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A Stepwise Optimal Design of a Dynamic Vibration Absorber with Tunable Resonant Frequency

Jiejian DI, Xiaowu DU, Qiang MA, Quanliang ZHAO, Fengbin LIU
College of Mechanical and Electronical Engineering, North China University of Technology,
Beijing, 100144, China
Tel.: +86 010-88802892
E-mail: dijiejian@ncut.edu.cn, duxiaowu789@163.com, maqiang856057@gmail.com,
zql-01@163.com, fbliu@ncut.edu.cn

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Abstract: A new kind of dynamic vibration absorber (DVA) with tunable resonant frequency is presented. The kinematics differential equation about it is built and the stepwise optimization is performed. Firstly, four main system parameters involving the ratios of mass μ, natural frequency f, vibration frequency g and damping ζ are solved by small-step-search method to obtain optimal steady state amplitude. Secondly, the sizing optimization of the dynamic vibration absorber is proceeded to search an optimal damping effect based on the optimal parameters (g, μ, ζ, f). And such the damping effect is simulated in a flat structure, and the results show that the working frequency band and damping effect of the DVA after optimization won 20 % of the effect of ascension compared with that before optimization. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Dynamic vibration absorber, Optimal design, Small step search method, Damping effect, Working frequency band.

1. Introductions

Modern industry has brought us a brand new lifestyle with unprecedented convenience and efficiency. However, vibration and correlated noise accompanied with those advance machines are negatively affecting living environment, human health and both accuracy and service life of equipment at the same time. As a sort of simple and effective device, Dynamic Vibration Absorber (DVA) has been widely utilized in engineering to alleviate vibration. Vibration absorber was firstly proposed by Frahm as a concept [1]. For hundred years, researchers from different countries have put plenty of efforts on developing and improving its structure, mechanism and control theory. Nayaya and Kurusu [2] proposed a vibration absorber which could adjust rigidity through varying fixing point of an overhang beam. Hill, Nagarajaiahand and Chinese scientist Li Jianfeng [3] respectively invented the semi-initiative vibration absorber which adjusting rigidity through a particular mechanism. Wang Lianhua [4] designed a vibration absorber with tunable resonant frequency based on magneto rheological elastomers with a satisfactory consequent on vibration reduction.

A new type of DVA was proposed in reference [5], as shown in Fig. 1. The upper and lower lead screws of the DVA have different directions on screw thread. When the DVA works, they are driven by a stepping motor and a lead screw. The shape of the spring leaf is deformed with the upand-down-motions of the lead screws, and then rigidity and the natural frequency of would be changed as well. Thus, the natural frequency of the DVA could be tuned continuously to fitting with the excitation with different frequency.

In order to improve the working frequency band and the damping effect of the DVA, two absorb masses are installed on the two sides of the DVA, and the weights of absorb masses are able to be adjusted to meet different requirements. In order to optimize the main parameters of the DVA, several efficient optimization methods have been proposed. Ronghua Su [6] proposed a method to minimum the vibration displacement of the main mass. Quanjuan Wang [7] proposed an optimization design method named as Power Flow. The optimization method mentioned above are much more difficult to bring about, so a method of small step search is proposed for the optimization of the four main system parameters involving the ratios of mass μ, natural frequency f, vibration frequency g and damping ζ the in this paper. This method has the advantages of convenience and fast running. The damping effect of the DVA after optimization is simulated in a flat structure, and the results show that the working frequency band and damping effect of the DVA after optimization are both improved.

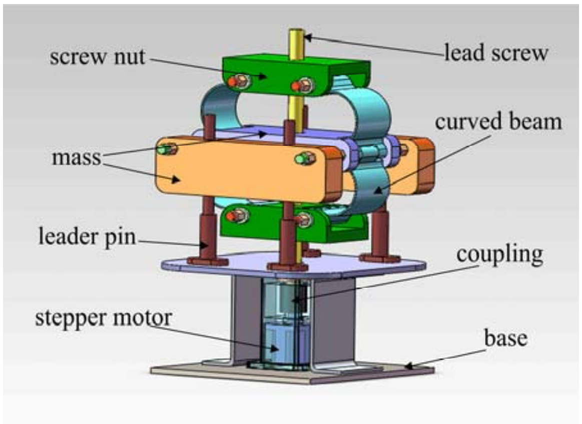

Fig. 1. 3D structure of DVA.

2. Mathematics Model of DVA

The two-degree of freedom vibration absorption system consisting of a main mass and a dynamic vibration absorber can be simplified as a physical model as shown in Fig. 2.

And, m_1, m_2, c_1, c_2, k_1, k_2 is the mass, damping and rigidity of the main mass and the DVA respectively. $p_0 \sin\varpi t$ is a harmonic force, and x_1, x_2 is the vibration displacement of the main mass and the DVA respectively.

The dynamics differential equation of the vibration absorption system can be established as:

$$\begin{cases} m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) = P_0 \sin\omega t \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) = 0 \end{cases}, \quad (1)$$

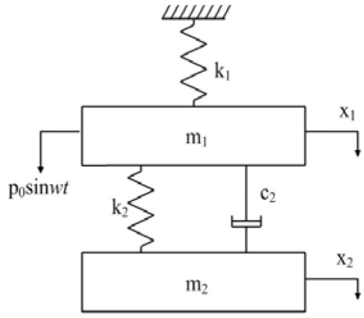

Fig. 2. Physical model of the vibration absorption system.

Its solution can be expressed as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sin\omega t, \quad (2)$$

The solution is:

$$\begin{cases} \dfrac{X_1}{\delta_{st}} = \left[\dfrac{(2\zeta g)^2 + (g^2 - f^2)^2}{A} \right]^{1/2} \\ \dfrac{X_2}{\delta_{st}} = \left[\dfrac{(2\zeta g)^2 + f^4}{A} \right]^{1/2} \end{cases}, \quad (3)$$

and

$$A = (2\zeta g)^2 (g^2 + \mu g^2 - 1) + \left[\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2) \right]^2, \quad (4)$$

where

$$\begin{cases} \delta_{st} = P_0 / k_1 - \text{static deformation} \\ \mu = m_2 / m_1 - \text{mass ratio} \\ f = \omega_a / \omega_n - \text{ratio of the natural frequency}, \\ \mathrm{g} = \varpi / \omega_{\mathrm{n}} - \text{ratio of vibration frequency} \\ \zeta = \mathrm{c}_2 / \mathrm{c}_{\mathrm{c}} - \text{damping ratio} \end{cases} \quad (5)$$

In order to obtain the good damping effects over the whole working frequency band of the DVA; the optimization method is a good solution with the minimum the maximum values of steady state amplitude X_1/δ_{st} as the objective.

3. System Parameters Optimization

To simplify, assuming $\alpha(X)$ denoted the steady state amplitude X_1/δ_{st}. It is well known that $a(\mathrm{X})$ is mostly affected by the ratios of mass μ, natural frequency f, vibration frequency g and damping ζ.

Therefore, the g, μ, ζ, f are chosen as design parameters for the optimization of the vibration absorption system, and be impressed by the vector $X = X(g, \mu, \zeta, f)$ with minimum the maximum values of $a(\mathrm{X})$ as the objective.

Then the mathematic model of optimization is given as follows.

$$\min \quad (\max a(X)), x \in X \subset R, \tag{6}$$

$$s.t. \begin{cases} \mu_l \le \mu \le \mu_u \\ f_l \le f \le f_u \\ g_l \le g \le g_u \\ \zeta_l \le \zeta \le \zeta_u \end{cases}, \tag{7}$$

where μ_l, μ_u are the lower and upper limits of the mass ratio, f_l, f_u are the lower and upper limits of the ratio of the natural frequency, g_l, g_u are the lower and upper limits of the ratio of the vibration frequency, ζ_l, ζ_u are the lower and upper limits of the damping ratio.

It is a nonlinear optimization problem, which is more difficult to solve. A small-step-search method is proposed in this paper, the optimization flow chart seen in Fig. 3. In order to obtain a variety of combinations, the parameters g, μ, ζ, f can be changed with a small increment by the method of small step search through the optimization. The maximum value of $\alpha(X)$ corresponding to each parameter will be obtained by small step searching. The maximum values under the different parameters will be compared with each other, and the minimum one within the maximum values pool of $a(\mathrm{X})$ will be picked out, thus, the corresponding parameters g, μ, ζ, f are the optimal values of the system.

In this vibration absorption system, the limits of variable g, μ, ζ, f are given as follows:

$$\begin{cases} \mu_l = \zeta_l = f_l = g_l = 0 \\ \mu_u = 0.1, \zeta_u = 1.3, f_u = 0.5, g_u = 1 \end{cases}, \tag{8}$$

where the initial values are: $\mu = g = \zeta = f = 0$, the step $e = 0.001$ and m=0, n=9999. By mean of the method mentioned above, the optimized values can be obtained through inputting the initial parameters into the optimization program. It can be calculated that $\mu = 0.1, g = 0.909, \zeta = 0.16, f = 1.5$ and the corresponding steady state amplitude $a(\mathrm{X}) = 4,601$.

A formulation [8] that calculating static deformation of the main mass is given as following:

$$\left(\frac{X_1}{\delta_{st}}\right)_{optimal} = \sqrt{1 + \frac{2}{\mu}}, \tag{9}$$

The steady state amplitude calculated by this formulation $\alpha(X)$ is 4.58, which is very nearly to value obtained by the proposed method in this paper. It can be concluded that the proposed optimization method is fairly valid and effective.

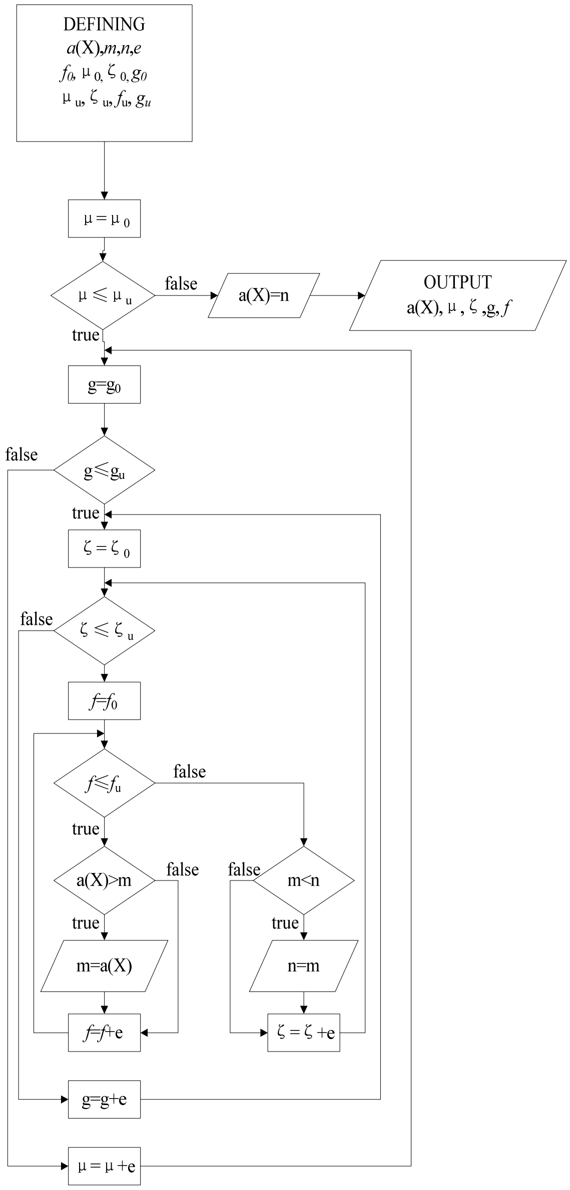

Fig. 3. The flow chart of the optimization.

4. Sizing Optimization of DVA

Since the optimal parameters μ, g, ζ, f of the vibration absorption system have been obtained by small step search method, then $a(\mathrm{X})$ is obtained as 4.66, and the mass of DVA after optimization can be achieved as following: $m_2 = \mu m_1 = 9.42\ kg$. For the stepwise optimization, then the initial structure of the DVA, described in reference [5] should be adjusted from the initial values mass $\mu = 0.07$, $g = 0.85$ natural frequency $f = 1.01$, vibration frequency and damping $\zeta = 0.2$ to the optimal values.

It is difficult to calculate the rigidity of the C-shape spring directly, thus an indirect method of measuring the deformation of the springs under a constant force will be used. It is obvious that the smaller the deformation of the C-shape spring is, the higher the rigidity of spring is with a constant force. Thus, to widen the frequency band, the rigidity of the springs needs to be enhanced as can as possible keeping the system optimal parameters unchanged. For the C-shape spring (Fig. 4), the rigidity and mass are affected by the dimension b and h as well. That is to say, the mass of C-shape spring is the function of the width *h* and thickness *b*, expressed in $m_2(b,h)$, the rigidity C-shape spring is the function of the width *h* and thickness *b* and mass $m_2{}^c$, expressed in $k(m_2(b,h),b,h)$. To balance them, the optimization technology is taken, and the optimization model is as follows:

$$\max K = (m_2(b,h),b,h)\,, \tag{10}$$

$$s.t.\begin{cases} m_2(b,h) = m_2{}^c \\ b_l \le b \le b_u \\ h_l \le h \le h_u \end{cases}, \tag{11}$$

where K is the rigidity of the C-shape spring, b_l, b_u are the lower and upper limits of the thickness of the C-shape spring, h_l, h_u are the lower and upper limits of the width of C-shape spring.

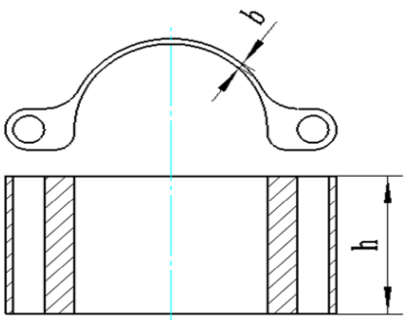

Fig. 4. Geometry of the C-spring in DVA.

For calculation in this paper, the limits of variable m_2, b, h are:

$$\begin{cases} m_2{}^c = 0.15 \\ b_u = 60, h_u = 5 \\ b_l = h_l = 0 \end{cases}, \tag{12}$$

The 3D model of the spring can be modeled using 3D modeling software, and the optimal dimensions of the C-shape spring can be achieved with the finite element analysis software, such as Ansys.

It can be shown from Fig. 5 that the deformation and the mass of the springs are much more sensitive to the width h than the thickness b. Actually the dimension b almost has little effect on the optimization result, so dimension h will be only concerned.

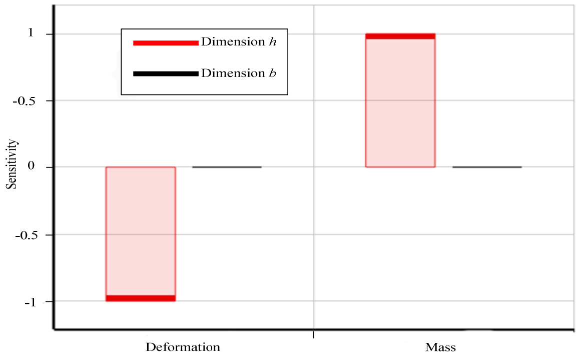

Fig. 5. Sensitivities analysis of the sizing h and b.

The effects of the sizing h on the mass and the deformation of the C-shape spring are shown in Fig. 6 and Fig. 7. It can be seen from Fig. 6 that the deformation is generally decreased with the larger size of width *h*, but the deformation becomes larger after the width is over 54 mm, so the width *h* limit of the springs is set from 40 mm to 53.5 mm, and the thickness *b* from 2.5 mm to 2 mm.

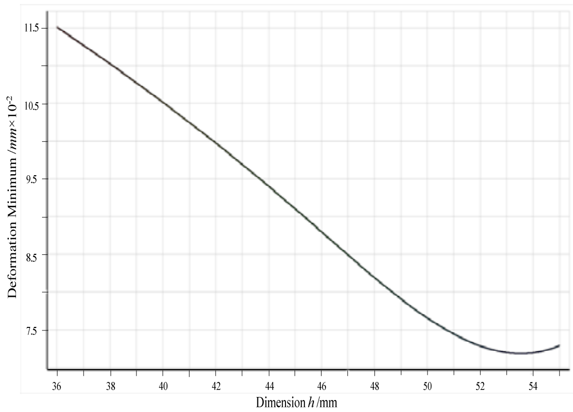

Fig. 6. The width-deformation relationship curve.

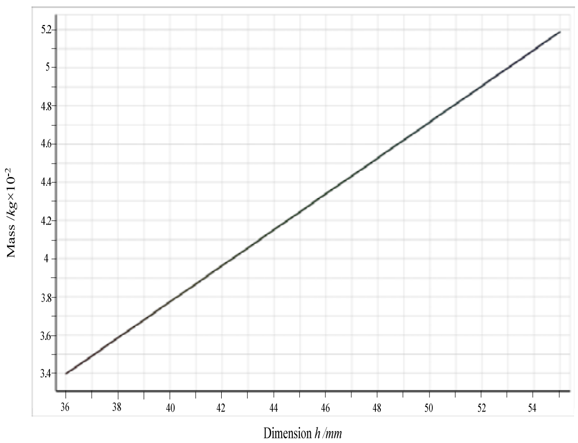

Fig. 7. The width-mass relationship curve.

To verify the vibration absorbing ability of the DVA, the method described in reference [9, 10] is adopted also. The acceleration amplitudes on the four measuring points located on the main mass is measured, and the ratio value of their square roots is taken to express the vibration response "L" of the main mass (Eq.13). The formulation is as follows:

$$L = 20\lg\frac{L_1}{L_2} = 20\lg\frac{\sqrt{\sum_{i=1}^{n} l_1^2 \Big/ n}}{\sqrt{\sum_{i=1}^{n} l_2^2 \Big/ n}}, \tag{13}$$

where L_1 and L_2 are the vibration response of the main mass with and without DVA respectively, l_1 and l_2 are the acceleration amplitude of measuring point i with and without DVA respectively, and n is the number of measuring points. The calculation results are shown in Fig. 8. It can be seen that, the acceleration amplitude response is serious under the excitation without DVA (Fig. 8 (a)), especially at the frequency about 50Hz, but the acceleration amplitude response is fairly good with DVA (Fig. 8 (b)), and more, the acceleration amplitude response is much better over the whole working frequency band after optimization (Fig. 8 (c)).

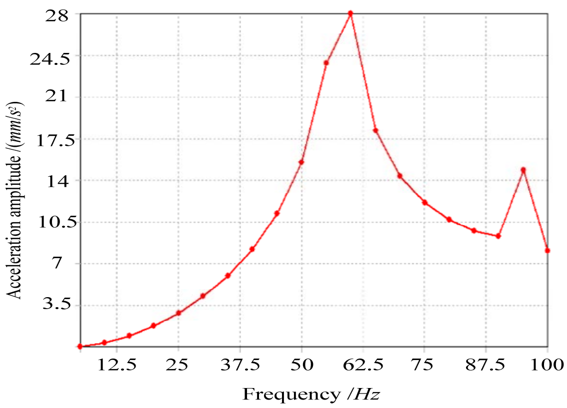

Fig. 8 (a). Comparison of the damping effect in acceleration amplitude: without the DVA.

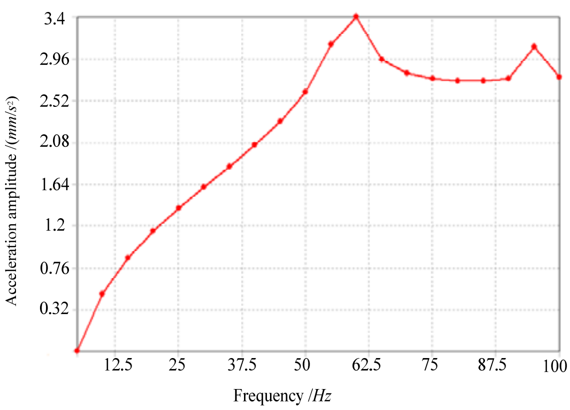

Fig. 8 (b). Comparison of the damping effect in acceleration amplitude: before the optimization.

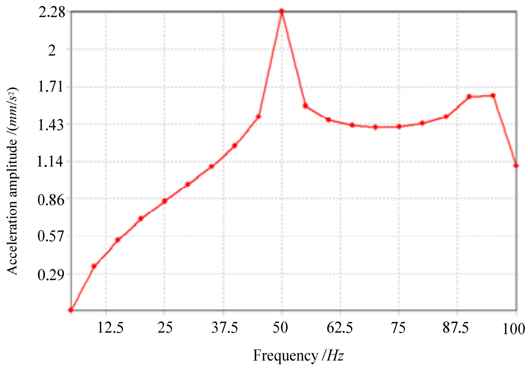

Fig. 8 (c). Comparison of the damping effect in acceleration amplitude: after the optimization.

Fig. 9 is the comparison of vibration absorbing ability of the DVA before and after optimization with the DVA installed at the measuring point that nearest to the vibration source. As shown in the figure, the DVA receives a remarkable vibration absorbing ability within the whole working band. The acceleration amplitude of the main mass has been improved significantly. Compared with the results before optimization, the vibration response after optimization reaches the peak value of 11 dB, and the damping effect of the DVA won 20 % of the effect of ascension.

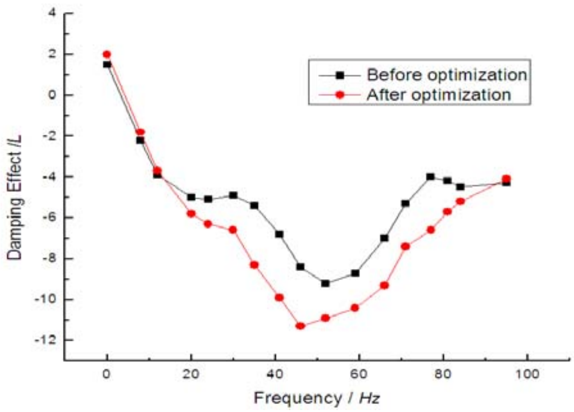

Fig. 9. Comparison of the damping effect before and after optimization.

5. Conclusions

A new kind of dynamic vibration absorber (DVA) with tunable resonant frequency is presented. To obtain a good damping effect, a step wise optimization is proposed. The main system parameters g, μ, ζ, f are firstly optimized by the small-step-search method, the minimum steady state amplitude X_1/δ_{st} within the maximum values, and the corresponding optimal system parameters are obtained. Based on the optimal system parameters obtained by first step, the sizing optimization is further performed on the C-shape springs with the width h and thickness as design variables. Compared to simulation results before optimization, the vibration-absorbing ability of the DVA is improved by about 20 %, it is shown that the proposed stepwise optimization is fairly feasible and effective.

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