Sensors & Transducers, Vol. 164, Issue 2, February 2014, pp. 233-241

Sensors & Transducers

Source Signals Separation and Reconstruction Following Principal Component Analysis

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Received: 20 November 2013 /Accepted: 28 January 2014 /Published: 28 February 2014

Abstract: For separation and reconstruction of source signals from observed signals problem, the physical significance of blind source separation modal and independent component analysis is not very clear, and its solution is not unique. Aiming at these disadvantages, a new linear and instantaneous mixing model and a novel source signals separation reconstruction solving method from observed signals based on principal component analysis (PCA) are put forward. Assumption of this new model is statistically unrelated rather than independent of source signals, which is different from the traditional blind source separation model. A one-to-one relationship between linear and instantaneous mixing matrix of new model and linear compound matrix of PCA, and a one-to-one relationship between unrelated source signals and principal components are demonstrated using the concept of linear separation matrix and unrelated of source signals. Based on this theoretical link, source signals separation and reconstruction problem is changed into PCA of observed signals then. The theoretical derivation and numerical simulation results show that, in despite of Gauss measurement noise, wave form and amplitude information of unrelated source signal can be separated and reconstructed by PCA when linear mixing matrix is column orthogonal and normalized; only wave form information of unrelated source signal can be separated and reconstructed by PCA when linear mixing matrix is column orthogonal but not normalized, unrelated source signal cannot be separated and reconstructed by PCA when mixing matrix is not column orthogonal or linear.

Keywords: Source signals separation and reconstruction, Principal component analysis, Linear and instantaneous mixing model, Statistically unrelated, Wave form and amplitude.

1. Introduction

Separation and reconstruction of source signals from observed signals is one of research hotspots in signal processing field, widely used in fetal heart monitoring [1, 2], communication [3], EEG [4], microphone speech separation [5], radar signals [6], fault diagnosis [7] and other problems.

Currently, the widely used blind source separation model (BSS) [8] assumes that there are linear instantaneous mixed and statistically independent among the source signals. Independent component analysis (ICA) [9], which is the main and most widely used solving method of BSS, makes the most Gauss of source signals as the goal, and uses optimization algorithm in solving process. But the physical meaning of BSS and ICA is not very clear and ICA easily falls into local extremum. ICA does not guarantee the uniqueness of separated source signals, and ultimately can only estimate the waveform, losing amplitude information of source signals. Using the non-correlation of each chaotic signal Li Xuexia and Feng Jiuchao [10] proposed to isolate multiple mixing chaotic signals from noise background with the eigenvalue decomposition method, but the existence, uniqueness and application conditions of their method do no clearly discussion.

PCA is one of the most classic in multivariate analysis and data mining. The idea was first proposed by the Pearson in 1901, and it was used for the field of biology [11]. Principal components are linear combinations of the original data which help to visualize similarities in an ensemble of signals. Because all principal components are orthogonal and ordered according to the variance contribution of sample, the largest two or three principal components provide an excellent representation of variability within a set of data [12].

2. Description and Hypothesis of Source Signals Separation and Reconstruction from Observed Signals Problem

2.1. Linear and Instantaneous Mixing Model

Considering emitting multiple signals from multiple physical sources but being received by a plurality of sensors, as shown in Fig. 1, signals from n sources signal $\mathbf{S}(t) = [\mathbf{s}_1(t), \mathbf{s}_2(t), \cdots, \mathbf{s}_n(t)]^T$ are received by m sensors and generated outputs $\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \cdots, \mathbf{x}_m(t)]^T$. Assuming that the transfer is instantaneous, namely the arrival time difference of different signals to each sensor can be neglected, and each signal source received by the sensor is linear mixture.

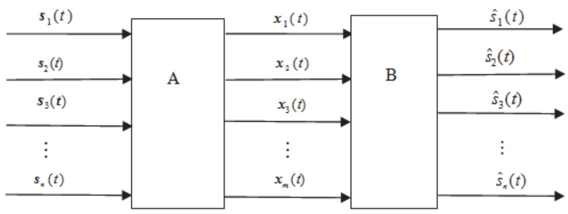

Fig. 1. Linear and instantaneous model of separation and reconstruction of source signals from observed signals problem.

When there is no measurement noise, linear and instantaneous model of separation and reconstruction of source signals from observed signals problem can be expressed as:

$$\mathbf{X}(t) = \mathbf{AS}(t) \tag{1}$$

In the presence of measurement noise, this model can be expressed as:

$$\hat{\mathbf{X}}(t) = \mathbf{AS}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{A} \in \mathbf{R}^{m \times n}$ is the linear mixing matrix, $\mathbf{n}(t)$ is the observation noise which meet Gauss distribution. The source signals separation and reconstruction problem can be expressed as: looking for a linear separation matrix $\mathbf{B} \in \mathbf{R}^{n \times m}$ only from observed signals $X(t)$, when the linear mixing matrix A and the number of source signal vectors $\mathbf{S}(t)$ are unknown, such that:

$$\hat{S}(t) = BX(t), \tag{3}$$

where $\hat{S}(t)$ is the reliable estimation of the source signal vector $\mathbf{S}(t)$.

Reliable estimates are the information of source signal sequence $\mathbf{S}(t)$ contained in $\hat{S}(t)$, including:

1) Number of sources sequence *n*;
2) Waveform of sources sequence;
3) Amplitude of sources sequence;
4) order of sources sequence.

Separation and reconstruction of source signals only from linear and instantaneous mixing observed signals is an underdetermined, uncertainty and inverse problem. In order to solve this problem, different assumptions must be added into it, which forms different models.

2.2. Assumptions of Traditional Blind Source Separation Model

The traditional model of blind source separation made the following hypothesis for source identification in order to ensure the existence and solving problems, and generally need:

a) Linear mixed matrix $\mathbf{S}(t)$ is a column matrix of full rank, i.e.

$$rank(\mathbf{A}) = n, \tag{4}$$

$\mathbf{A} \in \mathbf{R}^{m \times n}$ the left pseudo-inverse $\mathbf{A}^{\#} \in \mathbf{R}^{n \times m}$ do exists, which made

$$\mathbf{A}^{\#}\mathbf{A} = \mathbf{I}_{n \times n}, \tag{5}$$

b) Source signal vector $\mathbf{S}(t)$ is a stationary random process with zero mean vectors, and each component are mutually independent, and component obey the Gauss distribution is not more than one.

c) Noise vector $\mathbf{n}(t)$ is a random vector with zero mean, and are statistical independent with source signal vectors; or suppose noise vector can be neglected, i.e.:

$$\mathbf{n}(t) = 0 \tag{6}$$

2.3. Source Signals Unrelated Model

Different from assumption of the traditional blind source separation model b) and c), this new model only assumes that the various components of source signals $\mathbf{S}(t)$ are not related, noise vector $\mathbf{n}(t)$ and source signal vectors $\mathbf{S}(t)$ are not related.

Then:

$$E[\mathbf{s}_i(t)\mathbf{s}_j(t)^T] = 0 \quad 1 \le i, j \le n, i \ne j \tag{7}$$

$$E[\mathbf{s}_i(t)\mathbf{n}_j(t)^T] = 0 \; 1 \le i \le n, 1 \le j \le m \tag{8}$$

Assumption of this new model is statistically unrelated rather than independent of source signals, which is different from the traditional blind source separation model.

3. PCA of Linear Instantaneous Mixed Problem

3.1. PCA of Observation Signals

Definition 1: There are m observed signals $\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \cdots, \mathbf{x}_m(t)]^T$ in $\mathbb{R}^m$. And $\mathbf{X}(t)$ is formed by combining n irrelevant unknown uncorrelated latent variables $\mathbf{Y}(t) = [\mathbf{y}_1(t), \mathbf{y}_2(t), \cdots, \mathbf{y}_n(t)]^T$ in $\mathbb{R}^n$ with a matrix of a linear transformation $\mathbf{W} \in \mathbb{R}^{m\times n}$, where $\mathbf{W}^T\mathbf{W}=\mathrm{I}_{n\times n}$. So the n irrelevant unknown uncorrelated latent variables $\mathbf{Y}(t)$ are called the principal components of m observed signals $\mathbf{X}(t)$.

$$\mathbf{X}(t) = \mathbf{W}\mathbf{Y}(t) \tag{9}$$

$$\mathbf{W}^T\mathbf{W}=\mathrm{I}_{n\times n} \tag{10}$$

$$\mathbf{E[Y(t)Y(t)^T]} = \Lambda_{\mathbf{n\times n}} \tag{11}$$

Definition 2: There are m observed signals $\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \cdots, \mathbf{x}_m(t)]^T$ in $\mathbb{R}^m$. The matrix of a linear transformation $\mathbf{W} \in \mathbb{R}^{m\times n}$ and the principal components $\mathbf{Y}(t)$, which meet Eq. (9), (10) and (11) at the same time, is called principal component decompositions of $\mathbf{X}(t)$.

Theorem 1: From a statistic point of view, principal component decompositions of given m observed signals $\mathbf{X}(t)$, which meet Eq. (9), (10) and (11), are existed and unique.

Proof: Autocorrelation matrix definition of observed signals $\mathbf{X}(t)$ is:

$$\begin{aligned}\mathbf{C_{XX}} &\triangleq E[\mathbf{X}(t)\mathbf{X}(t)^T]=E[(\mathbf{W}\mathbf{Y}(t))(\mathbf{W}\mathbf{Y}(t))^T] \\ &=E[\mathbf{W}\mathbf{Y}(t)\mathbf{Y}(t)^T\mathbf{W}^T]=\mathbf{W}E[\mathbf{Y}(t)\mathbf{Y}(t)^T]\mathbf{W}^T \\ &=\mathbf{W}\Lambda_{n\times n}\mathbf{W}^T\end{aligned} \tag{12}$$

$$\begin{aligned}&rank(\mathbf{C_{XX}})=rank(\mathbf{W}\Lambda_{n\times n}\mathbf{W}^T) \\ &\le rank(\Lambda_{n\times n}) = n\end{aligned} \tag{13}$$

$$\mathbf{W}^T\mathbf{C_{XX}}\mathbf{W}=\mathbf{W}^T\mathbf{W}\Lambda_{n\times n}\mathbf{W}^T\mathbf{W}=\Lambda_{n\times n} \tag{14}$$

$$\begin{aligned}&rank(\mathbf{C_{XX}}) \ge rank(\mathbf{W}^T\mathbf{C_{XX}}\mathbf{W}) \\ &=rank(\Lambda_{n\times n}) = n\end{aligned} \tag{15}$$

Form Eq. (13) and (15),

$$rank(\mathbf{C_{XX}}) = n \tag{16}$$

After sorting eigenvalues of $\mathbf{C_{XX}}$ in descending order by value, its eigenvalues $\lambda_i, i = 1, 2, ..., n, n+1, ..., m$ meet:

$$\begin{aligned}&\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0 \\ &\lambda_{n+1} = \lambda_{n+2} = \cdots = \lambda_m = 0\end{aligned} \tag{17}$$

Set the normalized eigenvector $v_i \in \mathbb{R}^m$ corresponding to the eigenvalue $\lambda_i > 0, i = 1, 2, ..., n$, and $v_i \in \mathbb{R}^m$ meets:

$$C_{XX}v_i=\lambda_i v_i \tag{18}$$

Therefore, according to the qualities of the eigenvalue of the matrix, $\mathbf{C_{XX}}$ can be uniquely divided into:

$$\mathbf{C_{XX}} = \mathbf{V}\Lambda'\mathbf{V}^T, \tag{19}$$

where $\Lambda' \in \mathbb{R}^{n\times n}$ is the diagonal matrix formed by n nonzero, eigenvalues $\lambda_i > 0, i = 1, 2, ..., n$ of $\mathbf{C_{XX}}$ are sorted in descending order.

$$\Lambda' = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \in \mathbf{R}^{n\times n}, \tag{20}$$

where $\mathbf{V}$ is the transformational matrix which is composed of eigenvectors, $\mathbf{v}_i \in \mathbb{R}^m$.

$$\mathbf{V} = [\mathbf{v}_1 \mathbf{v}_2 \cdots \mathbf{v}_i \cdots \mathbf{v}_n] \tag{21}$$

and

$$\mathbf{V}^T\mathbf{V} = \mathbf{I}_{n\times n} \tag{22}$$

So $\mathbf{X}(t)$ can be uniquely divided into:

$$\mathbf{X}(t)=\mathbf{V}(\mathbf{V}^T\mathbf{X}(t)) \tag{23}$$

When eigenvalues of $\mathbf{C}_{\mathbf{XX}}$ are sorted in descending order by value in Eq. (17), $\mathbf{V}$ in Eq. (21) is unique. $\mathbf{V}$ meets requirement of linear transformation $\mathbf{W} \in \mathbb{R}^{m\times n}$, while $\mathbf{V}^T\mathbf{X}(t)$ just meets requirement of the principal components $\mathbf{Y}(t)$.

Proof ends.

3.2. Relationship between PCA and Source Signals Separation and Reconstruction from Observed Signals Problem Observation Signals

According to the hypothesis of 2.3 and Eq. 7, the autocorrelation matrix of source signal $\mathbf{C}_{\mathbf{SS}} \triangleq \mathrm{E}[\mathbf{S}(t)\mathbf{S}(t)^T] \in \mathbf{R}^{nxn}$ satisfies the properties:

1) Symmetry matrix;
2) All eigenvalues are positive;
3) Nonsingular diagonal matrix, namely $\mathbf{C}_{\mathbf{SS}} = \Lambda''$;
4) The rank is n, namely $rank(\mathbf{C}_{\mathbf{SS}}) = n$.

Take Eq. (1) into the m dimensional observation signal $\mathbf{X}(t)$ autocorrelation matrix $\mathbf{C}_{\mathbf{XX}} \triangleq E[\mathrm{X}(t)\mathrm{X}(t)^T] \in \mathbf{R}^{mxm}$, and then get:

$$\begin{aligned} \mathbf{C}_{\mathbf{XX}} &\triangleq E[\mathbf{X}(t)\mathbf{X}(t)^T] = E[\mathbf{AS}(t)\mathbf{S}(t)^T\mathbf{A}^T] \\ &= \mathbf{A}E[\mathbf{S}(t)\mathbf{S}(t)^T]\mathbf{A}^T = \mathbf{A}\mathbf{C}_{\mathbf{SS}}\mathbf{A}^T = \mathbf{A}\Lambda''\mathbf{A}^T \end{aligned} \tag{24}$$

According to the definition of the autocorrelation matrix, expression (24), assumption a) and proof process of theorem 1, it is clear that $\mathbf{C}_{\mathbf{XX}}$ need to satisfy the following properties:

1) Symmetric matrix;
2) All eigenvalues are non-negative;
3) The rank is rank is n, namely $rank(\mathbf{C}_{\mathbf{XX}}) = n$

Of course, the feature values $\lambda_i' > 0, i = 1,2,\cdots,n$ of the diagonal matrix is not arranged from large to small by value. Then, Eq. (19) can also be decomposed into:

$$\mathbf{C}_{\mathbf{XX}} = (\mathbf{V}\,\mathbf{P})\Lambda'''(\mathbf{V}\,\mathbf{P})^T \tag{25}$$

Eq. (23) corresponds:

$$\mathbf{X}(t) = \mathbf{V}\,\mathbf{P} * ((\mathbf{V}\,\mathbf{P})^T\,\mathbf{X}(t)), \tag{26}$$

where

$$\Lambda' = \mathbf{P}\Lambda'''\mathbf{P}^T, \tag{27}$$

where $\mathbf{P} \in \mathbb{R}^{n\times n}$ is any permutation matrix and meets:

$$\mathbf{P}^T\mathbf{P} = \mathrm{I}_{n\times n} \tag{28}$$

Compared Eq. (26), (22) and (11) with Eq. (1), (3) and (7), the PCA of observation signals is a special case of source signals separation and reconstruction problem, shown in Fig. 2. Linear compound matrix $\mathbf{V}$ is the transpose of linear separation matrix $\boldsymbol{B}^T$ in Eq. (5), and principal component $\boldsymbol{V}^T\boldsymbol{X}(t)$ is the estimation of the sources signals $\hat{\boldsymbol{S}}(t)$. Based on this theoretical link, source signals separation and reconstruction problem is changed into PCA of observed signals then.

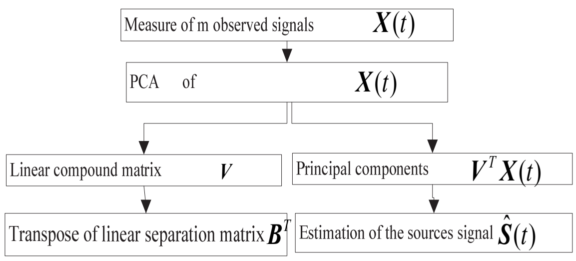

Fig. 2. Relationship between PCA and source signals separation and reconstruction from observed signals problem observation signals.

3.3. Different Types of Linear and Instantaneous Mixing Matrix

Under the assumption of a), it makes classification discussion to the different types of linear and instantaneous mixing matrix.

3.3.1. Linear Mixing Matrix with Normalized Orthogonal Columns

$\mathbf{A}$ meets :

$$\mathbf{A}^T\mathbf{A} = \mathrm{I}_{n\times n} \tag{29}$$

Contrasting Eq. (22) and (29), as shown in Fig. 2, the linear separation matrix $\mathbf{B} = \mathbf{V}^T$, an estimate on $\mathbf{S}(t)$ of the source signal vectors $\hat{\mathbf{S}}(t) = \mathbf{BX}(t)$ retains wave forms and amplitude information of unrelated source signals $\mathbf{S}(t)$. But Eq. (28) shows that, the non-uniqueness of P makes the order uncertainty of sequence of source signal $S(t)$ and $\hat{\mathbf{S}}(t)$. Because the eigenvalues of $\hat{\mathbf{S}}(t)$ and eigenvectors are arranged from large to small by value, each separated source signal is arranged according to the variance contribution from large to small by value.

1) Number of sources sequence n;
2) Waveform of sources sequence ;
3) Amplitude of sources sequence;
4) No order of sources sequence.

3.3.2. Linear Mixing Matrix with Orthogonal Columns but not Normalized

$\mathbf{A}$ meets:

$$\mathbf{A}^T \mathbf{A} = \Lambda'_{n\times n}, \tag{30}$$

where $\Lambda'_{n\times n} = \begin{bmatrix} \lambda'_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda'_n \end{bmatrix}$ is the diagonal matrix. And:

$$[\mathbf{A}\Lambda'^{-1/2}_{n\times n}]^T \mathbf{A}\Lambda'^{-1/2}_{n\times n} = I_{n\times n}, \tag{31}$$

where $\Lambda'^{-1/2}_{n\times n} = \begin{bmatrix} \lambda'^{-1/2}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda'^{-1/2}_n \end{bmatrix}$, Eq. (25) can be described as:

$$\begin{aligned} \mathbf{C}_{\mathbf{XX}} &= \mathbf{A}\Lambda\mathbf{A}^T \\ &= [\mathbf{A}\Lambda'^{-1/2}_{n\times n}]\Lambda\Lambda'_{n\times n}[\mathbf{A}\Lambda'^{-1/2}_{n\times n}]^T \end{aligned} \tag{32}$$

$\Lambda_{n\times n}\Lambda'_{n\times n}$ is a diagonal matrix composition $\mathbf{C}_{\mathbf{XX}}$ composed of nonzero eigenvalue $\lambda_i \neq 0, i = 1,2,\cdots,n$ and $\mathbf{A}\Lambda'^{-1/2}_{n\times n}$ is the eigenvalue matrix composed of orthogonal vector $\mathbf{V}_i \in \mathbf{R}^m$ $i = 1,2,\cdots n$ corresponding to eigenvector λ_i, which could be obtained by Eq. (18).

At this point, if we take the linear separation matrix $\mathbf{B} = [\mathbf{A}\Lambda'^{-1}_{n\times n}]^T$, an estimation $\hat{\mathbf{S}}(t) = \mathbf{B}\mathbf{X}(t)$ of the source signal vectors $\mathbf{S}(t)$ retains many unrelated source waveform and amplitude information of $S(t)$. But because $\lambda'_i \neq 0, i = 1,2,\cdots,n$ is unknown, $\Lambda'^{-1}_{n\times n}$ also remains unknown. As shown in Fig. 2, takes the linear separation matrix $\mathbf{B} = \mathbf{A}\Lambda'^{-1/2}_{n\times n}$, then $\hat{\mathbf{S}}(t) = \mathbf{BX}(t)$ estimates waveform of multiple uncorrelated source signals $S(t)$, but loses the amplitude information. At the same time, not only the uncertainty of λ'_i, but non-uniqueness of P determines the amplitude uncertainty of and order of source signals sequence $\hat{\mathbf{S}}(t)$ and $S(t)$.

1) Number of sources sequence n ;
2) Waveform of sources sequence ;
3) No amplitude of sources sequence;
4) No order of sources sequence.

3.3.3. Linear Mixing Matrix with Non-orthogonal Columns

$$\mathbf{A}^T \mathbf{A} \neq \Lambda'_{n\times n} \tag{33}$$

At this time, it is an underdetermined, uncertainty and inverse problem and different assumptions must be added into it. Linear separation matrix B could not be got by PCA of observation signals $\mathbf{X}(t)$. However, only the number of uncorrelated signal sources could be estimated by PCA.

1) Number of sources sequence n;
2) No waveform of sources sequence;
3) No amplitude of sources sequence;
4) No order of sources sequence.

3.3. Effects of Measurement Noise in Observation Signals

When there not exist measurement noise in observation signals, the separation and reconstruction of source signals from observed signals problem, as described in Section 2.2, the number of the irrelative source signals in model (1) could be determined by the number of the non zero eigenvalue of the observation signal in value decomposition of eigenvalue of the autocorrelation matrix. In engineering application, there certainly exist Gauss measurement noise in observation signals, when decomposing model (2) with autocorrelation matrix eigenvalue of observation signal, eigenvalues $\hat{\lambda}_i$ $i = n+1,...,m$ of autocorrelation matrix $C_{\hat{\mathbf{X}}\hat{\mathbf{X}}}$ in Eq. (17) are also greater than zero.

$$\hat{\lambda}_{n+1} > \hat{\lambda}_{n+2} > \cdots > \hat{\lambda}_m > 0 \tag{34}$$

Because the Gauss measurement white noise is uncorrelated, the relative feature values are small [13]. According to expression (17), eigenvalues of the correlation matrix $\mathbf{C}_{\hat{\mathbf{X}}\hat{\mathbf{X}}}$ are sorted in descending order by value. Taking the first n largest eigenvalues of $\mathbf{C}_{\hat{\mathbf{X}}\hat{\mathbf{X}}}$ to constitute a diagonal matrix $\hat{\Lambda}$, then the corresponding eigenvectors constitute the transform matrix $\hat{\mathbf{V}}$. In the cutting processing, $\mathbf{C}_{\hat{\mathbf{X}}\hat{\mathbf{X}}}$ can be approximately divided into eigenvalue equation:

$$\mathbf{C}_{\hat{\mathbf{X}}\hat{\mathbf{X}}} \approx \hat{\mathbf{V}}\hat{\Lambda}\hat{\mathbf{V}}^T \tag{35}$$

The observation signal $\hat{\mathbf{X}}(t)$ is decomposed in approximation into:

$$\hat{\mathbf{X}} \approx \hat{\mathbf{V}} * (\hat{\mathbf{V}}^T \hat{\mathbf{X}}) \tag{36}$$

At this point, the cumulative variance σ_S^2 of $(\hat{\mathbf{V}}^T\hat{\mathbf{X}})$ is:

$$\sigma_S^2 = \sum_{i=1}^{n} \lambda_i \tag{37}$$

The cumulative variance $\hat{\mathbf{X}}(t)$ of observation signal $\hat{\mathbf{X}}(t)$ is:

$$\sigma_X^2 = \sum_{i=1}^{m} \lambda_i \tag{38}$$

Assume:

$$\eta = \sigma_S^2 / \sigma_X^2 = \sum_{i=1}^{n} \lambda_i / \sum_{i=1}^{m} \lambda_i \tag{39}$$

$$\xi = 1 - \sum_{i=1}^{n} \lambda_i / \sum_{i=1}^{m} \lambda_i = \sum_{i=n+1}^{m} \lambda_i / \sum_{i=1}^{m} \lambda_i \tag{40}$$

η in expression (39) is the accumulated variance contribution rate of the n source signals, ξ in expression (40) is the error is caused by the truncation effect, namely the effects of the observation noise. When the number of irrelative sources n is unknown, we can set the threshold of expression (39) [13] (for example $\eta \geq 95\ \%$) and make the truncation of n, then use expression (40) to compute percentage ξ of the variance error of truncation, the specific algorithm is shown in Fig. 3.

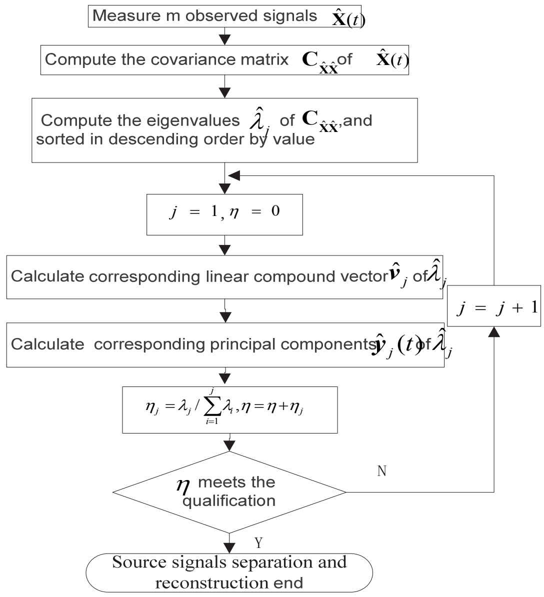

Fig. 3. Source signals separation and reconstruction process Following PCA of observed signals with measurement noise.

4. Simulation Verification

4.1. Mixing Matrix A with Orthogonal Column and Normalized

Supposing five observation signals $\mathbf{X}(t) = [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)]^T$ with 10 % Gauss measurement noise have been awarded, as shown in Fig. 4, which are results of a source signal $\mathbf{S}(t)$ aliasing through an unknown mixing matrix A with orthogonal column and normalized. Estimates $\hat{\mathbf{S}}(t)$ of the solution of source signals $\mathbf{S}(t)$ should be obtained only from $\mathbf{X}(t)$.

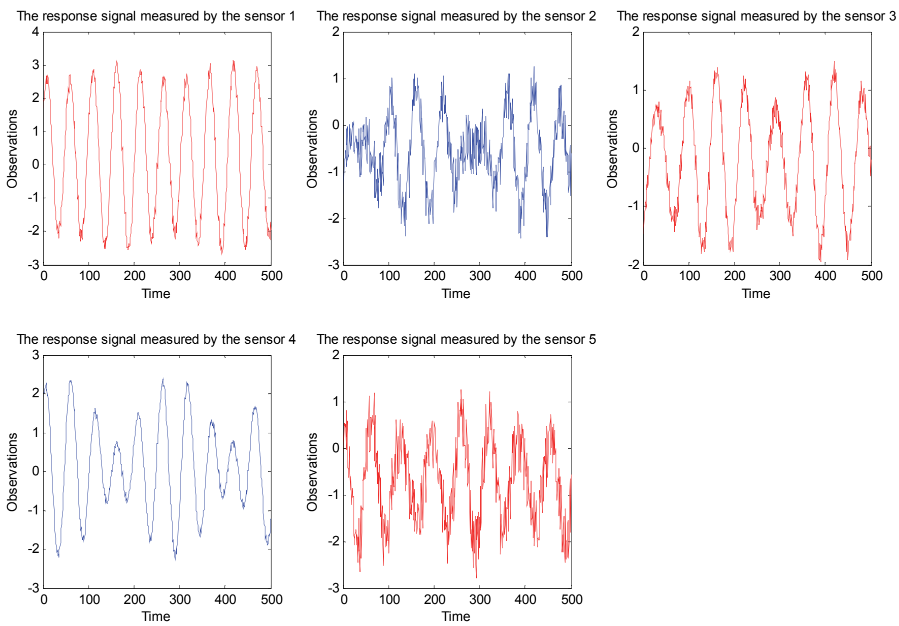

Fig. 4. Observed signals $\mathbf{X}(t)$ with 10 % Gauss measurement noise awarded by sensor.

Decompose the autocorrelation matrix eigenvalue of the observation signals, from the fold in Fig. 5, the cumulative contribution rate is more than 99 % to the third principal components, and can be truncated. Therefore, the number of source signal irrelative is three.

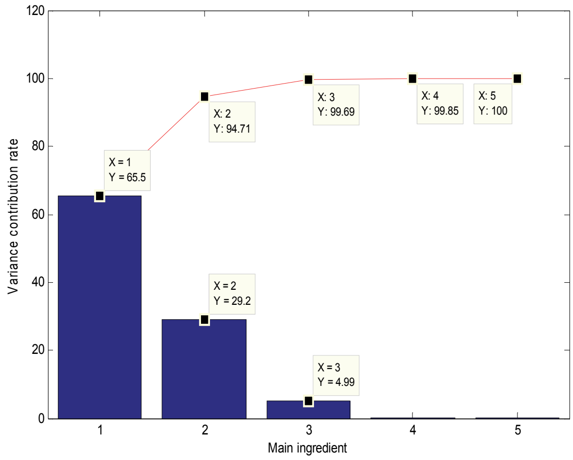

Fig. 5. Pareto chart of PCA contribution rate and the cumulative contribution rate.

The observation signal $\mathbf{X}(t)$ is actually composed of unknown source signals $\mathbf{S}(t)=[s_1(t),s_2(t),s_3(t)]^T$ of the row normalized orthogonal mixing matrix A, while add 10 % white noise in the observation signal.

S1(t) = 3*sin (5/128*pi*t+1/6*pi);

S2(t) = 2*rand (1, K);

S3(t) = 2*sin (3/97*pi*t-1/2*pi);

$$A=\begin{bmatrix} 7/sqrt(71) & 2/sqrt(70) & 1/sqrt(65.92) \\ 2/sqrt(71) & -5/sqrt(70) & 3/sqrt(65.92) \\ 1/sqrt(71) & -2/sqrt(70) & 5/sqrt(65.92) \\ 4/sqrt(71) & 1/sqrt(70) & -3.4/sqrt(65.92) \\ 1/sqrt(71) & -6/sqrt(70) & -4.4/sqrt(65.92) \end{bmatrix}$$

Fig. 6 shows, PCA of the observation signals $\boldsymbol{X}(t)$ can separate and reconstruct multiple uncorrelated waveforms of source signals, but also identify the amplitude information when mixing matrix A with orthogonal column and normalized. This method can tolerate Gauss measurement of strong noise.

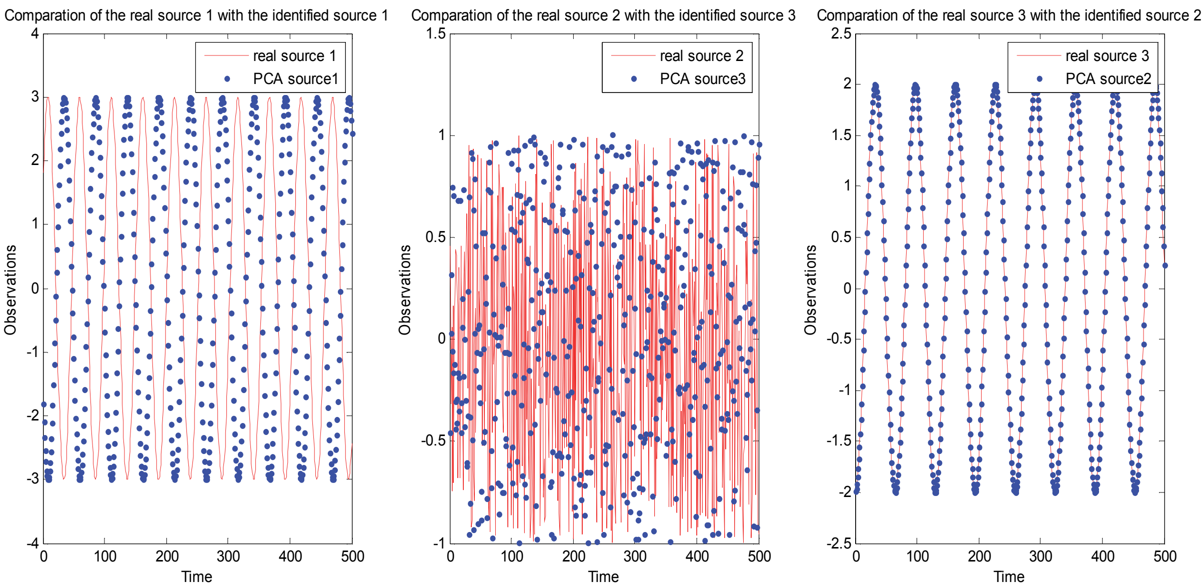

Fig. 6. Comparison of the estimates source signals with the real source signals.

4.2. Mixing Matrix A Column Orthogonal but not Normalized

Supposing four observation signals $\mathbf{X}(t)=[x_1(t),x_2(t),x_3(t),x_4(t)]^T$ with 10 % Gauss measurement noise have been awarded, as shown in Fig. 7, which are results of a source signal $\mathbf{S}(t)$ aliasing through an unknown mixing matrix A with orthogonal column but not normalized. Estimates $\hat{\mathbf{S}}(t)$ of the solution of source signals $\mathbf{S}(t)$ should be obtained only from $\mathbf{X}(t)$.

Decompose the autocorrelation matrix eigenvalue of the observation signals, from the fold in Fig. 8, the cumulative contribution rate is more than 99 % to the third principal components, and can be truncated. Therefore, the number of source signal irrelative is three.

The observation signal $\mathbf{X}(t)$ is actually composed of unknown source signals $\mathbf{S}(t)=[s_1(t),s_2(t),s_3(t)]^T$ of the row orthogonal but not normalized matrix A, while add 10 % Gauss measurement white noise in the observation signal.

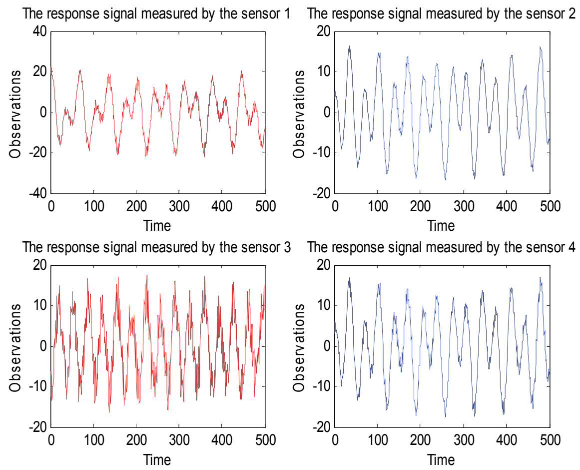

Fig. 7. Observed signals $\mathbf{X}(t)$ with 10 % Gauss measurement noise awarded by sensor.

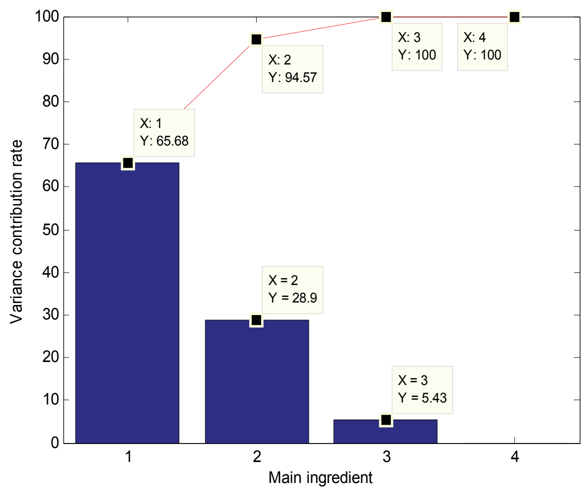

Fig. 8. Pareto chart of PCA contribution rate and the cumulative contribution rate.

S1=2*rand (1, K);

S2=2*sin (3/97*pi*t-1/2*pi);

S3=5*sin (1/17*pi*t+1/3*pi);

$$A = \begin{bmatrix} 3 & -5 & 2 \\ 1 & 3 & 2 \\ 6 & 1 & -2 \\ 2 & 3 & 2 \end{bmatrix}$$

Fig. 9 and Fig. 10 show, PCA of the observation signals $X(t)$ can only separate and reconstruct multiple uncorrelated waveforms of source signals, but not identify the amplitude information when mixing matrix A with orthogonal column but not normalized. This method can tolerate Gauss measurement of strong noise.

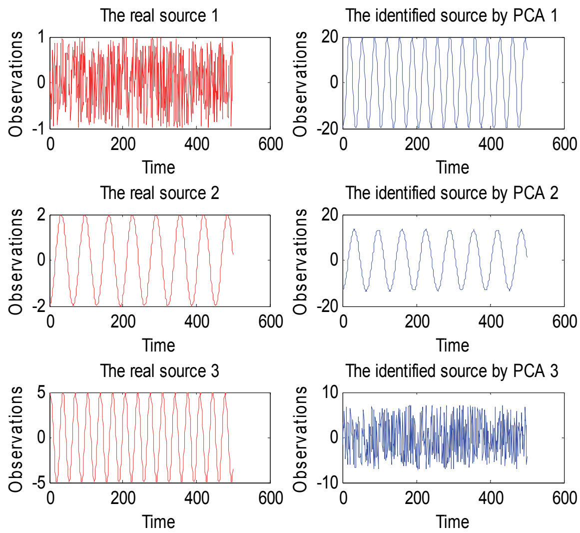

Fig. 9. The real source signals and estimate source signals.

5. Conclusion

To the linear and instantaneous model of separation and reconstruction of source signals from observed signals problem, this paper only hypothesized the source signals are not related, and introduced the observation signal PCA method. For the linear mixing matrix in different situations, the paper discussed in classification, strictly argued and made simulation verification of the existence, uniqueness and application conditions. For different types of linear and instantaneous mixing matrix, the information of multiple uncorrelated signals can be obtained by this method is also different. Results of simulation verification show that this method can tolerate strong Gauss measurement noise.

Model and method of source signals separation and reconstruction from observed signals problem when mixing matrix is not column orthogonal, or not linear, or not instantaneous, such as convolution, need further studying.

Acknowledgements

This work was financially supported by National Natural Science Foundation of China (Grant No. 51305142) and introduction of talents Huaqiao University Scientific Research Projects (Project No. 12BS217).

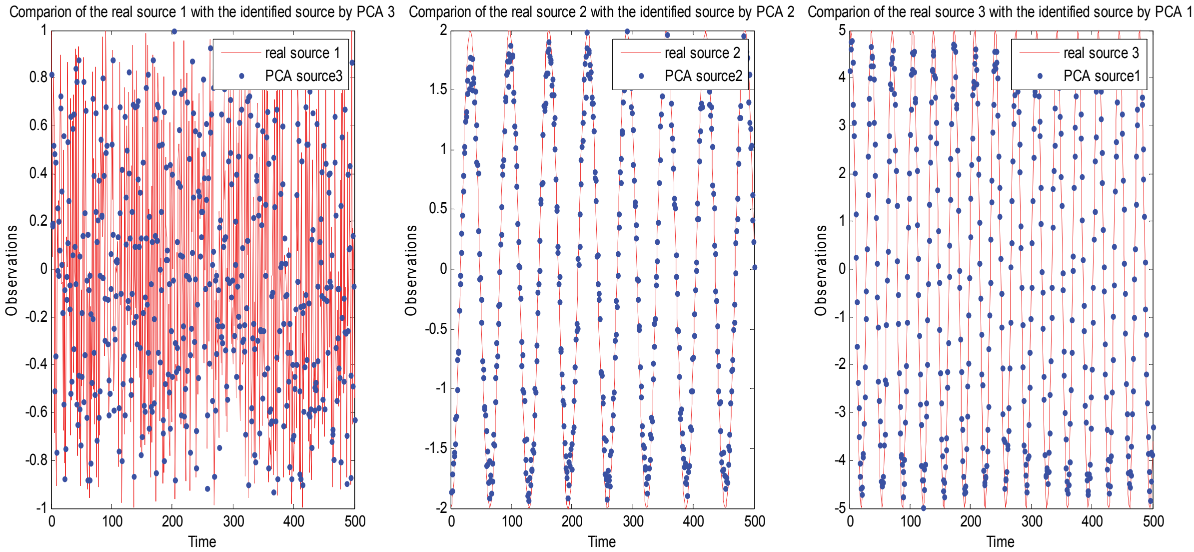

Fig. 10. The waveform comparison of the estimates source signals with real source signals.

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