

Study and Application of Wavelet-based Denoising Method of Seismic Effect Signals of Colluvium Accumulation Slope

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Abstract: Stability analysis of ground motion topography effect is one of the important topics in geotechnical and earthquake engineering. In order to solve the major problems in seismic effect signals of colluvium accumulation slope in different target positions: weak echo signal and large dynamic range, we proposed an improved wavelet denoising analysis method. Through denoising experiments, we calculated signal-noise ratio and root mean square error, which could be used as denoising evaluation index. In this way, we obtained optimal wavelet basis, decomposition scale, and threshold allocation rules and achieved the better wave filtering effect and the increased signal-noise ratio. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Round motion, Topography effect, Dynamic response, Denoising, Wavelet basis function.

1. Introduction

The earthquake destruction role shows the significant topographic amplification effect, which can be essentially interpreted as the coupling effect between earthquake power and topography. Therefore, stability analysis of ground motion topography effect is one of the important topics in geotechnical and earthquake engineering [1].

For the exploration targets at different positions, seismic effect signals of colluvium accumulation slope give weak echo signals, which are easy to be overwhelmed by Gaussian white noise. Therefore, it is necessary to perform denoising treatment so that the exploration target signals can show the neat, clear and less distorted waveform characteristics. The

frequency-domain characteristics of such ground-penetrating signal vary with time. Ground-penetrating signals belong to typical time-varying non-stationary signals, which are unpredictable signals with sudden transient nature. Due to the diversity of such seismic effect signals and interference noises and capture the diversity, the traditional signal processing techniques based on Fourier Transform and Function analysis cannot accurately describe the local characteristics of such signals.

As a signal analysis method based on time-frequency relation, wavelet transform shows multi-resolution analysis capability and characterization capability for the local signal characteristics in time domain and frequency domain. Wavelet transform is

very suitable to analyze transient anomalous signals contained in seismic effect signals of colluvium accumulation slope. Therefore, the application of wavelet analysis in decomposition, denoising, and reconstruction of seismic exploration signal has always been one of the hotspots [2, 3]. Uniform thresholding denoising algorithms of wavelet independent normal variables, especially the classic soft and hard thresholding denoising algorithms, are currently widely used. In the application of hard thresholding algorithm, the local characteristics of the signals can be maintained, but its estimated wavelet coefficient (EWC) shows the poor continuity, which may lead to the reconstructed signal ringing, Pseudo-Gibbs effect, and signal distortion. Soft thresholding algorithm allows the good continuity and easy signal processing procedure, but when the wavelet coefficient is large, compared with the original signal wavelet coefficient, EWC shows the fixed deviation, which will lead to the errors of the reconstructed signals as well as the blurred signal edges [4, 5]. The floating threshold algorithm, Symelets transform maximum estimation algorithm of the high-frequency component modulus, and other improved algorithms were adopted in the denoising, but the final denoising results show large errors under short-term pulse interference. Moreover, the completely rebuilt fixed filter was used in encoding and processing of signals. But the selected filter and common processing algorithm cannot provide the optimal filtering effect for seismic effect signals of colluvium accumulation slope [6, 7].

In this paper, according to the characteristics of noise-containing signals obtained during the exploration of seismic effect of colluvium accumulation slope, with the denoising evaluation indexes of signal-noise ratio (SNR) and root mean square error (RMSE), we improved thresholding denoising algorithm based on the classic soft and hard thresholding denoising algorithms. Through the signal reconstruction according to low-frequency and high-frequency coefficients of different wavelet decompositions, combined with simulation experiment, the optimal matching combination of wavelet basis function and threshold rule was obtained. The simulation experiments verified that the optimized algorithm could provide the better denoising effect and the higher SNR.

2. Wavelet Denoising Principle and Evaluation Method

The model of an observed signal containing noise can be expressed as follows:

$$f(t) = s(t) + \sigma^2 n(t), t = 0, 1, \dots, n-1, \quad (1)$$

where $s(t)$ is the true signal; $n(t)$ is the Gaussian white noise of the variance σ^2 . From the perspective of signal processing, the wavelet denoising is a noise

filtering process to restore the true signal. In general, the signal denoising process can be divided into three steps shown in Fig. 1.

1. Through discrete sampling of signal $f(t)$, wavelet coefficients ($w_{j,k}$, $u_{j,k}$, and $v_{j,k}$) of discrete signals from N sampling points are obtained;
2. Through the special threshold processing of $w_{j,k}$, the wavelet estimation coefficient $\hat{w}_{j,k}$ is obtained so that $\|\hat{w}_{j,k} - u_{j,k}\|$ is as small as possible;
3. Wavelet reconstruction is performed with $\hat{w}_{j,k}$ to obtain the denoised signal $\hat{f}(t)$.

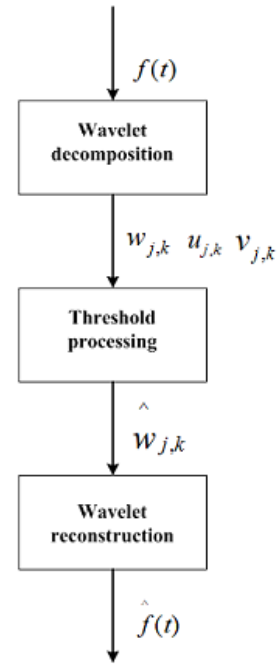


Fig. 1. Wavelet denoising flow diagram.

The denoising performance is generally evaluated according to SNR, RMSE, entropy principle of information, and other indexes. In this paper, based on different wavelet bases and threshold selection algorithm, SNR and RMSE were respectively used as the first and the second evaluation indexes to evaluate the difference among various thresholding rules and various reconstruction rules of wavelet basis low-frequency and high-frequency coefficients and provide the optimal rule combination. SNR and RMSE are defined as follows:

$$SNR = 10 \lg \left\{ \frac{\sum_{t=1}^N s^2(t)}{\sum_{t=1}^N [s(t) - n(t)]^2} \right\}, \quad (2)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N [s(t) - n(t)]^2}, \quad (3)$$

For the noise-containing signal, $f(t)$, after filtering using different threshold algorithms, SNR and RMSE of the de-noised signal were calculated. The larger SNR and the smaller RMSE indicate the better denoising effect.

During the wavelet denoising process, wavelet basis function, decomposition, threshold selection rule, and threshold function design are the key factors affecting the final noising effect.

3. Selection of Base Function

During the analysis and processing process of seismic effect signals of colluvium accumulation slope, for different wavelet basis functions have different time-frequency characteristics, different wavelet basis functions give different analysis results. For seismic effect signals of colluvium accumulation slope are characterized by unpredictable property, sudden transient property, diversity, seismic signal source diversity, and noise signals diversity, the selection of wavelet basis function should be based on the following considerations:

1. Signal reconstruction should be successfully performed so that more seismic source information can be obtained;
2. The selection of wavelet basis function should be adapted to sudden transient property and rapid decay property of seismic effect signals of colluvium accumulation slope;
3. The useful signals should be highlighted under the noise interference.

In common wavelet bases, including Haar, Morlet, Mexicanhat, Symlets, Biorthogonal, Coiflets, and Daubechies, according to the requirements of discrete wavelet variation, time-domain compact support property, and highlighting signal, Symlets, Coiflets, and Daubechies are applicable. Symlets, Coiflets, and Daubechies showed largely similar wavelet spectrum coefficient and could accurately reflect the feature that energy was concentrated in the vicinity of the main frequency in corresponding analysis results [8]. Therefore, Symlets, Coiflets, and Daubechies show the high analysis capability for the characteristics of seismic effect signals of colluvium accumulation slope and can be used as the alternative test wavelet bases.

4. Determination of Maximum Decomposition Scale

The deep analysis of seismic effect signals of colluvium accumulation slope requires multi-level decomposition. If the decomposition number is too low, SNR cannot be greatly improved; if the

decomposition number is too large, the signal frequency range is more finely divided and the calculation workload is increased considerably. Moreover, in practical engineering applications, the frequency resolution of discrete-time sequence signal is limited. Therefore, it is necessary to determine the maximum wavelet decomposition scale according to the characteristics of seismic effect signals of colluvium accumulation slope.

If the sampling frequency of discrete signal $f(t)$ is f_s and the sampling point number is N , for J wavelet signal transforms, according to the sampling theorem, we get:

$$\frac{f_s}{2N} \leq \frac{f_s}{2^{J+1}} \leq \frac{f_s}{2}, \quad J \leq \log_2^N \quad (4)$$

Mallat closely combined filter banks with wavelet and proposed the two-scale equation and tower decomposition algorithm [8, 9]. The close combination was realized through constructing impulse response function for the high-throughput and low-throughput filters and signal convolution. For the J^{th} wavelet decomposition, the coefficient number of impulse response function is $2^{J-1}L_f$. If the coefficient number of impulse response function is no more than the signal length, we get:

$$J \leq \log_2^{\frac{N}{L_f}} + 1, \quad (5)$$

Combining Eq. (4) with Eq. (5), maximum decomposition scale based on Mallat wavelet decomposition can be expressed as:

$$J_{\max} = \min[\text{int}(\log_2^N), \text{int}(\log_2^{\frac{N}{L_f}} + 1)], \quad (6)$$

For a typical seismic effect signal of colluvium accumulation slope, the acquisition process of ground motion topography effect is short and one wave period is approximately 20-30 s. The sampling frequency is $f_s = 1000 \text{ Hz}$. For the analysis and processing of seismic effect signals, only 20,000~30,000 data points are enough. According to the decomposition requirements, $N = 32768$ is adopted. Therefore, Eq. (6) indicates $J \leq 15$.

The decomposition scale was analyzed according to the relationship between sampling frequency and the frequency corresponding to the each wavelet decomposition scale. For the sampling frequency of $f_s = 1000 \text{ Hz}$ and J wavelet decompositions, the signal can be decomposed into signals of $J + 1$ frequency ranges and each frequency range is calculated as:

$$[0, \frac{f_s}{2^{j+1}}], [\frac{f_s}{2^{j+1}}, \frac{f_s}{2^j}] \quad j = 1, 2, \dots, J, \quad (7)$$

The frequency ranges corresponding to 6~8 decomposition scales are shown in Table 1.

Table 1. Frequency range of each decomposition layer.

$J = 6$	Frequency (Hz)	$J = 7$	Frequency (Hz)	$J = 8$	Frequency (Hz)
				a_8	0 ~1.9
		a_7	0~3.9	d_8	1.9~3.9
a_6	0~7.8	d_7	3.9~7.8	d_7	3.9~7.8
d_6	7.8~15.6	d_6	7.8~15.6	d_6	7.8~15.6
d_5	15.6~31.2	d_5	15.6~31.2	d_5	15.6~31.2
d_4	31.2~62.5	d_4	31.2~62.5	d_4	31.2~62.5
d_3	62.5~125	d_3	62.5~125	d_3	62.5~125
d_2	125~250	d_2	125~250	d_2	125~250
d_1	250~500	d_1	250~500	d_1	250~500

According to waveform and power spectrum distribution characteristics of common seismic effect signals of colluvium accumulation slope, signal energy is distributed in the frequency range [9]. The frequency band is mainly distributed in the middle region of the wavelet decompositions. The resolution is enough for 8 wavelet decompositions. If the decompositions are increased, computational effort is greatly increased and the low-frequency band is further divided. Therefore, the increase of decomposition is not meaningful for distinguishing the high-frequency noise. In the paper, considering timeliness and effectiveness, 8 wavelet decompositions are adopted.

5. Denoising Threshold Settings

In engineering applications, the wavelet thresholding denoising method is usually improved in two aspects: the selection of the appropriate threshold function and reasonable threshold limit value. Four common threshold limit value determination rules include rigrsyre estimation criteria based on the Stein unbiased risk adaptive threshold, heursure optimal estimation threshold selection criteria based on the heuristic threshold, sqtwolog criteria of fixed threshold limit value, and the minimaxi threshold criteria.

In the paper, based on the threshold function of Stein's Unbiased Risk Estimate (SURE), we improved the selection of the threshold [10]. If a threshold T is too small, the reconstructed signal contains a lot of noise and the required denoising effect is not reached. If the selected threshold is too large, the useful information contained in the signals

will be mistakenly filtered out. For the seismic exploration signal is essentially different from the noise signal in transmission characteristics, the zooming factor and the threshold effect of translation function used in wavelet transform correspond to transmission characteristics of the noise and can be used in denoising of signal reconstruction. The threshold limit value adopted in the conventional method is fixed and does not vary with time window and wavelet decomposition, the optimal denoising effect is not available.

In order to achieve the better denoising effect, based on engineering experiences, we proposed a threshold selection formula:

$$T = 2 \ln N \times \left(\frac{2\sigma_j^2}{\sqrt{N}} \right) \times 2^{\frac{J-j}{2}}, \quad (8)$$

$$j = J, J-1, \dots, 1$$

where N is the number of total sampling points; σ_j^2 is the standard deviation of noise at the j^{th} wavelet scale. The transform grade with the higher resolution of N indicates the larger decomposition scale and the larger threshold limit value. For ground motion topography effect signals, with the increase of the resolution, the noise deviation σ_j^2 increases exponentially.

6. Analysis of Simulation Results

In the simulation, according to the characteristics of seismic effect signals of colluvium accumulation slope, approximate simulation signal containing noise was generated firstly. Then, various thresholding rules were matched with wavelet basis functions for denoising, thus obtaining final denoised signals. Then SNR and RMSE of denoised signals were calculated and used to determine the denoising effect according evaluation rules. In this way, optimal matching parameters were obtained.

The heuristic fixed threshold based on heursure in Matlab was selected. Then three wavelet basis functions were used to filter simulation signals containing noise. The comparison between the denoised signal and original signal is shown in Fig. 2.

Table 2 shows the SNR and RMSE values obtained with the heuristic fixed threshold based on heursure. Based on the evaluation rules, denoising effects were ranked.

Different wavelet basis functions allow different SNRs and RMSEs of denoised signals. The SNR of denoised signals obtained with Db3 was the highest and RMSE is the smallest, indicating the optimal denoising effect. The effect is related to the regularity, the higher vanishing moment, and the close Symmetry of Daubechies wavelet.

The Db3 wavelet basis function was selected. Then soft, hard, and corrected thresholds were used

to filter simulation signals containing noise. The comparison between the denoised signal and original signal is shown in Fig. 3.

Table 3 shows the SNR and RMSE values obtained with different threshold rules. Based on the evaluation rules, denoising effects were ranked.

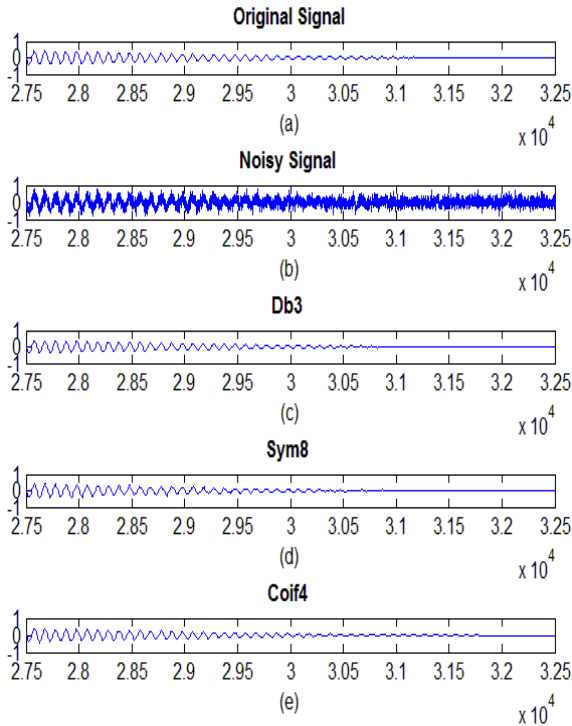


Fig. 2. Denoising waveforms obtained with three wavelet basis functions.

Table 2. Denoising effects of various wavelet functions.

Wavelet basis functions	SNR	RMSE	Denoising effect ordering
Noisy Signal	1.80 dB	0.2003	
Db3	22.5 dB	0.0184	1
Sym8	22.3 dB	0.0189	2
Coif4	20.9 dB	0.0223	3

Through the simulation experiment and SNR comparison, for seismic effect signals of colluvium accumulation slope, Db3 wavelet basis function of Daubechies series was selected to match with and correct the threshold rule and allowed the better denoising effect.

Table 3. Denoising effects obtained with different thresholds.

Wavelet basis functions	SNR	RMSE	Denoising effect ordering
Noisy Signal	1.80 dB	0.2003	
Improved	22.7 dB	0.0171	1
Soft	19.6 dB	0.0203	2
Hard	18.8 dB	0.0239	3

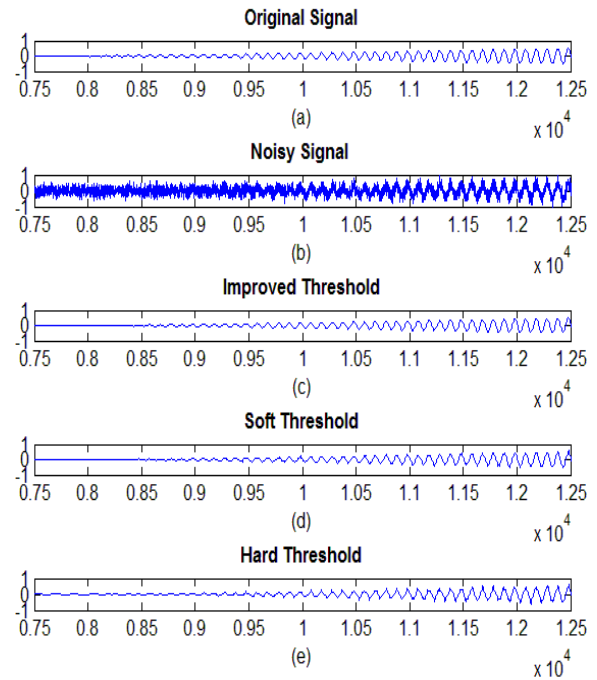


Fig. 3. Denoising waveforms obtained with different threshold rules.

7. Conclusions

In this paper, based on wavelet theory, we selected wavelet basis function which was applicable to seismic effect signals of colluvium accumulation slope and then determined maximum decomposition scale according to the sampling theorem and Mallat Algorithm. Moreover, combined with the decomposition scale, threshold denoising method was corrected. Finally, we obtained the optimal denoising method for seismic effect signals of colluvium accumulation slope. In the simulation experiments, the denoising indexes of SNR and RMSE indicated that the proposed denoising method could be easily realized. With certain advantages and feasibility, the proposed denoising method is of great values in seismic survey projects.

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