

## Kinematics Analysis and Simulation on Transfer Robot with Six Degrees of Freedom

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**Abstract:** Study focuses on transfer robot with Six Degrees of Freedom, establishing kinematic equation by D-H method, analyzing forward kinematics and obtaining inverse kinematics by using method of inverse transform. Based on vector product, it develops velocity Jacobian matrix of robot. The geometric model of robot virtual prototype is established by SolidWorks software and generates parameters such as mass and moment. Kinematic simulation for robot is performed by Mathematica software and develops curve graph of displacement, velocity and accelerated speed in x, y and z direction in end executor center of robot with measurement, analysis and assessment, which provides foundation for further kinematics analysis and structure optimization as well as motion control of robot. Copyright © 2014 IFSA Publishing, S. L.

**Keywords:** Transfer robot, Kinematics, Simulation, Optimization, SolidWorks, Mathematica.

### 1. Preface

In recent years, with deep development of technical research on robot and wider study fields, kinematic simulation analysis of robot plays more important role in spatial planning, trajectory control, motion control, off-line programming, optimization design and other aspects. Robot kinematic studies kinematic relation between each joint and rigid body, position of parts involved and relation of velocity and accelerating speed with time. Robot kinematics includes: one is direct kinematics where joint variable is given and calculate hand appearance, the other is inverse kinematics where hand appearance is given and calculate joint variable.

Transfer robot is an advanced automatic device which could increase productivity and improve working condition.

### 2. Robot Kinematics Model

Articulated robot with 6-DOF has very complicated mechanical structure, consisting of base, waist joints, upper arm, forearm, wrist joint and hands, every motion unit of which is composed of a small mechanical system, such as shaft, bearing, bushing, key, gear, motor as well as reducer. As shown in Fig. 1.

Motion arm of 6-DOF consists of 6 arms and 6 joints. Each arm is described with 4 parameters of  $a_{i-1}$ ,  $\alpha_{i-1}$ ,  $d_i$ ,  $\theta_i$ , where  $a_{i-1}$  and  $\alpha_{i-1}$  means feature of arm i-1;  $d_i$  and  $\theta_i$  means relation of arm i-1 and arm i. For rotary joint, joint angle is joint variable and other parameters are constant; for prismatic joint, polarization is joint variable and other parameters are constant. This method to describe

mechanism kinematics was brought up by Denavit and Hartenberg in 1955, called D-H method [1, 2].

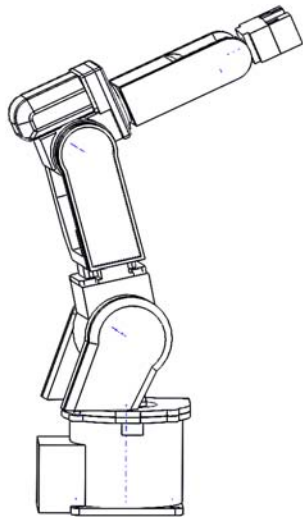


Fig. 1. Transfer robot model in six degrees of freedom.

Establish link coordinate system by D-H method, as shown in Fig. 2. Kinematics and structure parameters of related arms are shown in Table 1.

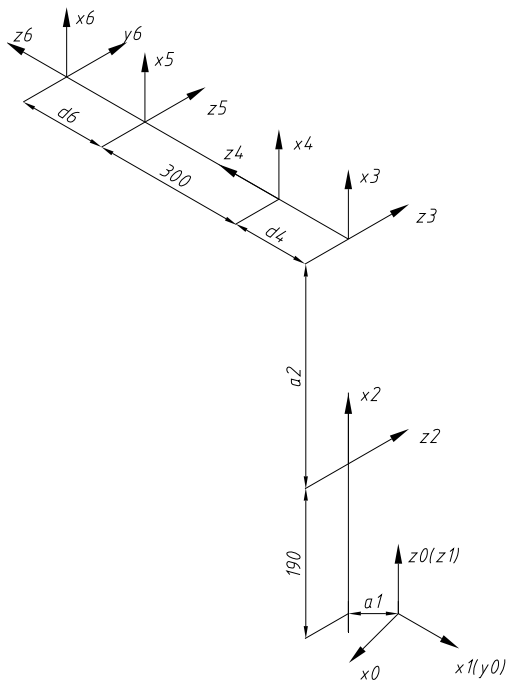


Fig. 2. Joint arm coordinate system.

Where  $a_i$  means distance from  $z_i$  to  $z_{i+1}$  along  $x_i$ ;  $\alpha_i$  means angle from  $z_i$  to  $z_{i+1}$  around  $x_i$ ;  $d_i$  means distance from  $x_{i-1}$  to  $x_i$  along  $z_i$ ;  $\theta_i$  means angle from  $x_{i-1}$  to  $x_i$  around  $z_i$  [3].

Table 1. D-H parameters.

| Joint $i$ | $a_{i-1}$ | $\alpha_{i-1}$ | $d_i$     | $\theta_i$           |
|-----------|-----------|----------------|-----------|----------------------|
| 1         | 0         | 0              | 0         | $\theta_1=90^\circ$  |
| 2         | $a_1=100$ | $-90^\circ$    | 0         | $\theta_2=-90^\circ$ |
| 3         | $a_2=450$ | 0              | 0         | $\theta_3=0$         |
| 4         | 0         | $+90^\circ$    | $d_4=160$ | $\theta_4=0$         |
| 5         | 0         | $-90^\circ$    | 0         | $\theta_5=0$         |
| 6         | 0         | $+90^\circ$    | $d_6=100$ | $\theta_6=0$         |

### 2.1. Direct Kinematics

Refer to link parameter to make sure arm transformation matrix expression in accordance with the "left to right" principle:

$${}_{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

Put parameters in Fig. 1 into transmission matrix (1), to calculate each arm homogeneous transformation matrix  ${}^0T_1, {}^1T_2, {}^2T_3, {}^3T_4, {}^4T_5, {}^5T_6$ . Multiply above matrixes to get robot end component homogeneous transformation matrix:

$${}^0T_6 = {}^0T_1({}^1T_2)({}^2T_3)({}^3T_4)({}^4T_5)({}^5T_6), \quad (2)$$

Put parameters in Fig. 1 into equation (1) and (2) to get position matrix  ${}^0T_6$  of robot end relative to basic coordinate system:

$${}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

Equation (3) means position relationship of end arm coordinate system {6} relative to basic coordinate system {0}, which is the foundation of each joint kinematics analysis of robots.

Initial position:  $\theta_1 = 90^\circ$ ;  $\theta_2 = -90^\circ$ ;  $\theta_3 = 0$ ;  $\theta_4 = 0$ ;  $\theta_5 = 0$ ;  $\theta_6 = 0$ .

Put initial value of  $\theta_i$  into equation (3) to get:

$${}^0T_6 = \begin{bmatrix} 0 & 0 & 0 & 660 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 640 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It accords with the position in Fig. 2, which proves above calculation process is correct.

By above analysis, establish kinematics equation of relation between transfer robot end actuator position and other joint variable in rectangular coordinate system, which gets forward solution of robot kinematics equation [4].

## 2.2. Reverse Kinematics

Reverse kinematics is to get joint variable by hand position, which is used to make sure motor parameters to drive joint and plan trajectory [5].

Common reverse kinematics methods include counter transformation method, geometric method and Pieper method and so on. The paper applies for counter transformation method to solve robot reverse kinematics, that is, to multiply a reverse matrix on the left of 6 joint transformation matrixes and calculate to find the invariable element in the right, and then make these variables equal with those in the left to get a trigonometric function. According to position of vectorial combined with robot arm parameter  $d$  to get joint variables  $\theta_1, \theta_2, \dots, \theta_6$ .

Inverse transformation  ${}^0T^{-1}(\theta_1)$  left multiply function (2) on left and right,

$${}^0T^{-1}(\theta_1) {}^0T = {}^1T(\theta_2) {}^2T(\theta_3) {}^3T(\theta_4) {}^4T(\theta_5) {}^5T(\theta_6), \quad (4)$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1T$$

Given function is equal on right and left sides to get  $\theta_1, \theta_3$ , there are two possibilities for each variable.

Inverse transformation  ${}^0T^{-1}$  left multiply function (2) on left and right,

$${}^0T^{-1}(\theta_1, \theta_2, \theta_3) {}^0T = {}^3T {}^4T {}^5T {}^6T, \quad (5)$$

Given function is equal on right and left sides to get  $\theta_2, \theta_4$ , there are two possibilities for each variable.

Inverse transformation  ${}^0T^{-1}$  left multiply function (2) on left and right,

$${}^0T^{-1}(\theta_1, \theta_2, \theta_3, \theta_4) {}^0T = {}^4T {}^5T {}^6T, \quad (6)$$

Given function is equal on right and left sides to get  $\theta_5$ , which is closed form. In the same way to get closed form of  $\theta_6$ .

Above analysis shows inverse of kinematics function has uncertainty and has many groups of solution which are possible different space position of kinematic joint.

## 2.3. Calculate Jacobian Matrix

Jacobian matrix of motion arm is its linear transformation of operation speed and joint speed, which can be seen as transmission ratio of movement from joint space to operation space [6], that is:

$$V = \dot{X} = J(q)\dot{q}, \quad (7)$$

where  $\dot{X}$  means operation velocity vector,  $\dot{q}$  means joint velocity vector. The number of movement dimensions of robot in operation space is equal to that of rows of Jacobian matrix and joints are equal to column. Above analysis shows Jacobian matrix of robot with six degrees of freedom is a  $6 \times 6$  matrix. The paper applies for vectorial method to calculate robot Jacobian matrix.

The line  $i$  of Jacobian matrix is:

$$J_i = \begin{bmatrix} Z_i \times {}^iP_n^0 \\ Z_i \end{bmatrix} = \begin{bmatrix} Z_i \times ({}^0R^i P_n) \\ Z_i \end{bmatrix}, \quad (8)$$

where  ${}^iP_n^0$  means robot end origin of coordinate in basic coordinate system  $\{0\}$  relative to position vectorial in coordinate system  $\{i\}$ ,  $Z_i$  means unit vectorial of  $z$  axis of coordinate system  $\{i\}$  in basic coordinate system  $\{0\}$ .  ${}^0R^i$  means direction cosine matrix. From above transformation matrix, get  ${}^0R^i, Z_i, {}^iP_n^0$  ( $i = 1, 2, \dots, 6$ ), and put them into function (8) to get line  $i$   $J_i$  ( $i = 1, 2, \dots, 6$ ) of Jacobian matrix, so as to make robot Jacobian matrix.

Jacobian matrix of robot is shown as below:

$${}^T J(q) = [{}^T J_1(q) \quad {}^T J_2(q) \quad {}^T J_3(q) \quad {}^T J_4(q) \quad {}^T J_5(q) \quad {}^T J_6(q)] \quad (9)$$

## 3. Kinematics Simulation

Put model into Mathematica, which includes 7 parts and each means base, vertical shaft, base turntable, upper arm, forearm, paws and wrist. Base is fixing part and rotating joints are set between other parts around  $Z_i$  axis. Given variable to each joint, simulate, analyze and compare kinematics.

### 3.1. Analysis on Simulation Result

Analyze point in the middle of paws, which is midpoint of paw and coincides with original point of coordinate system. Output data is shown as Fig. 3.

Fig. 3 shows that when time  $t=0$ , operation arm is in initial position, displacement in  $x, y, z$  direction is  $-500, 610, 1300$  (unit: mm), which is the same as calculation result.

Fig. 4 shows that when time  $t=0$ , velocity in x, y, z direction is -297, 0, 245, and maximum velocity is 297, 200, 245 (unit: mm/s) which is almost the same as calculation result.

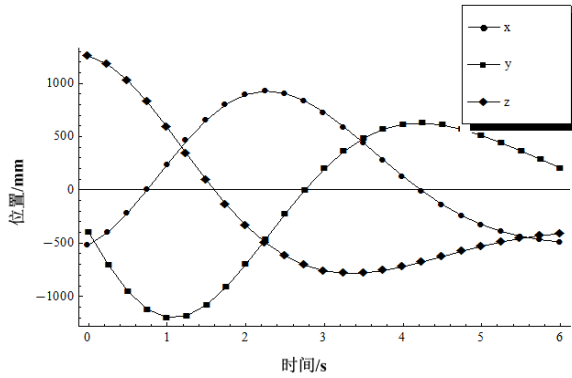


Fig. 3. Displacement curve of paw midpoint as time changes.

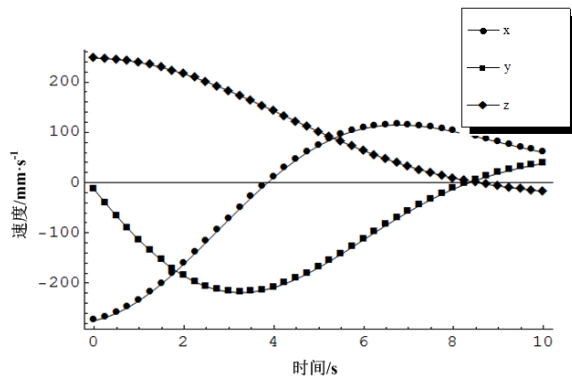


Fig. 4. Velocity curve of paw midpoint as time changes.

Fig. 5 shows that when time  $t=0$ , accelerated speed in x, y, z direction is 20, -113, 0 and maximum accelerated speed is 95, 113, 46 (unit: mm/s<sup>2</sup>), which is the same as theoretical analysis.

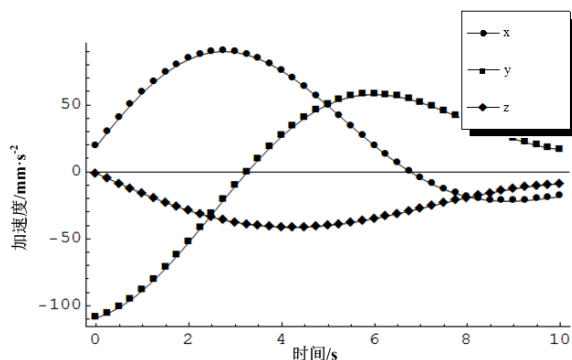


Fig. 5. Accelerated speed curve of paw midpoint as time changes.

The same verification is got from Fig. 6 and Fig. 7.

Change length of each arm and movement parameter of each joint to get different displacement, velocity and accelerated speed of arm in different conditions.

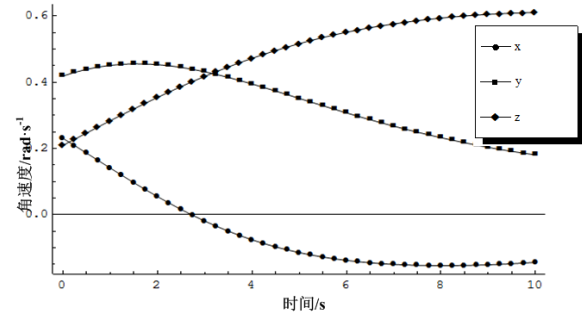


Fig. 6. Angular velocity curve of paw midpoint.

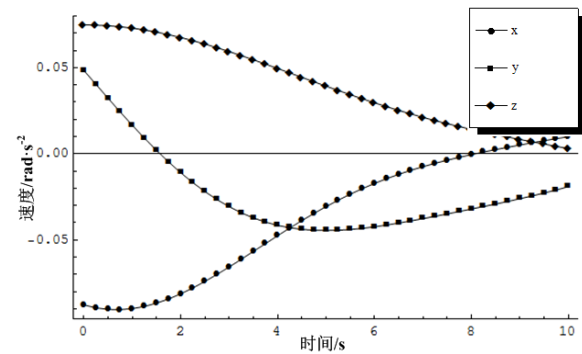


Fig. 7. Angular accelerated speed curve of paw midpoint.

#### 4. Conclusions

1) Based on robot D-H matrix theory, establish robot kinematics mathematic model, deduce homogeneous transformation matrix of transfer robot kinematics function by combing with robot structure, by which angular of every joint and paw position can be calculated mutually and get direct and inverse kinematics.

2) Vector product method is applied to deduce speed Jacobian matrix, which could calculate displacement and speed of points in paw with each joint angular and speed mutually.

3) Simulate by software to verify given structure parameters reasonable.

4) Mathematic simulation software is applied to simulate kinematics of transfer robot with 6-degrees of freedom, measure velocity, accelerated speed, displacement curve of robot end, analyze dynamic movement features of robot. Simulation result shows: robot moves steadily in working condition which can meet working requirements.

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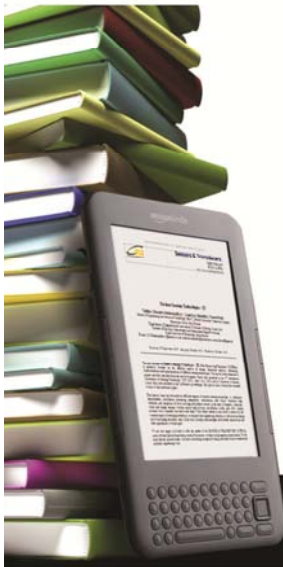
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