Sensor Calibration Method Based on Numerical Rounding

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Abstract: The rounding and operation rules of significant figures in tests are introduced. The method for determining the number of significant figures, the rounding interval based on the precision of test instruments and sensors calibration calculation and analysis are studied in this paper. The detailed analysis process of the various stages of the testing examples is given as well. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Numerical rounding, Significant figures, Precision, Error, Sensor calibration.

1. Introduction

In experiments and tests, a large number of original data are recorded. After complicated data computation and processing by using a variety of algorithms, the required test results can be finally obtained. The precision of the original data and intermediate computation results would directly influence the precision of the final result. Thus, the precision of data would greatly affect the judgment as to whether a system has reached the design requirements.

The original data are obtained from various measuring instruments. Their precisions are limited by the precision of instruments. Theoretically, only the approximated measured value can be obtained, and are usually represented by a value with a certain number of significant figures. The significant figures of a number are those digits indicating the precision of measurement. During the recording and computation, the number of significant figures should be determined according to the precision of measuring instrument. In general, most experimental results are submitted after a series of computation of data. Therefore, the choice of number of significant figures is crucial in the process of computation [1].

All tests and computations for tests involve choices of number of significant figures. A correct recording, rounding and computation of the significant figures are required for all researchers. It is also an important factor in scientific researches [2], but is often ignored by most researchers, which results in deviation of experimental results, or even an error in judgment of the truth [3, 4].

2. Numerical Rounding and the Operation Rule of Significant Figures

The choice of number of significant figures means numerical rounding, which is specified in the latest Chinese national standard GB/T 8170-2008 "Rules of Rounding off for Numerical Values & Expression and Judgment of Limiting Value". A detailed illustration of the definition of numerical rounding and the rules of rounding is given in the standard document. Different from the conventional rule of "rounding half up", the new rule adopted in the standard is the so-called "banker's rounding" rule: when the leftmost digit of the digits to be dropped is a single digit, it is dropped, and the rest
digits left to it are retained; when the leftmost digit of the digits to be dropped is 5 with a non-zero digit on its immediate right, the digit is rounded up; when the leftmost digit of the digits to be dropped is 5 with no more digits or all zeros on its right, if the rightmost digit of the digits to be retained is an old number, then the leftmost digit to be dropped is rounded up, or else, it is rounded down [5]. According to statistics, the traditional "rounding half up", according to which a digit is rounded up as long as the last figure is equal to or greater than 5, would lead to a larger result; whereas the "banker's rounding" adopted in the new standard is more scientific[1].

Significant figures consist of reliable digits and doubtful (estimated) digits [6-8], and the operation rules are listed below [9].

- In operations of addition and subtraction, the datum with the least number of decimal places should be treated as a reference. Other data can be rounded to the number of decimal places more than the least number by one. However, the final result should be rounded to the least number of decimal places.

- In operations of multiplication and division, the datum with the least number of significant figures should be treated as a reference in rounding other data. The data with number of significant figures greater than the least number obtained previously should be rounded to the number greater than the least number by one. The final result should be rounded to the least number of significant figures.

- In square or square root operations, the same treatment as in operation of multiplication is applicable to the former, whilst the same treatment as in operation of division is applicable to the square root operation, as an inverse operation of the square operation.

- In logarithmic operation, data with n significant figures should be dealt with n-digit logarithmic table, or (n+1)-digit logarithmic table, with a purpose of avoiding loss of precision.

- In trigonometric operation, the number of significant figures to be rounded to for the function value should increase with the reducing of angular error. When the angular error is 10", 1", 0.1", and 0.01", the corresponding number of significant figures to be rounded to for the function value should be 5, 6, 7, and 8, respectively.

- If the calculation result needs to be operated again (as an intermediate result), then the number of significant figures could be increased by one temporarily.

3. Precision of Measuring Instrument and Determination of Significant Figures in the Experimental Result

The rule in determining the number of figure retained in the test result is as follows: the last digit is unreliable, while the last but one digit should be a reliable [9]. Hence, the number of significant figures is determined by the precision of measuring instrument. That is, the last digit should be of the same order of magnitude with the precision of testing equipment. For the same target, the number of significant figures adopted would be different with instruments having different precisions (minimum scale).

For example, the precision of some pressure transducers is expressed with percentage of scale span. If its precision is 1.5 % FS, and full scale is 0-200 KPa, then the measuring precision would reach 200*1.5/100=3 kPa. If 168.73 kPa is the measuring result of some test, then the actual pressure would be 168.73±3 kPa. It is seen, the unit digit ‘8’ is already an unreliable digit, letting alone the two decimal digits after the decimal point. According to the precision of instrument, the test result should be rounded to unit digit, with the final result being 169 kPa. A total of three significant figures are retained, including two reliable digits and one doubtful digit.

4. Experiment Method of Sensor Calibration and Determination of Rounding Interval

A static pressure calibration experiment is performed on four air pressure sensors. The experiment method is as follows: air with pressure of 0 kPa, 10 kPa, …, 100 kPa is generated by the air pressure controller, and then the air pressure sensors convert the air pressure into a voltage signal. The voltage signal is subsequently converted to digital signal by an A/D data acquisition instrument and enters a computer to be stored for computation. Finally the relationship between air pressure and the output voltage of sensor would be obtained. The process stated above is adopted in calibration of many sensors. Hence, analysis and calculation of the process is of great importance. Fig. 1 is the schematic diagram of the calibration experiment of air pressure sensor.

![Fig. 1. Schematic diagram of calibration experiment of air pressure sensors.](image)
Table 1 shows the main technical indices of testing instruments.

Table 1. Main technical indices of testing instrument.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Test scale</th>
<th>Output scale</th>
<th>Accuracy</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Pressure Controller</td>
<td>0-100 kPa</td>
<td>0-100 kPa</td>
<td>0.02 %FS</td>
<td>Gauge Pressure</td>
</tr>
<tr>
<td>Air Pressure Sensor</td>
<td>0-100 kPa</td>
<td>1.5-4.5 V (analog)</td>
<td>0.5 %FS</td>
<td>Gauge Pressure</td>
</tr>
<tr>
<td>A/D Data Acquisition Instrument</td>
<td>0-5 V (analog)</td>
<td>0-5 V (digital)</td>
<td>0.2 %FS</td>
<td>12 bit, 0-150 kHz</td>
</tr>
</tbody>
</table>

The input and output of each section during the testing are calculated as follows:

4.1. Air Pressure Controller

Assuming the maximum measuring span of the air pressure controller is $S_1$, and the precision is $\sigma_1$, the maximum error would be $S_1\sigma_1$.

When the air pressure is controlled to be $P$, the actual output air pressure is $P \pm S_1\sigma_1$.

4.2. Air Pressure Sensor

Assuming the maximum input of the air pressure sensor is $S_{2p}$, the maximum output is $S_{2y}$, and the precision is $\sigma_2$, the maximum error of output would be $S_2\sigma_2$.

The input of air pressure sensor is just the output of the air pressure controller, which is $P \pm S_1\sigma_1$.

Without considering the non-linear conditions, the output voltage is $(P \pm S_1\sigma_1) \cdot \frac{S_{2y}}{S_{2p}} = S_2\sigma_2$.

If the maximum output of air pressure controller and the maximum input of air pressure sensor is the same $S_1 = S_{2p}$, then the equation above would be simplified as $P \frac{S_{2y}}{S_{2p}} = S_2\sigma_2$ (1).

4.3. A/D Data Acquisition Instrument

A/D data acquisition simply converts the analog signal to digital signal. The scale of its input and output are equal. Assuming the full scale of A/D data acquisition instrument is $S_3$, and the precision is $\sigma_3$, the maximum error of output would be $S_3\sigma_3$.

Its input is the input voltage of air pressure sensor:

$(P \pm S_1\sigma_1) \cdot \frac{S_{2y}}{S_{2p}} = S_2\sigma_2$ (1)

Its output (a value rounded to a specified number of significant figures) is:

$(P \pm S_1\sigma_1) \cdot \frac{S_{2y}}{S_{2p}} = S_2\sigma_2 + S_3\sigma_3$ (2)

The maximum error of test result is:

$\frac{S_{2y}}{S_{2p}} \cdot S_1\sigma_1 + S_2\sigma_3 + S_3\sigma_3$ (3)

If $S_1 = S_{2p}$, $S_{2y} = S_1$, then the equation would be simplified as $P \frac{S_{2y}}{S_{2p}} = S_2\sigma_2 \cdot \left(\sigma_1 + \sigma_2 + \sigma_3\right)$.

According to the technical indices of the test instrument, $S_1 = S_{2p}$. Thus, the maximum error of test data is as follows:

$S_{2y} \cdot \left(\sigma_1 + \sigma_2 + \sigma_3\right) = 4.5 \ast (0.0002 + 0.005) + 5 \ast 0.002 \ast 0.0334(V)$ (4)

Namely, it is appropriate that the original voltage value measured by A/D is rounded to 0.01 V.

5. Sensor Calibration Method Based on Numerical Rounding

5.1. Rounding of Original Data

Table 2 shows a part of the original data when the four air pressure sensors are at 30 kPa. The original data have 5 significant figures, and need to be rounded to 3 significant figures.

Table 2. Original data of output voltage with 1-4# sensors being at 30 kPa (unit: V).

<table>
<thead>
<tr>
<th></th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.3705</td>
<td>2.3338</td>
<td>2.3900</td>
<td>2.3582</td>
</tr>
<tr>
<td>2</td>
<td>2.3705</td>
<td>2.3314</td>
<td>2.3900</td>
<td>2.3558</td>
</tr>
<tr>
<td>3</td>
<td>2.3631</td>
<td>2.3265</td>
<td>2.3875</td>
<td>2.3509</td>
</tr>
<tr>
<td>4</td>
<td>2.3582</td>
<td>2.3216</td>
<td>2.3778</td>
<td>2.3460</td>
</tr>
<tr>
<td>5</td>
<td>2.3631</td>
<td>2.3265</td>
<td>2.3802</td>
<td>2.3509</td>
</tr>
<tr>
<td>6</td>
<td>2.3558</td>
<td>2.3216</td>
<td>2.3778</td>
<td>2.3460</td>
</tr>
</tbody>
</table>

Table 3 shows a part of the original data when the four air pressure sensors are at 30 kPa.
Table 3. Rounding result of output voltage with 1-4# sensors being at 30 kPa (unit: V).

<table>
<thead>
<tr>
<th></th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>1#</td>
<td>2.37</td>
<td>2.33</td>
<td>2.39</td>
<td>2.36</td>
</tr>
<tr>
<td>2#</td>
<td>2.37</td>
<td>2.33</td>
<td>2.39</td>
<td>2.36</td>
</tr>
<tr>
<td>3#</td>
<td>2.36</td>
<td>2.32</td>
<td>2.38</td>
<td>2.35</td>
</tr>
<tr>
<td>4#</td>
<td>2.36</td>
<td>2.32</td>
<td>2.38</td>
<td>2.35</td>
</tr>
</tbody>
</table>

5.2. Elimination of Rough Error

The error exceeding the anticipated error under specified conditions caused by carelessness of experiment staff, or sudden changes of environmental conditions, is defined as a rough error. The value of rough error is relatively large and would obviously distort the test results. Therefore, errors of this type must be eliminated. When the data volume is large, the $3\sigma$ rule is normally exploited, meaning that the datum whose difference with the mean is larger than 3 times of the standard deviation is eliminated. Generally, the $3\sigma$ rule is utilized for several times, until the condition is satisfied.

The waveform comparison among original data, rounded data and the data after rough error being eliminated is shown in Fig. 2.

5.3. Calibration of Air Pressure Sensor

After the original data have been rounded and the rough errors are eliminated, the output mean of the four sensors under 11 pressures is selected for subsequent operation. Because these means are intermediate results, there would be one more significant number retained, namely, the rounding interval is 0.001 V, according to the operation rules. The results can be seen in Table 4.

Table 4. Outputs of 1-4# air pressure sensors under 11 pressures.

<table>
<thead>
<tr>
<th>Test Pressure</th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 kPa</td>
<td>1.540</td>
<td>1.510</td>
<td>1.569</td>
<td>1.548</td>
</tr>
<tr>
<td>10 kPa</td>
<td>1.815</td>
<td>1.782</td>
<td>1.839</td>
<td>1.815</td>
</tr>
<tr>
<td>20 kPa</td>
<td>2.090</td>
<td>2.058</td>
<td>2.111</td>
<td>2.080</td>
</tr>
<tr>
<td>30 kPa</td>
<td>2.367</td>
<td>2.332</td>
<td>2.386</td>
<td>2.356</td>
</tr>
<tr>
<td>40 kPa</td>
<td>2.644</td>
<td>2.609</td>
<td>2.659</td>
<td>2.626</td>
</tr>
<tr>
<td>50 kPa</td>
<td>2.922</td>
<td>2.888</td>
<td>2.927</td>
<td>2.898</td>
</tr>
<tr>
<td>60 kPa</td>
<td>3.204</td>
<td>3.162</td>
<td>3.202</td>
<td>3.168</td>
</tr>
<tr>
<td>70 kPa</td>
<td>3.481</td>
<td>3.438</td>
<td>3.472</td>
<td>3.437</td>
</tr>
<tr>
<td>80 kPa</td>
<td>3.759</td>
<td>3.716</td>
<td>3.745</td>
<td>3.709</td>
</tr>
<tr>
<td>90 kPa</td>
<td>4.035</td>
<td>3.991</td>
<td>4.017</td>
<td>3.980</td>
</tr>
<tr>
<td>100 kPa</td>
<td>4.312</td>
<td>4.270</td>
<td>4.289</td>
<td>4.256</td>
</tr>
</tbody>
</table>

In the test stated above, the input pressure of a certain sensor is assumed to be $P$, while the output voltage is assumed as $V$. When good linearity of the sensor is guaranteed, $P = kV + a$ where $k$, $a$ is the undetermined coefficient.
In the test, $\hat{k}$ and $\hat{a}$ so excessive and does not reflect accurately the appropriate relationship between variables because of the inherent error of $V$ and $P$. So $\hat{k}$ and $\hat{a}$ should also be rounded and the rounding interval should be determined by the error of $V$ and $P$ as follows.

\[
\hat{k} = \frac{\sum_{i=0}^{10} (V_i - \bar{V}) P_i}{\sum_{i=0}^{10} (V_i - \bar{V})^2},
\]

(6)

\[
\hat{a} = \frac{\sum_{i=0}^{10} \frac{10}{i} (V_i - \bar{V}) P_i}{\sum_{i=0}^{10} (V_i - \bar{V})^2},
\]

(7)

From the above results, $\hat{k}$ and $\hat{a}$ error can be seen in Table 5.

Table 5. Errors of $\hat{k}$ and $\hat{a}$ of 1-4# air pressure sensors.

<table>
<thead>
<tr>
<th></th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{k}$</td>
<td>-2.09</td>
<td>-2.09</td>
<td>-2.23</td>
<td>-2.24</td>
</tr>
<tr>
<td>$\hat{a}$</td>
<td>6.10</td>
<td>6.04</td>
<td>6.54</td>
<td>6.48</td>
</tr>
</tbody>
</table>

According to the result above, the rounding interval of $\hat{k}$ and $\hat{a}$ should be taken to be 1. The calculation result of $\hat{k}$ and $\hat{a}$ for four sensors is shown in Table 6.

Table 6. $\hat{k}$ and $\hat{a}$ of 1-4# air pressure sensors.

<table>
<thead>
<tr>
<th></th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{k}$</td>
<td>36</td>
<td>36</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>$\hat{a}$</td>
<td>-55</td>
<td>-55</td>
<td>-58</td>
<td>-57</td>
</tr>
</tbody>
</table>

During the current calculation, the rounding interval of $V$ is 0.001 V, then the error of $P$ can be obtained from the following formula.

\[
dP = k dV,
\]

(10)

The result of error of $P$ is shown in Table 7.

Table 7. Error of $P$ of 1-4# air pressure sensors.

<table>
<thead>
<tr>
<th></th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dP$</td>
<td>0.036</td>
<td>0.036</td>
<td>0.037</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Therefore, in the calculation process, the rounding interval of the pressure $P$ is should be 0.01.

Each sensor has a group of optimal solutions $\hat{k}$, $\hat{a}$ and the linear fitting result is shown in Fig. 3.

![Fig. 3. Linear approximation result of inputs and outputs of the sensor.](image-url)
6. Conclusions

The test data be properly rounded, can make the final test results more in line with the actual situation. On the other hand, data bits after rounding reduced to decrease the amount of calculation, it is great convenience for the calculation contains lots computation and some computing requirements of real-time.

In this study, the problem of data rounding according to the precision of test instrument is discussed. With the method, a sensor calibration is performed in experiment, and scientifically reliable results are obtained.

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