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Methods of Evaluation of Output Signals from Resonance Accelerometers

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Abstract: In this paper two novel modifications of Weighted Least Square Method (WLSM) of frequency estimation suitable for evaluation of rapid changes of frequency occurring in resonance sensors are introduced and experimentally verified. The efficiency of the WLSM methods is compared to the nano-counting approach representing a substantial improvement of well-known start-stop methods of frequency measurement and estimation. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Resonance sensors, Accelerometers, Regression analysis, Frequency estimation, Weighted least square method, Nano-counting.

1. Introduction

Resonance accelerometers based on a quartz crystal resonator allow reaching the sensitivity and stability in order of nano-g (10^{-9} g). Typical examples are products designed by Paroscientific, Quartz Sensors Inc., and Quartz Seismic Sensors [1], which allow to measure variation of gravity caused by a relative movement of Moon. In general, the nano-g resolution feature is only useful if a measured quantity is stable enough over the subsampling period. Digiquartz Intelligent devices produced also by Paroscientific allow setting the integration time from 0.001 up to 270 seconds in 0.001 second increments. A practical bandwidth of interest is in the infrasound and deep infrasound (10 to 0.001) Hz.The sensitivity in order of nano-g would correspond to the resolution of A/D converters being approximately 30 bits. However, such a convertor cannot be practically with this resolution realized. Nevertheless, there fortunately exist advanced methods of frequency measurement, e.g. a nano-counting method, evaluating the changes of resonance frequency (typically around 30 kHz), which correspond to aforementioned nano-g resolution, and thus even when the A/D converter cannot have such a resolution there is still possibility to observe nano-g changes. Therefore, a direct measurement of frequency is thus preferable instead of using a frequency-to-voltage converter or current-to-voltage converter located inside the body of the sensor followed by A/D converter. In some resonance accelerometers a current is used as an output quantity (e.g. INNALABS INN-204 accelerometers QTC) and the scale factor equals to 1.361214 mA/g which corresponds to 10^3 times smaller resolution, i.e. approximately 1 μg [6]. In reality the actual resolution is diminished due to quantization errors and interfering signals (SNR) of A/D convertors.

The implementation of direct measurement of resonant frequency changes is complicated by the necessity to respect the speed of frequency changes corresponding to variation of a measured quantity, i.e. acceleration. Only a few methods of frequency measurements are able to fulfill the contradictory requirements on the speed of changes while preserving the accuracy of measurements. Those methods are a nano-counting method and Weighted Least Mean Square (WLMS) frequency estimation method. Applicability and subsequent accuracy of both methods are partially limited by uncertainties caused by triggering and rounding errors (nano-counting method) or by the shape of measured signals and quantization errors caused by A/D convertor (WLMS). These limitations were our main motivations. The contribution of this paper is then in the modification WLMS method to reach better performance and confirm a proposed approach.

The remainder of this paper is organized as follows: In Section 2, frequency measurement methods suitable for rapid frequency changes with preserving accuracy are described and studied. In Section 3, there are proposed modifications of the WLMS frequency estimation method with confirmed improvements based on simulations. In Section 4, experimental results are provided. Section 5 concludes this paper.

2. The Methods of Frequency Estimation Suitable for Evaluating Rapid Frequency Changes

2.1. Nano-counting Method using a Regression Analysis

The nano-counting method is one of suitable methods which can be used frequency measurements of resonance accelerometers with nano-g resolution. This resolution is required for measurements of for instance gravity variations caused by a relative movement of Moon. The character of the variations is shown in Fig. 1. Obtained data were measured within 6 days with an absolute 3-g accelerometer and evaluated by the nano-counting method. The deviations from 1 g are about 40 nano-g's [1].

In accordance with the principle of regression analysis the method evaluates time intervals between zero crossings of the signal which is performed by counting the number of pulses from an etalon generator providing a referential frequency. This evaluation is performed per each subsample which corresponds to a particular window in which a constant frequency is expected. The longer the subsample, the higher the accuracy of the evaluation can be reached. The mentioned principle suffers from uncertainties caused by triggering and rounding errors. Therefore, obtained results, i.e. the content of the counter) are subsequently processed using five-stage low pass digital filters of FIR or better IIR types acting on the sub-samples. In the end it filters all frequencies with a -100 dB/decade roll-off above a user selectable cutoff frequency and acts as an effective anti-aliasing filter. The inherent resolution depends on the cutoff frequency and not on the sampling period. Typically the sampling rate is set at the Nyquist limit, or twice the cutoff frequency.

After the filtering zero crossings are evaluated which is followed by determination of the clock counts between nearby zero crossings corresponding to subsamples. Since the number of clock counts is determined more times, the regression analysis estimates their slope as depicted in Fig. 2. This approach gives a better measure of a signal period compared to a classical start-stop method. The numbers and errors in the plot of Fig. 2 serve only for illustration of the mentioned principle.

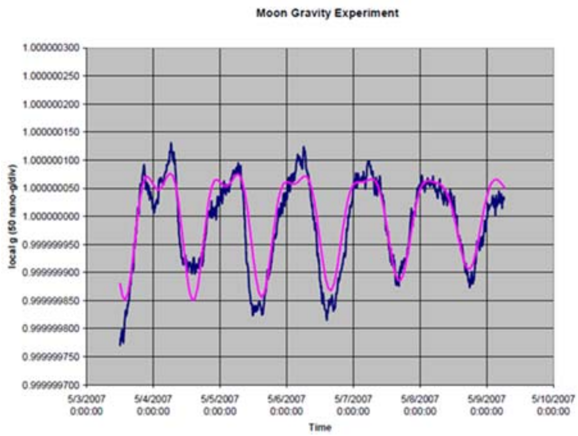

Fig. 1. Local vertical gravity changes using the regression FIR counting method and compared to predictions from the positions of the Sun and the Moon [1].

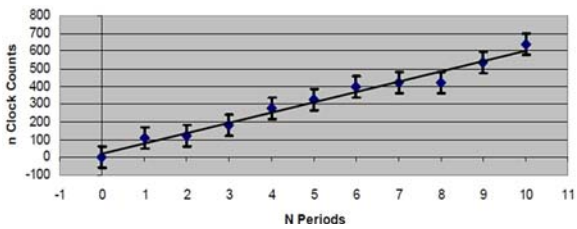

Fig. 2. The regression method used for the number of clock counts evaluated in subsamples.

The relative standard deviation of frequency estimation is given by [1]

$$\frac{\sigma_T}{T} = \sqrt{12}\,\frac{\sigma_t}{\sqrt{N_S^3 hT}}, \qquad (1)$$

where T represents a signal period, h is the number of signal periods per a subsample, N_S denotes the number of subsamples measured, σ_t is a standard deviation of the error caused by rounding and triggering in each subsample.

2.2. Estimation of Frequency using a Weighted Least Mean Square Method (WLSM)

The WLSM method [3] is based on finding the slope of tangent to a sinusoidal or cosinusoidal representation of the signal whose frequency is the subject of the measurement. In fact it is the modification of known methods of linear regression for nonlinear functions [4, 5]. In the case of a signal sinusoidal shape its values need to be transformed to the arcsin*x* function and then the slope of transformed function can be evaluated, see Fig. 3 for a cosinusoidal function. The values of the signal $s(t_i)$ are weighted by the multiplication with the coefficients W_i in order to respect the course of a sinusoidal shape of the signal. The samples lying closer to zero are multiplied by larger weight than those located around the signal maximums.

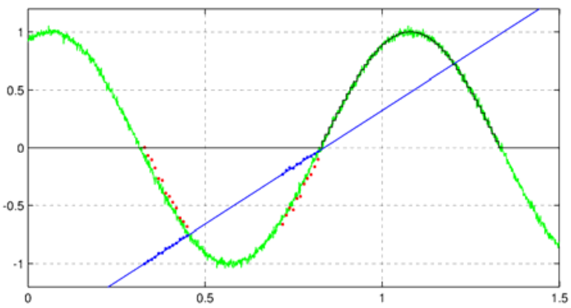

Fig. 3. Transformation of a sampled cos(2πx/T) waveform phase shifted by 2πx0/T (green line) to linear (blue line) by the inverse function. The scale of the horizontal axis corresponds to x/T.

For the estimated value $\hat{f}$ of frequency following relation is valid

$$2\pi \hat{f} = \frac{\sum_{i=1}^{N} W_i \sum_{i=1}^{N} t_i W_i z_i - \sum_{i=1}^{N} t_i W_i \sum_{i=1}^{N} W_i z_i}{\sum_{i=1}^{N} W_i \sum_{i=1}^{N} t_i^2 W_i - \left(\sum_{i=1}^{N} t_i W_i\right)^2}, \tag{2}$$

$$z_i = \arcsin \frac{s(t_i)}{A}, \tag{3}$$

$$W_i = \cos^2 z_i, \tag{4}$$

where $s(t_i)$ corresponds to the signal at time instances t_i, A is the amplitude of the signal, and σ^2 denotes the variance of the noise accompanying the signal.

The relative standard deviation σ_f of the measurement using WLSM method is given by [3]

$$\frac{\sigma_f}{f} = \frac{2.2\sigma}{(mn_p^3)^{1/2}}, \tag{5}$$

where n_p is the number of complete half periods in the observation window, f denotes a measured frequency, m is an average number of samples contained in one signal period.

The standard deviation for large n and *SNR* approximates the theoretically achievable value of CRB (Cramér Rao Bound) [5, 7]. This relation shows an important practical consequence: If σ means quantization noise of an A/D convertor used for digitalization of the signal, then its influence on accuracy of frequency determination is decreased by the factor

$$F_d = \frac{(mn_p^3)^{1/2}}{2.2}, \tag{6}$$

In other words, when a frequency carries information about a measured quantity, an achievable resolution using WLSM is higher than in the case when solely A/D conversion is used. According to (5) a relative (normalized) standard deviation of frequency evaluation does not depend on the frequency. That is a desirable property since a uniform error can be thus specified for a wide range of frequencies [3].

Rewriting (5) to a practical form by introducing the parameter N, which corresponds to the number of signal periods, i.e.

$$n_p = 2N \text{ and } m = \frac{n}{N},$$

where n is the total number of samples, it is possible to obtain the relative standard deviation defined as

$$\frac{\sigma_f}{f} = \frac{2.2\sigma}{(mn_p^3)^{1/2}} = \frac{2.2\sigma}{\sqrt{\frac{n}{N}(2N)^3}} = \frac{2.2\sigma}{\sqrt{8nN^2}} = \frac{0.78\sigma}{N\sqrt{n}}, \tag{7}$$

Based on (7) the standard deviation σ_f can be then expressed as

$$\frac{\sigma_f}{f} = \frac{0.78\sigma_f}{N\sqrt{n}}, \tag{8}$$

For practical calculations it is suitable to express σ in terms of SNR corresponding to an A/D convertor used for signal acquisition. In the case of frequency estimation for the signal amplitude A=1 and noise RMS n_{rms} the *SNR* is defined as

$$SNR = \frac{s_{rms}}{n_{rms}} = \frac{\sqrt{2}/2}{\sigma}, \tag{9}$$

By substituting (9) in (8) the standard deviation σ_f can be written in the form

$$\sigma_f = \frac{0.55 f}{N\sqrt{nSNR}}, \tag{10}$$

Furthermore, by substituting

$$N = t_m f \text{ and } n = t_m f_s,$$

where f_s denotes a sampling frequency and t_m is a measurement time interval, eq. (10) can be expressed as

$$\sigma_f = \frac{0.78\sigma_f}{N\sqrt{n}} = \frac{0.78\sigma_f}{t_m f \cdot \sqrt{t_m f_s}} = \frac{0.78\sigma_f}{\sqrt{t_m^3 f_s}}, \quad (11)$$

With respect to (11) the measurement time interval t_m has a strong influence on the accuracy of frequency evaluation.

2.3. Mutual Relation of a Standard Deviation between the WLSM and Nano-counting Method

A basic relation of the relative standard deviation for nano-counting approach defined in (1) can be further modified by substituting

$$N = N_s \text{ and } N_s = n$$

and then expressed as

$$\frac{\sigma_T}{T} = \sqrt{12}\frac{\sigma_t}{\sqrt{N_S^3}hT} = \sqrt{12}\frac{\sigma_t}{N_s h\sqrt{N_s}T} = \sqrt{12}\frac{\sigma_t}{N\sqrt{n}T}, \quad (12)$$

In contrast to the WLSM method the relative standard deviation of the nano-counting method is indirectly proportional to the period T.

3. Modification of the WLSM Frequency Estimation Method

In order to find optimal parameters of the WLSM method the analyses of the WLSM method were performed based on the study of influences of all particular key factors [2], i.e. weighted factors impacts as well as impacts of signal amplitude and offset variations and impact of length of a measurement time interval.

3.1. Influence of Weighted Factors W_i

To simplify the estimation procedure by replacing weighted factors in (2) originally defined by (4) with a simpler binary weighting following assumptions were made:

$$\begin{aligned} |s(t_i)| &\le \sqrt{1/2} \Rightarrow W_i = 1 \\ |s(t_i)| &> \sqrt{1/2} \Rightarrow W_i = 0 \end{aligned}, \quad (13)$$

where $\sqrt{1/2}$ represents the value of a decision level used for binary weighting.

Practically it means that samples lying in interval <-45°; +45°> are multiplied by W_i =1 and samples out of this interval are multiplied by W_i = 0. The results of simulation are shown in Fig. 4. A green line depicts the dependence of standard deviation when weighted coefficients are defined based on (4) (ideal WLSM method), in contrast a red line indicates the situation when a binary algorithm (BWLSM) using the assumption in (13) is implemented, dotted black line shows the case of minimal theoretically achievable uncertainty when a sinusoidal signal with additive white noise is used (Cramer-Rao Bound – CRB [7]). The green line shows the result of simulation of ideal WLSM method.

The progressions in Fig. 4 confirm a negligible difference (less than few percent) in standard deviation of frequency estimation between both methods WLSM and BWLSM. The more pronounced difference between methods in question appears in influence of decision level on standard deviation for case of changing the amplitude of the measured signal (see Fig. 5, blue and red dashed lines).

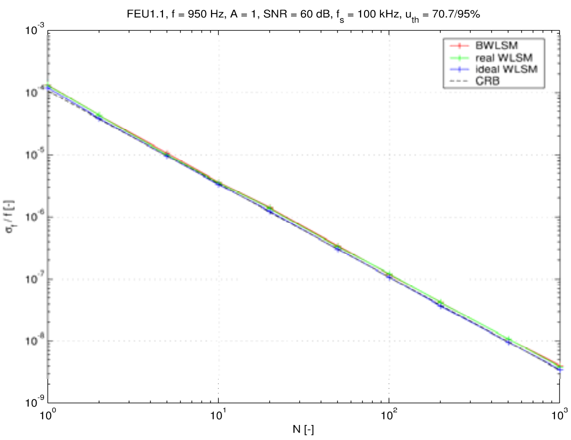

Fig. 4. Dependence of standard deviation on application of different weighted factors.

3.2. Influence of a Decision Level on the Accuracy of Frequency Estimation

Another important parameter of the BWLSM method is the decision (threshold) level u^{th} determining whether a particular sample is going to be used for the frequency estimation or not. The modification of the decision level allows reducing the influence of interfering effects on accuracy of frequency estimation. This dependence is shown for two types of a signal in Fig. 5. Solid lines correspond to the situation when the signal contains additive white noise with a normal distribution. In contrast, dashed lines depict the case when the signal is amplitude modulated with the depth of modulation equal to 1 %.

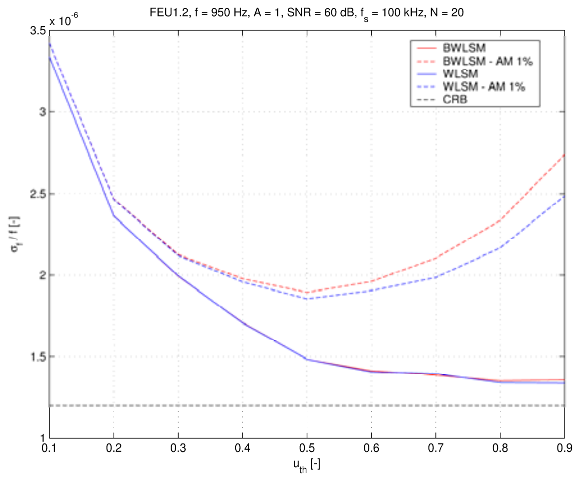

Fig. 5. Dependence of the relative standard deviation on a decision level u_{th}.

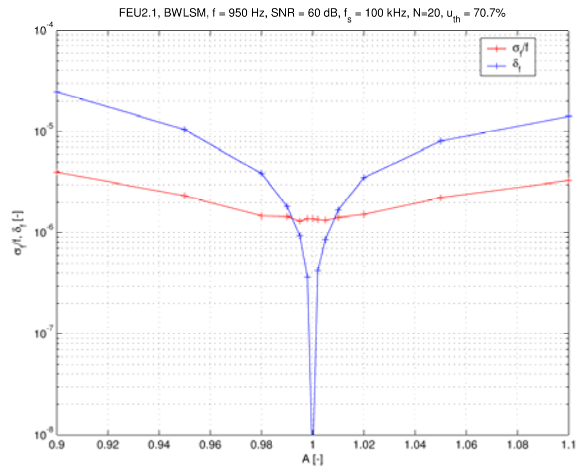

Fig. 6. Dependence of the relative standard deviation and a systematic error on a signal amplitude variation.

According to progressions in Fig. 5 it can be observed that in solid line cases the tendency of the standard deviation in declining with an increasing number of samples in contrast to the dashed line cases which correspond to certain types of signals with varying amplitude also occurring in practice.

These signals suffering from a harmonic distortion, non-unity amplitude, or non-zero offset corresponding curve have a convex shape with its minimum moving to zero for increasing deviation of the signal from an ideal sinusoidal waveform.

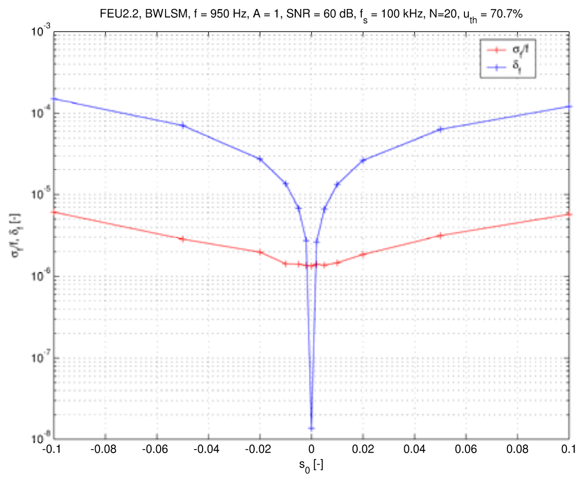

Fig. 7. Dependences of the relative standard deviation and a systematic error on a signal offset.

3.3. Effects of Signal Amplitude and Offset Variations on the Accuracy of Frequency Estimation

For a proper function of the WLSM frequency estimation method there is a condition which requires a unity/constant amplitude, i.e. A=1, and zero offset. When the amplitude A is constant but not equal to 1, it needs to be normalized. Unfortunately, this condition is hard to fulfill in practice.

In Fig. 6 and Fig. 7 there are shown the dependences of standard deviation of frequency estimation on a signal amplitude variation and offset. As can be seen even a small deviation of amplitude or a small offset can cause relatively large systematic errors of the frequency estimation.

According to Fig. 6 and Fig. 7 there can be observed that the variation of amplitude being up to 0.5 % and offset less than 0.1 % of the amplitude cause relative systematic errors in order of 10^{-7} which is in majority cases acceptable.

3.4. Influence of the Measurement Time Interval on the Accuracy of Frequency Estimation

With respect to (13) increasing the measurement time interval t_m leads to the suppression of disturbing effects on the frequency estimation. The dependence of a systematic error on the measurement time interval is shown in Fig. 8. The systematic errors caused by a non-unity amplitude and non-zero offset are depicted by red and blue dashed lines. The influence of error caused by noise equivalent to SNR = 60 dB, expressed by the standard deviation, is indicated by curves drawn in solid lines.

Trends of systematic errors caused by the variation of the amplitude (“err, A”) about 1 % while offset s_o is zero (dashed red line), and when offset s_o changes about 1 % of the amplitude, while A=1 (dashed blue line) can be characterized by the decrease proportional to the square of t_m. The increase of the t_m naturally leads to the reduction of the relative standard deviation with the slope of $t_m^{3/2}$, see (11).

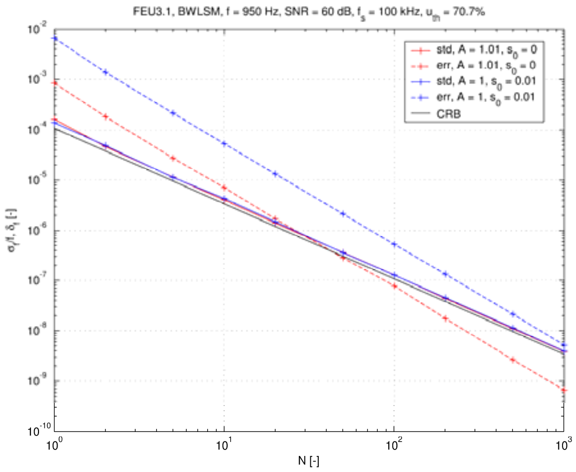

Fig. 8. Dependence of the relative standard deviation and the systematic error of the frequency estimation on the measurement time interval.

3.5. Reduction of the Systematic Error by a Double Estimation Method

As it was shown in previous analysis (Fig. 8), both systematic errors caused by non-unity amplitude and non-zero offset decline based on the square law. The method of double estimation exploits this law by using two estimations for two different measurement time intervals defined by numbers of periods N_1 and N_2. The values of frequencies f_1 and f_2, evaluated for those numbers N_1 and N_2, differ from a correct value of the frequency due to the existence of a total systematic error ε_T. Assuming that during both measurement and evaluation periods the ε_T is the same, the relation of a correct value of the frequency f_{cor} to frequencies f_1 and f_2 is given by

$$f_1 = f_{cor} + \frac{\varepsilon_T}{N_1^2} \text{ and } f_2 = f_{cor} + \frac{\varepsilon_T}{N_2^2}, \quad (14)$$

Based on (14) the corrected estimation of frequency f_{cor} can be calculated as

$$f_{cor} = \frac{N_1^2 f_1 - N_2^2 f_2}{N_1^2 - N_2^2}, \quad (15)$$

The influence of the correction using the double estimation method is shown in Fig. 9. This method is used as the extension of the BWLSM method, but can also be used for the original WLMS method. In Section 3.1 it was confirmed that performances of the WLSM and BWLSM are similar and their differences can be thus considered negligible. To confirm the advantageous applicability of the double estimation method the analyses and comparison of different methods, see Fig. 9, were carried out. The estimation was performed for two known frequencies and two numbers of periods N_1=N, where N corresponds to the number on x axis scale, and N_2 = $N/2$. It is important to notice that the double estimation method reduces only systematic errors (evoked mostly by variation of the amplitude and offset), while the standard deviation caused by stochastic processes occurring during the measurement is not influenced. However, in practice the systematic errors very often prevail, thus the double estimation method decreases substantially uncertainty of the overall frequency estimation method.

The efficiency of the double estimation method can be illustrated by the comparison of errors depicted by dotted lines in Fig. 8 and corresponding lines in Fig. 9. In both cases the signal parameters are the same. As shown in Fig. 9 for large N (N > 10) the frequency correction is very efficient and thus the systematic error caused by both amplitude variation about 1 % and offset equal to 0.01 V (1 % of the amplitude) is much smaller than the standard deviation. It is important to notice that decreasing number of periods leads to the increase of the error with respect to non-corrected estimation, see the lines near the origin in Fig. 9.

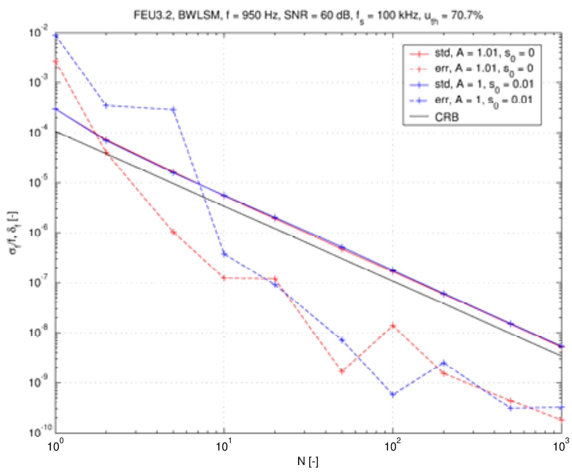

Fig. 9. Dependence of the relative standard deviation (std) and the relative systematic error (err) when the frequency corrected with respect to (15).

4. Experimental Verification of the Double Estimation Method

For experimental verification of the proposed double estimation method the algorithm used in the WLSM method was implemented into a microcontroller MSP430F1611 and a harmonic signal with anarbitrarily chosen frequency of 515 Hz with a unity amplitude and zero-offset was generated by a function generator [2]. The lines in Fig. 10 displaying relative systematic errors and standard deviations for the original WLSM and the WLSM extended by the double estimation method are in a good agreement with those in Fig. 9. This agreement confirms a proper function of the double estimation method.

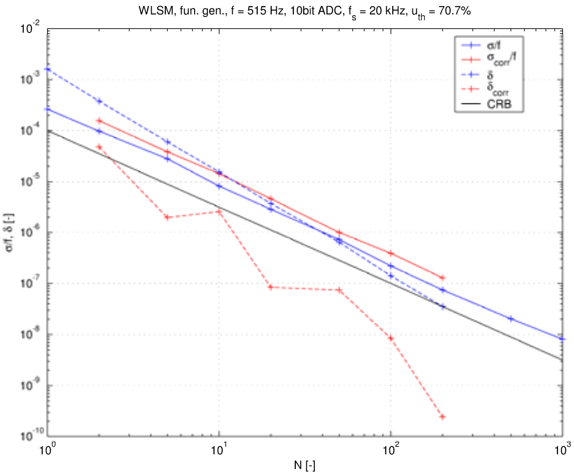

Fig. 10. Experimental verification and comparison of the WLSM and double estimation method.

Based on performed analyses and simulations the double estimation method modifying the original WLSM method as well as the BWLSM method thus appears as a suitable way to fulfill requirements for evaluation of a rapid variation of a resonance frequency while preserving the accuracy. Moreover, the proposed method in contrast to the nano-counting method requires only a small capacity of memory and possesses relatively simple processing algorithms.

5. Conclusion

The aim of this paper is to show the advantages of direct frequency estimation methods applied to output signal from resonance sensors carrying information about measured quantity instead of an intermediate transformation of frequency to voltage or current and subsequent A/D converter usage. The contribution of this paper arises from two already known and commonly used methods which are the nano-counting method and WLSM method. Novel methods modifying the WLSM method are proposed and their performances compared with the original WLSM method to confirm their suitability for estimating the frequency of resonance sensors with nano-g resolution.

A certain disadvantage of the WLSM methods lies in their limited application for harmonic signals only, while the nano-counting method is universally applicable. Fortunately, waveforms of resonance sensors' output signals are very close to sinusoidal due to resonance sensors inherently high quality Q factor. Nevertheless, a basic drawback of the WLSM method is generally the necessity to know the amplitude and offset of signal being estimated. Based on performed analyses of signal amplitude/offset variations, which affect the frequency estimation accuracy, there were proposed novel methods, i.e. binary weighted WLSM (BWLSM) method and the double estimation method enhancing the accuracy by frequency corrections. By performed analyses it was confirmed that proposed methods reduced the systematic error caused by variations in amplitude and offset which negatively affect the original WLSM method.

The BWLSM method further simplifies the weighting calculation used in the WLSM method by a single threshold leveling while preserving a comparable accuracy. This fact confirms the BWLSM method suitability for easier implementation to computational microcontroller units for real-time frequency estimation.

The nano-counting method with the resolution of 1.5×10^{-9} generally requires a 5 stage IIR filter with the total delay of 1.24 s and a corner frequency of 0.7 Hz [8]. A quantitative comparison of both nano-counting and WLSM methods can be illustrated on the example, when: the frequency of the signal is 35 kHz (a typical value for resonance sensors), the measurement time interval t_m equals to 1 s, the clock frequency in the nano-counting method equals to f_c=59 MHz and the sampling frequency in the WLSM method is f_s = 1 MHz. For the example a 16 bit A/D convertor is used which leads to SNR (SINAD) equal to 98 dB. According to (11) for the WLSM $\sigma_f=1.02\times10^{-13}$ while for the nano-counting method corresponding $\sigma_T=0.9\times10^{-9}$. To reach the resolution in order of 10^{-9} the signal needs to be additionally filtered, IIR filters are commonly used, and thus a caused delay would reach the value around 1 s. This fact limits the applicability of the nano-counting method for signals with rapid frequency changes. Furthermore, by combining the WLSM (BWLSM) with the double estimation method a final resolution is more than two orders higher than that of the nano-counting method while a computation time of the BWLSM method is only in order of several tens of ms.

Even if a certain drawback of WLSM/BWLSM method might seem to be the necessity to use A/D convertor, the convertor in this case does not require a high resolution about 30 bits. The requirements for the convertor resolution are low as long as a quantization error can be decreased proportionally to the value of $\sqrt{t_m^3 f_s}$. As the "by-product" WLSM method could be also used for testing of A/D convertors [9]. The WLSM and BWLSM methods further allow to estimate a frequency with limited accuracy even for t_m shorter than the signal period.

For a sinusoidal signal it might be just from its two samples.

Acknowledgements

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