is

$$\hat{x}(t+1 | t+1) = \hat{x}(t+1) + \sum_{i=1}^{2^n} \bar{y}_i(t+1) \beta_i(t) + \sum_{i=1}^{2^n} \bar{y}_i(t+1) \beta_i(t)$$

and $\bar{y}_i(t+1)$ can be given similarly as follows:

$$\bar{y}_i(t+1) = \begin{cases} P(t)C_{\hat{x},i}(t+1)D_{\hat{x},i}^T + R_{\hat{x},i} \end{cases}$$

where $P(t+1)$ is

$$P(t+1) = P(t) + \sum_{i=1}^{2^n} \bar{y}_i(t+1)$$

and $F(Q)$ can be given as follows:

$$F(Q) = \begin{cases} P(t)C_{\hat{x},i}(t+1)D_{\hat{x},i}^T + R_{\hat{x},i} \end{cases}$$

with the other variables can be referred in the above.

Proof: For $P_y(t+1)$, there are $2^n$ cases in sum

$$P_y(t+1) = \sum_{i=1}^{2^n} P_y(t) + \sum_{i=1}^{2^n} \bar{y}_i(t+1)$$

where $P_y(t+1)$ is

$$P_y(t+1) = F(Q_y(t)) + \gamma(t)|\gamma(t)$$

and $F(Q_y(t))$ can be given as follows:

$$F(Q_y(t)) = \begin{cases} P(t)C_{\hat{x},i}(t+1)D_{\hat{x},i}^T + R_{\hat{x},i} \end{cases}$$

with the other variables can be referred in the above.

3.4. Computation of $P_y(t+1)$

The cross covariance matrix $P_y(t+1)(t \neq j)$ will be given here, since $P_y(t+1)(t+1) = 0$, so $P_y(t+1)(t+1)$ only needs to be given.

**Theorem 3:** Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], the cross covariance matrix $P_y(t+1)$ can be given as

$$P_y(t+1) = \sum_{i=1}^{2^n} P(t)C_{\hat{x},i}(t+1)D_{\hat{x},i}^T$$

and $F(Q_y(t))$ can be given as follows:

$$F(Q_y(t)) = \begin{cases} P(t)C_{\hat{x},i}(t+1)D_{\hat{x},i}^T + R_{\hat{x},i} \end{cases}$$

with the other variables can be referred in the above.
For $e_j(t+1)$ of the j-th sub-system, there are also $2^n$ cases:

\[ (j-1)\gamma_j(t) = \cdots = \gamma_{jm}(t) = 1 \]
\[ e_j(t+1) = [A_j - \gamma_{j1} \cdots \gamma_{jm}(t)K_{j1 \cdots jm}(t)C_{j1 \cdots jm}]e_j(t) \]
\[ (j-1)\gamma_j(t) = \cdots = \gamma_{jm}(t) = 1 \]
\[ e_j(t+1) = [A_j - \gamma_{j1} \cdots \gamma_{jm}(t)K_{j1 \cdots jm}(t)C_{j1 \cdots jm}]e_j(t) \]

For the last case of $\gamma_{jm}(t) = 0$, it can be given as

\[ ej(t+1) = Aje_j(t)+Bjx_j(t)wj(t)+uj(t). \]  

Thus, from the definition of $P_{\varphi}(t)$, there are $2^{2m}$ cases as

\[ (ij-1)\gamma_i(t) = \cdots = \gamma_{im}(t) = 1, \quad \gamma_j(t) = \cdots = \gamma_{jm}(t) = 1 \]
\[ P_{\varphi}(t+1) = [A_i - \gamma_{i1} \cdots \gamma_{im}(t)K_{i1 \cdots im}(t)C_{i1 \cdots im}(t)]P_{\varphi}(t) \]
\[ \times [A_j - \gamma_{j1} \cdots \gamma_{jm}(t)K_{j1 \cdots jm}(t)C_{j1 \cdots jm}(t)]^T + Q_i(t). \]

\[ (ij-2^m) \gamma_i(t) = \cdots = \gamma_{im}(t) = 0, \quad \gamma_J(t) = \cdots = \gamma_{jm}(t) = 0 \]
\[ P_{\varphi}(t+1) = A_i P_{\varphi}(t)A_j^T + Q_i(t). \]  

The proof of $P_{\varphi}(t)$ is over.

For $P_{\varphi}(t+1)|t+1)$, also $2^{2m}$ cases are dealt with, for $e_j(t+1|t+1)$ when

\[ \gamma_{ij}(t) = \cdots = \gamma_{im}(t) = 1, \gamma_{j1}(t) = \cdots = \gamma_{jm}(t) = 1 \]

it can be given as

\[ P_{\varphi}(t+1|t+1) = E[e_j(t+1|t+1)e_j^T(t+1|t+1)] \]
\[ = [I - \Sigma_{i1 \cdots im}(t+1)C_{i1 \cdots im}]P_{\varphi}(t+1) \]
\[ \times [I - \Sigma_{j1 \cdots jm}(t+1)C_{j1 \cdots jm}]^T. \]  

For the last case of $\gamma_{ij}(t) = \cdots = \gamma_{im}(t) = 0, \gamma_{j1}(t) = \cdots = \gamma_{jm}(t) = 0$, and for $e_j(t+1|t+1)$, it can be given as

\[ P_{\varphi}(t+1|t+1) = E[e_j(t+1|t+1)e_j^T(t+1|t+1)] \]
\[ = P_{\varphi}(t+1), \]

The other parts are omitted, thus (45) can be given.

3.5. Kalman Information Fusion Filter

The main result which extends the case of [15] will be given in form of theorem in the paper

**Theorem 4:** For the state-space description (1)-(3) with $\gamma_{ij}(0), \ldots, \gamma_{ij}(t), (i = 1, \cdots, l, j = 1, \cdots, m)$, the similar corresponding assumptions and definitions in [15], the information fusion Kalman filter is given by (5), where $\hat{x}_i(t|t)$ is calculated in (39), $P_{\varphi}(t|t)$ is calculated in (40), and $P_{\varphi}(t|t)$ is calculated in (45).

4. Numerical Example

In the section, we will give an example to show the efficiency of the presented results. For the simplicity of discussion, let $l = m = 2$, then the optimal information fusion filter is

\[ \hat{x}_i(t|t) = c_1(t)\hat{x}_1(t|t) + c_2(t)\hat{x}_2(t|t) \]

where

\[ c_1(t) = \frac{trP_{\varphi}(t|t) - trP_{\varphi}(t|t)}{trP_{\varphi}(t|t) - trP_{\varphi}(t|t) + trP_{\varphi}(t|t)}, \]
\[ c_2(t) = \frac{trP_{\varphi}(t|t) - trP_{\varphi}(t|t)}{trP_{\varphi}(t|t) - trP_{\varphi}(t|t) + trP_{\varphi}(t|t)} \]

and the corresponding error covariance matrix is

\[ P_{\varphi}(t|t) = c_1^2(t)P_{\varphi}(t|t) + c_2^2(t)P_{\varphi}(t|t) + c_3(t)c_2(t)P_{\varphi}(t|t) \]

and $trP_{\varphi}(t|t) \leq trP_{\varphi}(t|t)$. Consider the system (1)-(3) with $N = 100$, and

\[ A = \begin{bmatrix} 0.4 & 0.1 \\ 0 & 0.3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.4 & 0.1 \\ 0 & 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 & 0.8 \\ 0 & 0.6 \end{bmatrix}, \]
\[ C_{i1} = [0.5 \ 0.1], \quad C_{i2} = [1 \ 0.5], \]
\[ C_{j1} = [0.5 \ 0.1], \quad C_{j2} = [2 \ 0.3], \]
\[ D_{1} = [0.5 \ 0.2], \quad D_{2} = [2 \ 0.1], \]
\[ D_{3} = [0.5 \ 0.2], \quad D_{4} = [0.1 \ 0.3]. \]

The initial state value $x_i(0) = x_j(0) = [1 \ 0.5]$, and noises $u_i(t)$ and $w_i(t)$ are white noises with zero means and unity covariance matrices $Q = Q_\varphi = 1, M_j = 1(i,j = 1, 2)$. Observation noise $v_i(t)(i,j = 1,2)$ is of zero means and with covariance matrix $R = 0.01$.

We simulate four cases for fixed i-subsystem of

1) $\gamma_{i1} = \gamma_{i2} = 0,$
2) $\gamma_{i1} = 1, \gamma_{i2} = 0,$
3) $\gamma_{i1} = 0, \gamma_{i2} = 1,$
4) $\gamma_{i1} = \gamma_{i2} = 1,$
Since the simulated example has 16 cases, it’s complicate to give all cases, so we only give the filters $\hat{x}(t \mid t)$ of the signal $x(t)$ based on observation $\{\hat{y}_i(s)\}_{i=0}$. The following four figures show the origin and its estimator under the case (4).

The tracking performance of Kalman filter $\begin{bmatrix} \hat{x}_1(t \mid t) \\ \hat{x}_2(t \mid t) \end{bmatrix}$ is drawn in Fig. 1 and Fig. 2 when $\gamma_1 = 1, \gamma_2 = 0$, and they are also given in Fig. 3 and Fig. 4 when $\gamma_1 = 1, \gamma_2 = 1$. Seen from the above figures, the filter are good, and the estimate performance when $\gamma_1 = 1, \gamma_2 = 1$ is better than them when $\gamma_1 = 1, \gamma_2 = 0$, since the available measurements are more.

5. Conclusions

Information fusion Kalman filter for multiple WSNs with multiplicative noises has been given. The filter is given by the scalar weighting criterion and the projection theorem. The proposed Kalman filtering formulae for each sub-system can obviously show the structure of results. The given examples show the efficiency of the proposed approach. In addition, the proposed approach can be used to deal with many practice problems, for example Intelligent Body Sensor Network [17].

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References


[7]. S. Sun, P. Cui, Multi-sensor optimal information fusion steady-state Kalman filter weighted by scalars, Control and Decision, 19, 2, 2004, pp. 208–211.

[8]. S. Sun, Multi-sensor information fusion white noise filter weighted by scalars based on Kalman predictor, Automatica, 40, 8, 2004, pp. 1447-1453.


