Sensors & Transducers, Vol. 183, Issue 12, December 2014, pp. 265-272

Sensors & Transducers

Effects on Mechanical Properties of Track Structure and Running Safety Caused by Uneven Settlement of Bridge Piers

Yang Song, * Xiao Hong, Huang Lu Wei
Bei Jing JiaoTong University, 100044, Bei Jing China
* Tel.: 010-51687247
* E-mail: xiaoh@bjtu.edu.cn

Received: 15 October 2014 /Accepted: 28 November 2014 /Published: 31 December 2014

Abstract: A static and a dynamic analysis model are separately established using ABAQUS finite element program to study the mechanical properties of track structure and running stability and safety of the vehicle under the condition that uneven settlement of bridge piers exists. The static model is aimed at studying the effects caused by uneven settlement alone and the dynamic one is for the effects induced by uneven settlement and train loads together. Conclusions are drawn as follows: 1) The influence range of uneven settlement on fastenings is 10 groups of fastenings on both sides of the beam joint. Among them the influence on the first group of fastenings is the largest. 2) The influence area of uneven settlement on rail stress is within the range of 15 m on both sides of the beam joint. The bigger uneven settlement is, the larger rail stress will be. 3) Uneven settlement of bridge piers has certain effects on running stability and safety, but every evaluating indicator is within the specification limits, showing that the operation has good emergency capacity. 4) The influence induced by uneven settlement is much greater on the mechanical characteristics of rail and fastenings than on running stability and safety, making it clear that the factor that determines the limit of uneven settlement of bridge piers is fastening force and rail stress, not the running stability and safety. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Uneven settlement of bridge piers, ABAQUS, Mechanical property, Tension of rail fastenings, Rail stress, Running stability and safety.

1. Introduction

The construction of urban rail transit in China has been developing very fast these years. According to the Twelfth Five-year Plan in Transportation, 7395 kilometers of subway lines of which the total cost reaches 3800 billion will be built in China in the next ten years. It is expected that 33 cities will have 177 subway lines in 2020. With large numbers of urban rail transit completed and put into operation, some structural injuries appear continuously. Among them, uneven settlement between adjacent piers is one of typical structural injuries, which is induced by failure of bridge bearings, extraction of groundwater, construction errors, etc. For traditional ballast track structure, this tiny displacement has little influence on track structure because of the properties of granularity and mobility of the ballast. But for ballastless track structure, small deformation of the bridge will have impacts on the mechanical behaviors of track structure because of the rigid connection between track structure and the bridge, especially for rail and fastenings [1, 2]. In addition, uneven settlement of adjacent piers will lead to additional track irregularity on rail surface, which will have certain effects on running stability and safety [3]. Therefore, it has important theoretical significance and engineering value to do researches on the effects on mechanical properties of track structure and running safety caused by uneven settlement of bridge piers.

Uneven settlement of bridge piers will lead to sinking and vertical corner at beam end. Thus, the problem of uneven settlement is actually the problem of beam end displacement. At present, lots of researches on beam end displacement relevant problems have already been carried out in many countries. In China, effects caused by uneven settlement on track structure, especially on fastening systems are studied by Zhao Pingrui, Wei Yahui [1, 2, 4] while the vibration response property of vehicle-track-bridge coupled system under differential settlement of piers is studied by Song Guohua [5]. In Japan, Railway Building Design Standards and Interpretations – Deformation Limit [6] is developed from existing researches, in which detailed provisions about the deformation of ballastless track at beam end is written. In Germany, two relevant criteria are issued, respectively the Basic Principles and General Requirements on Ballastless Bridge (DS504.5401) [7] and Check for Ballastless Track Locating on the Beam-end (DS504.5405) [8], in which the limit of fastening force, construction measures aimed to reduce fastening force, check method on ballastless track at beam end, etc are specified.

Although many scholars at home and aboard have done a lot of researches on uneven settlement problems, most of the current studies are based on the division of statics. Researches on mechanical properties of track structure under the combined action of dynamic train load together with uneven settlement of bridge piers are still not reported yet. To cover the shortage of current researches, a static and a dynamic analysis model are established in this paper using ABAQUS finite element program to study the mechanical behaviors of rail and fastenings and the running stability and security not only under the action of differential settlement of adjacent piers alone but also under the combined action of dynamic train load and uneven settlement. The research results can provide scientific basis for failure of rail and fastenings of track structure in bridge section in urban rail transit, and can offer suggestions for monitoring, maintenance and replacement of track structural parts.

2. Finite Element Model

In consideration of the superiority of ABAQUS in processing wheel/rail contact, as well as the powerful nonlinear solution ability, ABAQUS finite element program is intended to be used to establish the static and dynamic analytical models to do researches on the effects on mechanical behaviors of track structure and operation stability and security caused by uneven settlement of bridge piers.

2.1. Static Analysis Model

It is consist of rail, fastenings, longitudinal track girder and bridge. Rail is the type of CHN60, simulated by solid element. On rail surface German railway low interference spectrum is applied to simulate actual irregularity of the rail [9]. Longitudinal track girder is concrete structure, simulated by solid element. The bridge is simple-box simple-cell simply-supported box girder of which the span is 32 m. The bearing is 0.55 m away from the beam end. The cross section of the box girder is shown in Fig. 1. The material parameters of rail, longitudinal track girder and bridge are shown in Table 1.

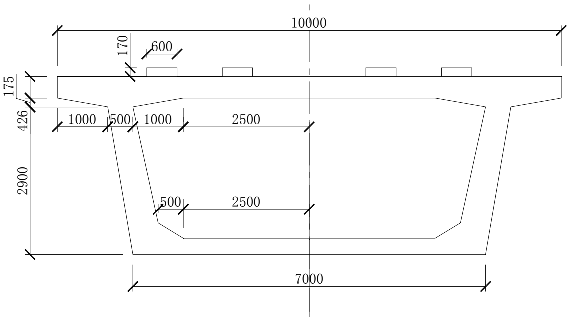


Fig. 1. Cross section of box girder (unit: mm).

Table 1. Material parameters of track and bridge structure.

Items	Modulus of elasticity (Pa)	Poisson ratio	Density (kg/m^3)
Rail	2.1×10^{11}	0.3	7830
Longitudinal track girder	3×10^{10}	0.167	2500
Box girder	3×10^{10}	0.167	2500

The type of fastenings is DTVII2 which are largely used on bridges in Beijing subway lines. The longitudinal and transverse supports of fastenings are simulated by linear spring elements while the vertical support is simulated by nonlinear spring element. The vertical stress status of fastenings can be divided into 3 stages, correspondingly, the vertical stiffness should be taken as three discrete values according to the following formula [4]:

$$K_F = \begin{cases} K_P & \Delta y < -F_0/2K_S \\ K_P + 2K_S & -F_0/2K_S < \Delta y < F_0/K_P \\ 2K_S & F_0/K_P < \Delta y \end{cases}, \quad (1)$$

where K_F is the joint stiffness of fastening in kN/mm, K_P is the stiffness of resilient tie pad in kN/mm, K_S is one sided stiffness of elastic rod in kN/mm, Δy is the vertical deformation of fastening in mm, F_0 is the initial toe load of fastening in kN. As to DTVII2 fastenings, K_P, K_S, F_0 are respectively adopted as 30 kN/mm, 0.36 kN/mm, 8 kN [10].

Bridge bearings are simulated by linear spring elements. Mechanical parameters of fastenings and bearings are shown in Table 2.

In order to eliminate the influence of boundary conditions, four spans of box girder are adopted. Uneven settlement of piers is realized by applying displacement boundary condition to No. 3 pier. Schematic diagram of the model is shown in Fig. 2.

Table 2. Mechanical parameters of fastening and bearing.

Mechanical index	Fastening	Bearing
Vertical stiffness(N/m)	–	3×10^9
Lateral stiffness (N/m)	2.4×10^7	3×10^9
Longitudinal stiffness (N/m)	2.4×10^7	3×10^9
Vertical damping (N·s/m)	1.25×10^4	1×10^5
Lateral damping(N·s/m)	1×10^4	1×10^5
Longitudinal damping(N·s/m)	1×10^4	1×10^5

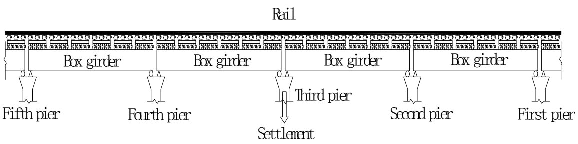


Fig. 2. Schematic diagram of static analysis model.

2.2. Dynamic Analysis Model

Vehicle-track-bridge three-dimensional coupled dynamic analysis model is established based on the static analysis model, which is purposed to study the mechanical behaviors of rail and fastenings and the running stability and safety under the combined action of dynamic train load and the uneven settlement.

The metro vehicle is B car-type metro in the model, of which the main parameters are as follows: The length is 23.69 m, the height is 3.0 m, the rigid wheelbase is 2.2 m, the length between truck centers is 12.6 m and the axle load is 14.1 t. The assumptions are shown below: car body, bogies and wheelsets are regarded as rigid bodies, not considering their elastic deformation. Three frame members are symmetrical, not considering the eccentric effect. The first suspension and secondary suspension are simulated by spring-damper element, not considering the nonlinear characteristics. 5 degrees of freedom that are sinking and floating, sideslip, nodding, rolling and yawing are considered for car body and the bogie. 4 degrees of freedom that are sinking and floating, sideslip, rolling and yawing are considered for wheelsets, not considering its rotation around the wheel axle. The whole vehicle model that established is shown in Fig. 3.

In vehicle-track-bridge coupled dynamic analysis model, wheel/rail contact relationship is the vital factor that directly affects the solution of wheel/rail force. Wheel/rail contact relationship includes two aspects: wheel/rail normal interaction and tangential interaction. As to the normal force, it is assumed to be calculated using Hertz non-linear contact theory, concretely by the equation below:

$$P(t) = [\frac{1}{G}\Delta Z(t)]^{3/2}, \tag{2}$$

where G is the wheel/rail contact constant in $m/N^{2/3}$, $\Delta Z(t)$ is the wheel/rail compression amount at t moment in m. As to worn-tread wheel, $G = 3.86R^{-0.115}\times10^{-8}(m/N^{2/3})$, where R is the wheel radius in m.

Wheel/rail tangential interaction is described by penalty function friction model, wheel/rail friction coefficient taken as 0.3.

Vehicle-track-bridge three-dimensional coupled dynamic analysis model that established is shown in Fig. 4.

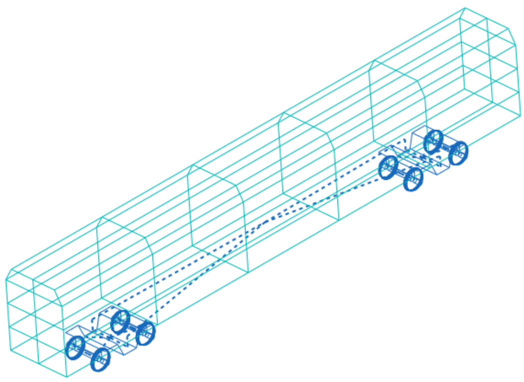


Fig. 3. Whole vehicle model.

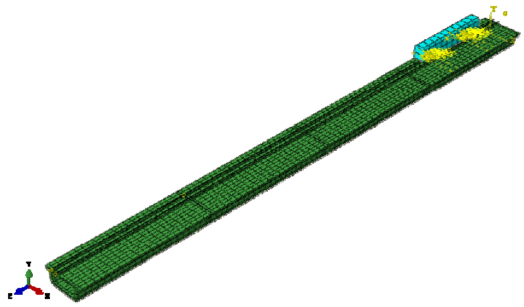


Fig. 4. Vehicle-track-bridge coupled dynamic analysis model.

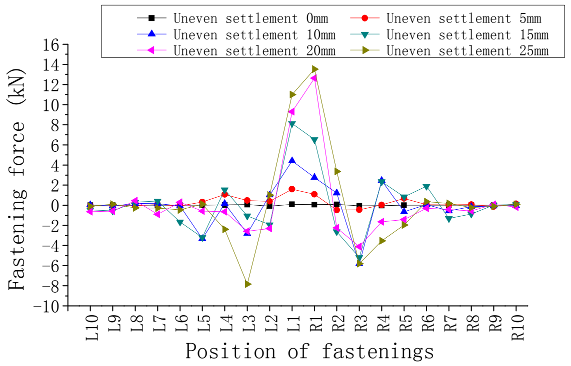
3. Influence Analysis on Mechanical Properties of Track Structure

According to *Code for Design of Metro* (GB50157-2013), uneven settlement between adjacent piers should not exceed 20 mm. But in order to obtain the influencing rule of uneven settlement on mechanical behaviors of track structure and operation security, the values of uneven settlement for calculation in this paper should be slightly more than the limit value in the code. Thus, six conditions are calculated for static and dynamic analysis: the values of uneven settlement are separately 0 mm, 5 mm, 10 mm, 15 mm, 20 mm and 25 mm.

3.1. Results of Static Analysis

1) Analysis on fastening force.

Force conditions of the fastenings on both sides of the beam joint under different conditions are shown in Fig. 5.



Note: L represents left side while R represents right side of the beam joint.

Fig. 5. Force conditions of the fastenings on both sides of the beam joint.

Conclusions can be drawn from Fig. 5: 1) Uneven settlement of bridge piers has large influence on the force condition of fastenings. Fastenings which are further away from the beam joint suffer smaller forces while those which are nearer to the beam joint suffer larger forces. 2) The force of the tenth group of fastenings is nearly 0, showing that the influence range of uneven settlement on fastenings is 10 groups on both sides of the beam joint. 3) The impact of differential settlement on the first group of fastenings on both sides of the beam joint is largest and the fastening force is tension. Fastenings except for the first group suffer much less forces than the first group and the fastening forces are tension or compression randomly. 4) With uneven settlement increasing, the tension of the first group of fastenings increases. Values of the tension of the first group of fastenings under different conditions are shown in Table 3.

It can be seen from Table 3 that the tensile forces of the first group of fastenings on both sides of the beam joint increase with uneven settlement increasing. In order to ensure the safe use of fastenings, fastening tension should not exceed the crimping force of the elastic rod. As to DTVII2 fastenings, the crimping force of one elastic rod is 4 kN, making the toe load of one group of fastenings 8 kN. When the uneven settlement reaches 15 mm, the tension of the first group of fastenings on left side of the beam joint has already exceeded 8 kN, causing the injuries like fracture of the elastic rod to happen. Therefore, the force condition of the first group of fastenings on both sides of the beam joint should be paid more attention to when uneven settlement of bridge piers exist. Fastenings at such positions are suggested being replaced of those whose toe loads are larger.

Table 3. Tension of the first group of fastenings on both sides of the beam end.

Conditions	First group of fastenings on left side (kN)	First group of fastenings on right side (kN)
Uneven settlement 0 mm	0.091	0.068
Uneven settlement 5 mm	1.595	1.080
Uneven settlement 10 mm	4.405	2.750
Uneven settlement 15 mm	8.130	6.540
Uneven settlement 20 mm	9.290	12.630
Uneven settlement 25 mm	10.995	13.520

2) Analysis on rail stress

The vertical displacement of rail when uneven settlement is 20 mm is shown in Fig. 6. It can be seen from Fig. 6 that when No. 3 pier goes down by 20 mm than adjacent piers the rail also sinks correspondingly, leading to the angle shape irregularity on rail surface. When flexural bending of rail appears, stress is produced inside the rail due to the constraints of undeformed region of rail and subjacent fastenings. Rail stress along the longitudinal direction of the route under different values of uneven settlement is shown in Fig. 7.

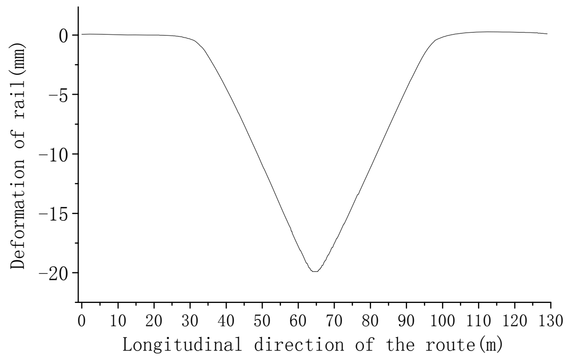


Fig. 6. Flexure curve of rail.

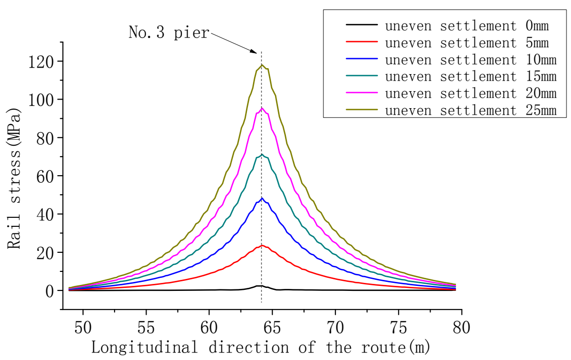


Fig. 7. Rail stress along the longitudinal direction of the route.

Conclusions can be drawn from Fig. 7: 1) Uneven settlement of bridge piers has large influence on rail stress. Stress of the rail which is further away from the beam joint is smaller while that is nearer to the beam joint is larger. 2) Rail stress goes down to 0 at the position of about 15 m away from the beam joint, indicating that the influence area of uneven settlement on rail stress is within the range of 15 m on both sides of the beam joint. 3) The larger differential settlement is, the larger the rail stress will be. The maximum value of rail stress when differential settlement is 5 mm is 23.5 MPa while that when differential settlement is 25 mm is 118 MPa. The latter is 4 times than the former, increasing significantly.

According to the researches by relevant references [4], as to continuously welded rail track using CHN60 rail, additional rail stress caused by deformation at beam end should not exceed 102 MPa. The additional stress of rail calculated in this section when uneven settlement is 25 mm reaches 118 MPa, which is more than the allowable value. Thus, monitoring of rail stress should be enhanced when uneven settlement of bridge piers reaches 20 mm and larger so as to avoid rail yielding.

3.2. Results of Dynamic Analysis

1) Analysis on fastening force.

Time-history curves of the force condition of the first group of fastenings on left and right sides of the beam joint when uneven settlement is 20 mm are shown in Fig. 8 and Fig. 9.

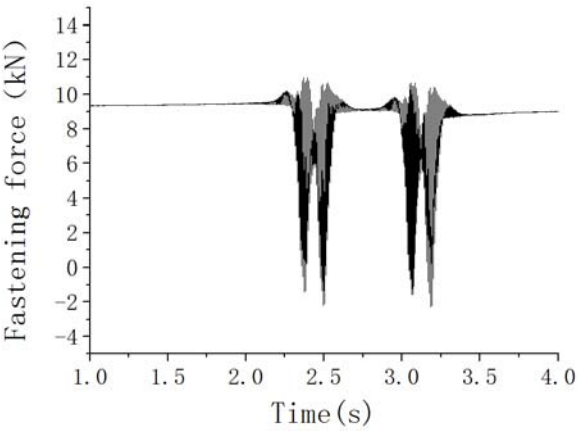


Fig. 8. Time-history curve of fastening force on left side of the beam joint.

As can be seen from Fig. 8, as to the first fastening on left side of the beam joint, the fastening is in tensile condition when 4 wheelsets has not yet reached its position while it is in compression condition when 4 wheelsets pass right by its position. The fastening tension reaches the maximum value between two wheelsets of one bogie. This is because the rail beneath the wheelsets has a downward displacement at this moment, making the rail above fastenings having the trend of going upward, which increases the fastening tension.

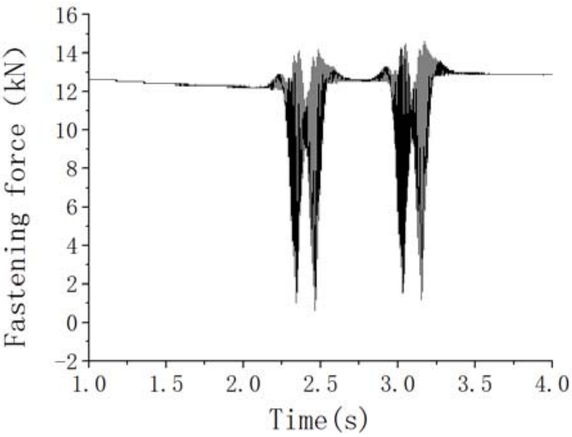


Fig. 9. Time-history curve of fastening force on right side of the beam joint.

As can be seen from Fig. 9, as to the first fastening on right side of the beam joint, it is in tensile condition in the whole time period. When 4 wheelsets pass right by the fastening position, fastening tension is the least while when fastening is between two wheelsets of one bogie, fastening tension is the largest.

The force condition of the first group of fastenings on both sides of the beam joint under the train dynamic load is shown in Table 4.

Table 4. Force condition of the first group of fastenings on both sides of the beam joint.

Positions / Conditions	Left side of the beam joint		Right side of the beam joint	
	T	C	T	C
Uneven settlement 0 mm	1.8	12.44	1.86	12.77
Uneven settlement 5 mm	3.2	10.51	2.63	10.83
Uneven settlement 10 mm	6	7.7	4.62	9.02
Uneven settlement 15 mm	9.79	3.86	8.39	5.16
Uneven settlement 20 mm	11	2.31	14.62	None
Uneven settlement 25 mm	12.73	0.92	15.78	None

Note: T represents tension while C represents compression

It can be seen from Table 4 that the tension of the first group of fastenings increases gradually while the compression decreases with the increase of uneven settlement. When uneven settlement reaches 15 mm, the tension of fastenings has exceeded the toe load of DTVII2 fastening. Therefore, the force condition of the first group of fastenings on both sides of the beam end should be focused on. It is proposed that fastenings at this position should be replaced by fastenings with larger crimping force.

2) Analysis on dynamic stress of rail

The dynamic stress of rail above No. 3 pier under different conditions is shown in Table 5:

Table 5. Dynamic stress of rail above No. 3 pier.

Condition	Rail dynamic stress (MPa)
Uneven settlement 0 mm	56.4
Uneven settlement 5 mm	103.8
Uneven settlement 10 mm	176.1
Uneven settlement 15 mm	251.1
Uneven settlement 20 mm	341.9
Uneven settlement 25 mm	433.2

As can be seen from Table 5, the dynamic stress of rail increases with the increase of differential settlement. The dynamic stress of rail is 56.4 MPa when differential settlement is 0 mm while it is 433.2 MPa when differential settlement is 25 mm. The latter increases nearly 7 times than the former, indicating that differential settlement has great impact on dynamic stress of rail under the dynamic train load.

According to Hot-rolled Steel Rails for Railway (GB2585-2007), the allowable stress of rail is 352 MPa for CHN60 rail in continuously welded rail track. When uneven settlement of bridge piers reaches 25 mm, the calculated dynamic stress of rail is 433.2 MPa, which is much larger than the allowable stress. It is thus clear that additional irregularity on rail surface is produced due to uneven settlement of bridge piers, which will sharply increase wheel/rail contact force, worsen wheel/rail contact relationship, cause oversize dynamic stress inside rail. Therefore, more attention should be paid to the monitoring of rail stress when uneven settlement reaches 20 mm and larger so as to prevent the rail from yielding.

4. Influence Analysis on Running Stability and Safety

4.1. Influence Analysis on Running Stability

The indexes that are used to evaluate running stability are vertical and lateral accelerations of the vehicle. Time-history curves of vertical and lateral accelerations of the vehicle are shown in Fig. 10 and Fig. 11.

As can be seen from Fig. 10 and Fig. 11, the vertical acceleration of the vehicle increases with the increase of uneven settlement. When vehicle passes by No. 3 pier which settles down (corresponding 2.35 s~3.24 s), the value of the vertical acceleration of the vehicle reaches maximum, indicating that the vertical acceleration of the vehicle will increase when uneven settlement exists. As to the lateral acceleration of the vehicle, the variety regulation with the increase of uneven settlement at different moments is different. The maximum value of lateral acceleration of the vehicle does not appear in the time period when the vehicle passes by No. 3 pier. This suggests that uneven settlement has great impact on the vertical acceleration of the vehicle while little impact on the lateral acceleration.

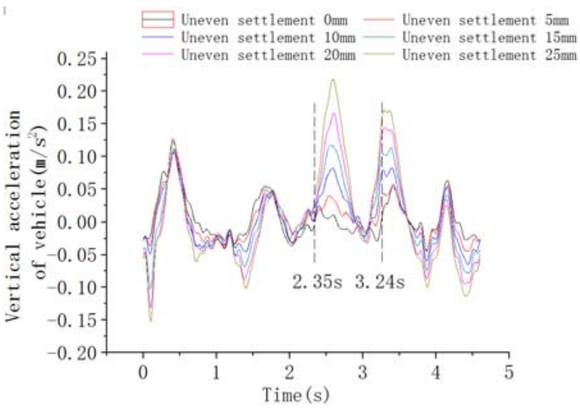


Fig. 10. Time-history curve of vertical acceleration of vehicle.

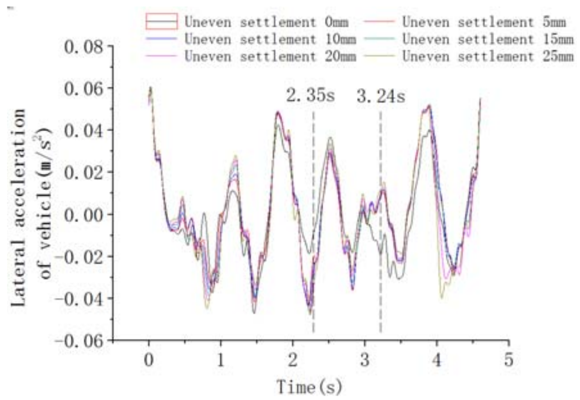


Fig. 11. Time-history curve of lateral acceleration of vehicle.

The maximum values of vertical and lateral accelerations of the vehicle under different conditions are shown in Table 6.

Table 6. Maximum values of vertical and lateral accelerations of the vehicle.

Conditions	Vertical acceleration of the vehicle (m/s^2)	Lertical acceleration of the vehicle (m/s^2)
Uneven settlement 0 mm	0.106	0.060
Uneven settlement 5 mm	0.108	0.044
Uneven settlement 10 mm	0.111	0.059
Uneven settlement 15 mm	0.121	0.060
Uneven settlement 20 mm	0.166	0.060
Uneven settlement 25 mm	0.218	0.059

It can be seen from Table 6 that the vertical acceleration of the vehicle increases with the increase of uneven settlement of bridge piers. The value when uneven settlement is 0 mm is 0.106 m/s^2 while it is 0.218 m/s^2 when uneven settlement is 25 mm, the latter increasing by one time than the former. Lateral acceleration of the vehicle does not increase when uneven settlement increases.

According to Railway Vehicles-Specification for Evaluation the Dynamic Performance and Accreditation Test (GB5599-85), the vibration acceleration of railway passenger car should meet the following requirements:

$$\begin{cases} a \le 0.13g & \textit{Vertical} \\ a \le 0.1g & \textit{Lateral} \end{cases}, \quad (3)$$

It can be seen that although the vertical acceleration increases with the increase of uneven settlement, the values are within the safe range specified in Equation 3.

4.2. Influence Analysis on Running Safety

Running safety indicators that calculated under different conditions are shown in Table 7.

Table 7. Calculated results of running safety indicators.

Conditions	Derailment coefficient	Rate of wheel load reduction
Uneven settlement 0 mm	0.166	0.435
Uneven settlement 5 mm	0.162	0.483
Uneven settlement 10 mm	0.164	0.485
Uneven settlement 15 mm	0.167	0.491
Uneven settlement 20 mm	0.170	0.496
Uneven settlement 25 mm	0.173	0.504

It can be seen from Table 7 that derailment coefficient and rate of wheel load reduction both have increasing trends with the increase of uneven settlement. The value of derailment coefficient when uneven settlement is 0 mm is 0.166 while it is 0.173 when uneven settlement is 25 mm, the latter increasing by 4.2 % than the former. The value of rate of wheel load reduction when uneven settlement is 0 mm is 0.435 while it is 0.504 when uneven settlement is 25 mm, the latter increasing by 15.9 % than the former. This indicates that uneven settlement has some impact on running safety.

According to Railway Vehicles-Specification for Evaluation the Dynamic Performance and Accreditation Test (GB5599-85),derailment coefficient and rate of wheel load reduction of the vehicle should meet the provisions listed in Table 8.

Comparing the calculated safety indicators in Table 7 with the safety standards in Table 8, values of derailment coefficient and rate of wheel load reduction both are less than the safety limit although they increase under differential settlements. It is thus clear that uneven settlement has a certain impact on running safety. But when the value of uneven settlement ≤ 25 mm, no safety problem will happen and then the operation safety is ensured.

Table 8. Safety standard of running safety indicators.

GB5599-85	First limit (Danger limit)	Second limit (allowable limit)
Derailment coefficient	≤1.2	≤1.0
Rate of wheel load reduction	0.65	0.60

It is indicated from the calculation in section 3 and 4 that the tension of the first group of fastenings on left and right sides of the beam end has exceeded the toe load of DTVII2 fastenings when uneven settlement reaches 15 mm, which may cause the elastic rod to fracture. The dynamic stress of rail above the sunken pier has exceeded the allowable stress of CHN60 rail when uneven settlement reaches 25 mm, causing the rail to yield with great possibility. But running stability and safety still meet the requirements specified in the standard when uneven settlement reaches 25 mm. It is thus clear from above mentioned results that the influence induced by uneven settlement is much greater on the mechanical characteristics of rail and fastenings than on running stability and safety, indicating that the factor that determines the limit of uneven settlement of bridge piers is fastening force and rail stress, not the running stability and safety.

5. Conclusions and Suggestions

A static analysis model for researches on the mechanical behaviors of track structure under the action of uneven settlement alone and a dynamic analysis model for researches on mechanical characteristics of track structure and running stability and safety under the combined action of uneven settlement and the dynamic train load are established using ABAQUS finite element program. Conclusions and suggestions are as follows:

1) The influence range of uneven settlement on fastenings is 10 groups of fastenings on both sides of the beam joint. Among them the influence on the first group of fastenings is the largest and the fastening force is tension. With uneven settlement increasing, the tension of the first group of fastenings increases. When uneven settlement reaches 15 mm, the tension has exceeded the toe load of DTVII2 fastening. It is proposed that fastenings at this position should be replaced by fastenings with larger crimping force.

2) The influence area of uneven settlement on rail stress is within the range of 15 m on both sides of the beam joint. The larger differential settlement is, the

larger rail stress will be. The dynamic stress of rail has substantially exceeded the allowable value of CHN60 rail when uneven settlement is 25 mm.

3) Uneven settlement has a certain impact on the vertical acceleration of the vehicle while little effect on the lateral acceleration. With the increase of the amount of differential settlement, the vertical acceleration of the vehicle increases, but far less than the specification limit.

4) Derailment coefficient and rate of wheel load reduction have increasing trends with the increase of uneven settlement, indicating that uneven settlement has a certain impact on running safety. But compared with the specification limits, the safety indicators are within the safe range and no safety problem will happen.

5) The influence induced by uneven settlement is much greater on the mechanical characteristics of rail and fastenings than on running stability and safety, indicating that the factor that determines the limit of uneven settlement of bridge piers is fastening force and rail stress, not the running stability and safety. Although Code for Design of Metro (GB 50157-2003) specifies that uneven settlement between adjacent piers should not exceed 20 mm, the monitoring of fastening force and rail stress should be strengthened when uneven settlement reaches 15 mm in order to protect track structure from damage.

Acknowledgements

This work was financially supported by the Fundamental Research Funds for the Central Universities (2014JBM080) and Beijing Nova Program (Z12111000250000).

References

[1]. Zhao Pingrui, Xiao Jieling, Liu Xueyi, The Influence of the displacement at bridge ends to the fastening systems used on ballastless tracks, *Journal of the China Railway Society*, Vol. 30, Issue 5, 2008, pp. 68-73.

[2]. Zhao Pingrui, Wang Ping, Research report on influence of the displacement at bridge ends to the fastening systems used on ballastless tracks, *Southwest Jiaotong University*, Chengdu, 2006.

[3]. Song Guohua, Gao Mangmang, Li Guoqing, Vehicle-track-bridge vertical system coupled vibration analysis in the case of uneven settlement of pier and abutment, *China Railway Science*, Vol. 31, Issue 2, 2010, pp. 29-33.

[4]. Wei Yahui, Research on beam end displacement relevant problems for high-speed railway ballastless track bridge, *China Academy of Railway Sciences*, Beijing, 2012.

[5]. Song Guohua, Vehicle-track-bridge vertical system coupled vibration analysis in the case of uneven settlement of pier and abutment, *China Academy of Railway Sciences*, Beijing, 2007.

[6]. Railway comprehensive technology institute of Japan. Railway building design standards and interpretations – deformation limit, *Maruzen Publisher*, Tokyo, 2004.

[7]. The basic principles and general requirements on ballastless bridge, DS504.5401.

[8]. Check for ballastless track locating on the beam-end, DS504.5405.

[9]. Li Zaiwei, Lian Songliang, Li Qiuling, Analysis of track irregularity spectrum of urban rail transit, *Journal of East China Jiaotong University*, Vol. 28, Issue 5, 2011, pp. 83-87.

[10]. Wu Jianzhong, Li Xiangjiu, Wu Jianghong. Design research on DTVII2 fastening on bridge, *Metro*, Issue 3, 2003, pp. 24-28.