Experiments of Phase Errors Estimation and Correction for High-resolution Airborne SAR Imaging

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Abstract: For the lack of high precision inertial measurement system, the experimental airborne L-band SAR system was badly affected by random phase error induced by motion instability. The resultant SAR images derived from the L-SAR system were out of focus by conventional SAR signal processing and imaging method. It is necessary to estimate and correct phase errors for high resolution SAR imaging. As a prospective scheme to refocus these images, the phase gradient autofocus (PGA) algorithm was studied in the SAR imaging processing. This paper focused on the calculation process of phase error estimation, which was first derived mathematically in detail. Then, the effectiveness of PGA algorithm was tested not only for isolated point scatter scene, but also for various distributed-object scenes to verify its performance. Particularly, a near-range scene, a mid-range scene, and a far-range scene were tested. Experiments show that the imaging algorithm based on phase gradient autofocus can effectively measure and compensate the phase error, thus obtain an obviously improved airborne SAR image in azimuth resolution. The processing results prove the effectiveness of the proposed procedure for high-resolution airborne SAR imaging. Copyright © 2013 IFSA.

Keywords: Coherent imaging, Phase error, Estimation, PGA, SAR signal processing.

1. Introduction

All coherent imaging techniques require extreme phase stability to avoid degrading image. In SAR systems, phase fluctuations are mainly due to two factors: propagation at GHz frequencies in turbulent atmosphere and/or troposphere, and unwanted deviations from the nominal trajectory of the platform carrying the antenna [1-3]. The first phase error source is dominant in the space-borne case, while in airborne applications the loss of synchronization is mainly due to, but not limited to, random deviations of the platform from the nominal trajectory [3]. These phase distortions can cause degradation in performance of the system, producing a blurred image. Thus phase error correction can’t be neglected for high-resolution airborne SAR imaging processing.

Methods have been developed to compensate for these effects [4-10]. They can mainly be grouped into two classes. In the first it is assumed that phase errors are a priori known by means of onboard instrumentation. These methods, requiring the accurate tracking of the antenna phase center, introduce very stringent specifications on the precision of the inertial measurement system. However, in many cases, the inertial navigation system (INS) is not precise enough to provide motion error parameters with required accuracy, and sometimes even cannot provide usable INS data [3, 4]. For example, due to the small size of light aircraft, light aircraft SAR usually has more serious
motion errors in air turbulence. However, limited by the cost or payloads, light aircraft SAR sometimes cannot carry high precision navigation systems [10]. So, there are always some residual errors left in the estimation of the actual flight path, especially in airborne SAR applications. Residual phase errors are the main cause of the degradations in high-resolution SAR images, limiting achievable performance especially in the azimuth direction. Moreover, they do not allow compensation of the phase perturbation due to other factors, such as propagation effects, that can still impair the accurate focusing of the image, even with perfect motion compensation [4-6].

The second class includes the methods assuming that the phase errors are not a priori known but they have to be determined by investigating some properties of the raw data [4-10], so also known as data-driven autofocus techniques. They are, in principle, independent of the source of phase errors. Various automatic focusing methods have been proposed and built to estimate and correct phase errors mainly induced by motion instability by using dedicated algorithms on raw SAR data [4, 7-10]. Most of these automatic focusing techniques and extensions are usually difficult to focus scenes without having strong isolated scatterer present. However, the method of phase gradient autofocus (PGA) makes a narrow beam assumption to estimate the phase errors by statistical method while the aircraft platform motion compensation system is not provided. It has been claimed to be very robust on a variety of scene contents and phase-error functions [4, 5, 10].

For the lack of high precision inertial measurement system, the experimental airborne L-band SAR imaging system was badly affected by random phase error induced by motion instability. As a prospective scheme to refocus these images, the phase gradient autofocus (PGA) algorithm was tested in the SAR imaging processing. This paper reveals the details of phase error calculation and presents the experiment of airborne L-SAR imaging from the phase-corrupted data based on PGA algorithm. The rest of this paper is organized as follows. In Section II, the calculation process of phase error estimation is derived mathematically in detail. In Section III, the imaging experiments are illustrated based on PGA phase error estimation and correction from raw SAR data. Also the refocusing results are presented. In Section IV, we conclude this paper and give some suggestions.

2. Calculation Process of Phase Errors Estimation

Previous publications [4, 6, 10] described the algorithmic steps of PGA and these are not repeated here in the same detail. The fundamental concept from which PGA was developed was to make a robust estimation of the derivative (gradient) of the phase error using only the defocused complex SAR image. In spotlight-mode SAR, the complex image and phase history are Fourier transform pairs [11]. The range-compressed phase-history domain data are obtained by a one-dimensional Fourier transform in the azimuth (or cross-range) direction. Let us denote the range-compressed phase-history domain data containing the phase error as

$$F_n(u) = |F_n(u)| \exp(j(\phi_n(u) + \theta_n(u))).$$

(1)

The subscript $n$ refers to the $n$th range bin, $u$ is the relative position along the synthetic aperture, $|F_n(u)|$ and $\phi_n(u)$ are the magnitude and phase, respectively, of the range-compressed data for range bin $n$. The uncompensated error $\phi_n(u)$ along the synthetic aperture is common to all range bins of interest and independent of $n$.

PGA assumes a narrow beam in range to exploit the redundancy of the phase error along scatters at different range bins. The critical steps of the PGA algorithm consist of center shifting, windowing and phase gradient estimation. It estimates phase errors using the data obtained by isolating many single defocused targets via center-shifting and windowing operations. Let us denote the shifted and windowed image data as $g_n(x)$ and

$$G_n(u) = \left| G_n(u) \right| \exp(j(\phi_n(u) + \theta_n(u))), \quad (2)$$

be the inverse Fourier transform. The scatterer-dependent phase function for each range bin is denoted by $\theta_n(u)$.

The derivative and conjugate of $G_n(u)$ can be calculated by

$$G'_n(u) = \left| G_n(u) \right| \exp(j(\phi_n(u) + \theta_n(u)))$$

$$= \left| G_n(u) \right| \exp\left(j(\phi_n(u) + \theta_n(u)) + j(\phi_n(u))
\right.

$$+ \theta_n(u)) \right| G_n(u) \exp(j(\phi_n(u) + \theta_n(u)))

$$= (\left| G_n(u) \right| + j(\phi_n(u))
\right.

$$+ \theta_n(u)) \right| G_n(u) \exp(j(\phi_n(u) + \theta_n(u)))$$

and

$$G^*_n(u) = \left| G_n(u) \right| \exp(-j(\phi_n(u) + \theta_n(u)) \right), \quad (3)$$

where $G'_n(u)$ and $G^*_n(u)$ denote the derivative and conjugate, respectively. Hence, we have

$$G'_n(u)G^*_n(u) = \left| G_n(u) \right| \left| G_n(u) \right|^2$$

$$+ j(\phi_n(u) + \theta_n(u)) \right| G_n(u) \right|^2.$$

The phase error is related with the imaginary part of the above equation, which is given by

$$\text{Im}\{G'_n(u)G^*_n(u)\} = (\phi_n(u) + \theta_n(u)) \right| G_n(u) \right|^2.$$  

By using the above equation, the following expression is obtained:

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\[ \sum_n \text{Im}\{G_n' (u)G_n^*(u)\} \]
\[ = \sum_n (\phi_e' (u) + \theta_n (u)) |G_n(u)|^2 \]
\[ = \sum_n \phi_e' (u) |G_n(u)|^2 + \sum_n \theta_n' (u) |G_n(u)|^2 . \]
\[ \phi_e' (u) \sum_n |G_n(u)|^2 = \sum_n \theta_n' (u) |G_n(u)|^2 . \]

A linear unbiased minimum variance estimate of the gradient of the phase error, \( \phi_e (u) \), is given by,

\[ \hat{\phi}_e (u) = \frac{\sum_n \text{Im}\{G_n' (u)G_n^*(u)\}}{\sum_n |G_n(u)|^2} \]
\[ = \phi_e' (u) + \frac{\sum_n \theta_n' (u) |G_n(u)|^2}{\sum_n |G_n(u)|^2} . \]

Thus, the estimate yields the gradient of the true phase error, plus an error term,

\[ \sum_n \theta_n' (u) |G_n(u)|^2 \]
\[ \sum_n |G_n(u)|^2 . \]

The PGA phase gradient estimation kernel is defined as the second term in the above equation. The mathematical expectation of random variable \( \theta_n (u) \) is 0. Accordingly, the mathematical expectation of the linear transformation (i.e. derivative) of \( \theta_n (u) \) is 0. Therefore, when \( n \) is sufficiently large, the error term will tend to 0, then the estimation of phase error, \( \hat{\phi}_e (u) \), becomes more accurate.

The estimated phase gradient, \( \hat{\phi}_e (u) \) is integrated to obtain \( \hat{\phi}_e (u) \), and any bias and linear trend is removed prior to correction. Phase correction is imposed by complex multiplication of the range compressed phase-history domain data by \( \exp[-j \hat{\phi}_e (u)] \). The estimation and correction process is repeated iteratively. As the image becomes more focused, individual scatterers become more compact, the signal-to-clutter improves, the circular shifting more precisely removes the Doppler offsets, and the algorithm is driven toward convergence. Removal of any linear trend in the phase error estimation prevents image shifting and the bias removal allows computation of the RMS phase error removed at each iteration as a means for monitoring convergence. The estimation and correction process is repeated iteratively until the algorithm converges to the true phase error. Once a reliable estimate for the phase error is available, the raw SAR data is corrected to obtain highly improved reconstructions. Generally, when the RMS error drops to a few tenths of a radian, the image is well focused and will not improve with additional iterations.

The estimated phase error for actual phase correction is given by

\[ \hat{\phi}_e (u) = \int \hat{\phi}_e (u) \]
\[ = \sum_n \text{Im}\{G_n' (u)G_n^*(u)\} \]
\[ \sum_n |G_n(u)|^2 \]
\[ = \phi_e' (u) + \frac{\sum_n \theta_n' (u) |G_n(u)|^2}{\sum_n |G_n(u)|^2} . \]

The mathematical expectation of the error term, \( \int \sum_n \theta_n' (u) |G_n(u)|^2 \)
\[ \sum_n |G_n(u)|^2 \], will tend to 0 for \( \sum_n \theta_n' (u) |G_n(u)|^2 \)
\[ \sum_n |G_n(u)|^2 \] is approaching 0. With the mathematical analysis, it can be seen that the estimation process exploits the redundancy of the phase-error information contained in the degraded image, independent of the underlying scene content.

### 3. Imaging Experiments

The whole imaging scheme can be divided into the following steps: SAR complex image creation; center cyclic shifting; windowing; Fast Fourier Transform in the azimuth direction; estimation of phase error; phase error correction and Inverse Fast Fourier Transform. The critical steps of the PGA algorithm consist of center shifting, windowing and phase gradient estimation. These steps are repeated iteratively until the algorithm converges to the true phase error. The first step in PGA is to select for each range bin n the strongest scatterer and shift it to the origin (center of the image), to remove the frequency offset due to the Doppler of the scatterer. This shifting operation is done as a circular buffer where samples that would be shifted off the left or right edge of the array, are instead wrapped around and shifted in from the opposite edge. The next important step is windowing the circularly shifted imagery. The width of window should cover the most part of energy of the point target. Windowing has the effect of preserving the width of the dominant blur and suppressing the pulse echo disturbance of neighboring targets for each range bin while discarding data that cannot contribute to the phase-error estimation. This allows the phase-error estimation to proceed using input data having the highest signal-to-noise ratio. After the image data is circularly shifted and windowed, the phase gradient is estimated. In Section II, the calculation process of phase error estimation is derived mathematically in detail.

The experimental signal data were acquired by the experimental airborne L-SAR system without any motion measurement instruments. The time of the whole aperture duration was about 20s for the L-SAR system. For such a long interval, the aircraft can’t
keep flying in a uniform beeline motion and the phase stability was badly affected in a whole aperture period. To obtain a refocused image, we adopted the above signal processing and phase error compensation method to perform imaging processing.

Firstly, we illustrate the processing of PGA algorithm with a distinct point target in the middle of SAR image by analyzing the azimuth time signal of the point target. Fig. 1 (a) shows a profile of imaging result along range samples before phase error compensation. We can find that there exists an obvious point target. Fig. 1 (b) shows the details of the point target of Fig. 1 (a), which is severely out of focus. Fig. 1 (c) shows the correction phase after phase gradient estimation. From Fig. 1 (c), we inferred that the aircraft velocity varied obviously in the 4000-6000 azimuth samples. For these variations, the point target was out of focus and had two peak values in the Fig. 1 (b). Fig. 1 (d) illustrates the refocused point target after phase error correction. We can easily see that the point target has achieved the radar extreme resolution from analyzing the azimuth time signal of point target after phase error compensation.

Fig. 1. A scene containing an isolated point target.
By analyzing the azimuth time signal of point target after center cyclic shifting, we knew that the sampling rate of radar system is not uniform. From Fig. 1 (a)–(d) with a distinct point target in the middle of SAR image, we estimated phase gradient and gave out the corrected phase after estimation. Before phase error correction, the point target is not well focused. After phase compensation, the quality of refocused SAR image scene is greatly improved. Fig. 1 (e) and (f) show the image scene with the distinct point target before and after phase compensation, respectively.

With the point scatter scene processing, it is relatively easy to verify the whole imaging procedure of PGA method. As mentioned above, the most important characteristic of PGA is that, PGA is independent on SAR scene content. So, we carried out the PGA imaging algorithm to various distributed-object scenes to test its effectiveness in the experiment. The experimental data were also acquired by the same experimental airborne L-band SAR system without any motion measurement instruments. The scenes without obvious isolated point target before and after PGA refocusing were compared. Particularly, a near-range scene, a mid-range scene, and a far-range scene were tested.

4. Conclusions

For the airborne L-SAR system without high precision inertial measurement instrumentation, it is necessary to use data-driven autofocus techniques for focusing the imagery. In our experiment, results showed that the imaging scheme based on phase gradient autofocus can effectively measures and compensate the phase error, thus obtaining an obviously improved airborne L-SAR image in azimuth resolution. We tested the effectiveness of PGA algorithm not only for isolated point scatter scene, but also for various distributed-object scenes to verify its performance. Experiment results indicated that PGA imaging is successful to provide near diffraction-limited performance in nearly all cases. Specifically, strong isolated point-like scatterers are not needed.

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