QPSK Carrier Signal Detection Based on Double Duffing Oscillators

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Abstract: The paper proposed a new method for the detection of QPSK carrier signal that based on double duffing oscillator to solve the problems as high attenuation and strong noise interference in the QPSK carrier communication of low voltage power line. This method has making use of the different phase of QPSK carrier signal to stimulate the double duffing oscillators to enter different states and through the judgment of the state of the double duffing oscillators to detect the phase information that the QPSK carrier signal carries. At the same time, through the study of the system solutions of duffing oscillator, obtained the change rule of them, based on these rules, the paper proposed a discriminant algorithm that used the difference distribution, and this algorithm is characterized by small calculating amount and low error judgment rate. The numerical simulation and experimental test show that the double duffing oscillator method can detect the phase information of QPSK carrier signal in low voltage power line accurately with the low SNR environment.

Keywords: Weak signal detection, Double duffing oscillators, QPSK carrier signal, Low voltage power line communication, Phase information.

1. Introduction

Low voltage power line communication used the existed power line as the transmission media to transmit data and signals. With this technology one needn’t to lay power line, it has the features as low investment and high-speed installation. But since the low voltage power line is not designed for the transmission of carrier signal. It needs to transmit power at the same time as it transmits carrier signals. Besides, there is noise interference caused by different device in the process of transmission [1-5]. The current studies reveal that the high noise level and signal attenuation are the two main factors that influenced the communication quality of power line carrier signal. The commonly used filtering method and spectrum method can filter out a part of noise, but when the signal is very weak, it is difficult to extract the signal accurately and have high quality communication on low voltage network. So it is necessary to study new signal detection method under this kind of low SNR.

Chaotic oscillator is a new method used in weak signal detection in recent years [6-15]. It has the characteristics as very sensitive to weak signal and has a strong immunity to noise. The lowest detected SNR of this method is much lower than the lowest detected SNR -10 dB of traditional methods. And thus it provides a new direction for the research of weak signal detection. The key to the detection technique is to determine the state of chaotic oscillator system. The current state discrimination
methods of chaotic oscillator have Melnikov method, Lyapunov index method [16], image recognition [17]. However, the algorithmically of these methods are complicated and the calculation is too large and has poor real-time performance.

Firstly, this paper constructed detection equation for double duffing oscillator and described the specific method to detect the phase of QPSK carrier signal with double duffing oscillator, analyzed the change rule of system solutions of duffing oscillator, proposed the new algorithm to determine the state of the system with the difference distribution of system solution, reduced the calculation amount for state determination. Finally, the paper demonstrated the effectiveness of this method with simulation and experimental data.

2. The Principle of Double Duffing Oscillator Detection

Currently Holmes duffing oscillator is one of the most widely studied oscillators, most weak signal detection systems of chaotic oscillator used the equation or its deformation equation. The equation of Holmes duffing oscillator is as following:

\[
\dot{x} + 2\xi \dot{x} - x + x^3 = \delta f \cos(\omega t) \tag{1}
\]

The equation (1) is a second-order differential equation which regards each order derivative as a new variable. The equation (1) can be transformed into a matrix form:

\[
\dot{X} = F(x, y) + \delta G(y, t) \tag{2}
\]

In which:

\[
X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad F(x, y) = \begin{pmatrix} y \\ -x - x^3 \end{pmatrix}, \quad G(y, t) = \begin{pmatrix} 0 \\ -\delta y + \delta f \cos(\omega t) \end{pmatrix}
\]

The formula (2) is the equation of state, can describe the change rule of the dynamical system, in which the expanded space of state variables x, y is called state space. Where the disturbance part G(y, t) is the periodic function of time t which includes damping and periodic driven parts (periodic driving force).

2.1. The Principle of Phase Detection Based on Single Duffing Oscillator

When \( \delta \neq 0 \), that is added perturbed parts in oscillator, including damping and periodic driving force, usually supposed \( \delta = 1 \), its equation is as follows:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -x + x^3 + f \cos(\omega t) \\
\end{align*}
\]

In the equation \( \xi \) is the damping ratio, \( f, \omega \) is the amplitude and frequency of periodic driving force.

When the external measured signal \( a \cos(\omega t + \theta) \) are added, we can regard the periodic driving force \( f \cos(\omega t) \) and external measured signal \( a \cos(\omega t + \theta) \) as the periodic driving force in the entire duffing oscillator equation. The detection equation is as following:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -x + x^3 + f \cos(\omega t) + a \cos(\omega t + \theta) \\
\end{align*}
\]

The simplified form of the right part of the equation is:

\[
\begin{align*}
&f \cos(\omega t) + a \cos(\omega t + \theta) \\
&= (f + a \cos(\theta)) \cos(\omega t) - a \sin(\theta) \sin(\omega t) \\
&= f_0 \cos(\omega t + \varphi) \\
\end{align*}
\]

In which:

\[
f_0 = \sqrt{f^2 + 2fa \cos(\theta) + a^2} \tag{6}
\]

\[
\varphi = \arctg \left( \frac{a \sin(\theta)}{f + a \cos(\theta)} \right) \tag{7}
\]

From the equation (6) we can see, once the measured signal are added, the amplitude \( f_0 \) of the periodic driving force in the entire equation is not only related to the amplitude a of the measured signal but also the phase \( \theta \) of the measured signal. Therefore, the change of the phase \( \theta \) of the measured signal can cause the change of the amplitude \( f_0 \) of the periodic driving force and thus stimulate the change of the oscillator’s state.

In actual detection, when the measured signal \( a \cos(\omega t + \theta) \) of the same frequency but different phased are added and if the phase \( \theta \) belongs to the \( \arccos(\frac{a}{2f}) - \pi \leq \theta \leq \pi - \arccos(\frac{a}{2f}) \), the measured signal will increase the amplitude \( f_0 \) of the periodic driving force and caused the change of the state of the duffing oscillator, and thus the duffing oscillator entered the large scale periodic state, as shown in Fig. 1(a); when the phase \( \theta \) is not belong to this range, the measured signal will weaken the amplitude \( f_0 \) of the periodic driving force, the duffing oscillator’s state won’t change and remains in the chaotic state, as shown in Fig. 1(b). Through the judgment of those two states of duffing oscillator, we can determine the phase range of the measured signal.
2.2. The Principle of Phase Detection Based on Double Duffing Oscillator

The detection equation of the double duffing oscillator was constructed with two single duffing oscillators. As shown in equation (8).

\[
\begin{align*}
  x_1 & = y_1 \\
  y_1 + \xi y_1 - x_1 + x_1^3 & = f \cos(\omega t + \alpha) + a \cos(\omega t + \tau) \\
  x_2 & = y_2 \\
  y_2 + \xi y_2 - x_2 + x_2^3 & = f \cos(\omega t - \pi / 2) + a \cos(\omega t + \tau)
\end{align*}
\]

The \( \cos(\omega t + \theta) \) in equation (8) is the carrier signal of QPSK and \( \tau \) has four phases as \( \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \). When the value of \( \tau \) is different, the two oscillators would enter into different state. Through the discrimination of the states of two duffing oscillators, the phase \( \tau \) of QSPK signal will be detected. The detection rule of the phase \( \tau \) of QSPK signal is as shown in Table 1.

![Fig. 1. System state of duffing oscillator.](image)

<table>
<thead>
<tr>
<th>Phase ( \tau ) of QSPK signal</th>
<th>State of the first oscillator</th>
<th>State of the second oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/4 )</td>
<td>Large scale periodic state</td>
<td>Chaotic state</td>
</tr>
<tr>
<td>( 3\pi/4 )</td>
<td>Chaotic state</td>
<td>Chaotic state</td>
</tr>
<tr>
<td>( 5\pi/4 )</td>
<td>Chaotic state</td>
<td>Large scale periodic state</td>
</tr>
<tr>
<td>( 7\pi/4 )</td>
<td>Large scale periodic state</td>
<td>Large scale periodic state</td>
</tr>
</tbody>
</table>

Therefore, the key of the detection of double duffing oscillator is to discriminate the state of the two oscillators accurately. However, the current discriminate algorithm of the duffing oscillator’s system state is large amount of calculations and poor real-time performance.

3. The Algorithm of Difference Distribution

This section used the fourth-order Runge-Kutta method to solve the duffing oscillator and discovered the change rule between the system solution \( x \) and \( y \) and proposed a method to distinguish the system’s state with the difference distribution of system solution thus to reduce the calculation amount of the state discrimination.

3.1. The Change Rule of System Solution

Deduced the solution formula of fourth-order Runge-Kutta method to find the solution for Holmes duffing oscillator:

\[
\begin{align*}
  x_{n+1} & = x_n + \frac{h}{6} (k_{11} + 2k_{12} + 2k_{13} + k_{14}) \\
  y_{n+1} & = y_n + \frac{h}{6} (k_{21} + 2k_{22} + 2k_{23} + k_{24}) \\
  k_{11} & = f(t_n, x_n, y_n) = y_1 \\
  k_{12} & = g(t_n, x_n, y_n) = -\xi y_1 + x_1 - x_1^3 + f \cos(\omega t_n) \\
  k_{13} & = f(t_n + \frac{h}{2}, x_n + \frac{h}{2} k_{11}, y_n + \frac{h}{2} k_{12}) \\
  k_{14} & = g(t_n + \frac{h}{2}, x_n + \frac{h}{2} k_{11}, y_n + \frac{h}{2} k_{12}) \\
  k_{21} & = f(t_n + \frac{h}{2}, x_n + \frac{h}{2} k_{21}, y_n + \frac{h}{2} k_{22}) \\
  k_{22} & = g(t_n + \frac{h}{2}, x_n + \frac{h}{2} k_{21}, y_n + \frac{h}{2} k_{22}) \\
  k_{23} & = f(t_n + h, x_n + h k_{21}, y_n + h k_{22}) \\
  k_{24} & = g(t_n + h, x_n + h k_{21}, y_n + h k_{22})
\end{align*}
\]

where \( h \) is step size selected in the calculation process. Through the solution of formula (9), found that rules existed between system solution \( x \) and \( y \) in large scale periodic state, the relations between \( x \) and \( y \) is periodic and the phase relations is fixed, as shown in Fig. 2(a); while in chaotic state, the changes of \( x \) and \( y \) is random, as shown in Fig. 2(b).

Select a difference for the system solution \( x \) and \( y \) with every other carrier signal period \( T = 2/\omega \). supposed the difference of system solution \( x \) and \( y \) is Z:

\[
Z = x(nT + \theta) - y(nT + \theta) \quad n = 0,1,2,...
\]

And \( \theta \) is an arbitrary starting time. Fig. 3 is the distribution of the difference values \( Z \), in a large scale state, the difference values \( Z \) is almost at the same line this indicates the distribution is stable and the distribution is random in chaotic state.
According to the above rule, whether the difference values between the system solutions $x$ and $y$ is constant or not can be used to judge the system state of duffing oscillator.

3.2. Algorithm Implementation

In detection, get the sample standard deviation of $N$ the difference values $Z$ to judge the system’s state.

The Determination of Mean Value

Improved the sample standard deviation $\sigma(Z)$ in order to enhance the discriminability of the difference value $Z$’s sample standard deviation in chaotic state and large scale periodic state. Supposed the mean is A in large scale periodic state and the mean is B in chaotic state, if $A-B=\Delta\tau$, then:

In large scale periodic state, the sample standard deviation of difference value $Z$ is:

$$\sigma(Z) = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (z_i - A)^2} \quad (11)$$

In chaotic state, the sample standard deviation of difference value $Z$ is:

$$\sigma(Z) = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (z_i - B)^2} \quad (12)$$

Improved the formula (12) and replaced B by A, then:

$$\sigma(Z) = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (z_i - A)^2}$$

After the replacement, the sample standard deviation $\sigma(Z)$ in chaotic state added constant term and has become larger, the difference of sample standard deviation $\sigma(Z)$ in chaotic state with large scale periodic state becomes more obvious.

The determination of $N$

The smaller $N$ is the fewer sample of the difference value $Z$ will be. In chaotic state, the dispersion of the $\sigma(Z)$ could leads to misjudgment as shown in Fig. 4(a).

With the increase of $N$, the sample quantity is enhanced and the fluctuation of $\sigma(Z)$ is smaller, but the calculation quantity increased, and when the signal phase has the changes, the state transition process will elongated. And there will be misjudgment of phase in the transition process as shown in Fig. 4(c). Through the experimental analysis, it is appropriate when $N=5$, in chaotic state, the fluctuation of $\sigma(Z)$ are very little and the state transition zone is relatively narrow as shown in Fig. 4(b).
Fig. 4. The standard deviation of the difference value Z when N=3, 5, 10.

Detection Steps
1) Obtained the mean A of \( N \) the difference values \( Z \) in the large scale periodic state and regards it as the mean in detection:

\[
A = \frac{1}{N} \sum_{i=1}^{N} Z_i \tag{14}
\]

2) In determining the state of a moment of the duffing oscillator system, taking \( \sigma(Z) \) of \( N \) the difference values \( Z \) that indicates the stability of \( N \) the difference values \( Z \):

\[
\sigma(Z) = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (Z_i - A)^2} \tag{15}
\]

3) The smaller the standard deviation \( \sigma(Z) \) is, the distribution of difference value \( Z \) will be more stable. If the \( \sigma(Z) \) is less than the threshold, then the distribution of \( Z \) will be supposed to be constant and regarded as in a large scale periodic state, otherwise, the duffing oscillator system will be regarded as in a chaotic state.

This method quantitatively analyzed the stability of difference value \( Z \) through the standard deviation of \( Z \). In detection, the discrimination each time is not only related with the difference value \( Z \) of the moment, but also related to the difference value \( Z \) of the previous \( N-1 \) times. Therefore, the short-term deviation of the system’s movement trajectory will not cause the misjudgment of the system’s state.

3.3. Performance Analysis of Noise Suppression

Taking single duffing oscillator for example, when noise is added to the measured signal, the trajectory of duffing oscillator will be affected, and then the detection formula (3) is transformed into:

\[
\begin{align*}
\dot{x}_n &= y_n \\
\dot{y}_n + \xi(y_n) - (x_n) + (x_n)' &= f \cos(\omega t) + a \cos(\omega t + \theta) + n(t)
\end{align*}
\tag{16}
\]

In the formula (16), \( n(t) \) is the noise, \( x_n, y_n \) are the system solution perturbed by noise, and the formula (16) is transformed into:

\[
\begin{align*}
x_n &= y_n \\
(y_n + \Delta y) + \xi(y_n + \Delta y) - (x_n + \Delta x) + (x_n + \Delta x)' &= f \cos(\omega t) + a \cos(\omega t + \theta) + n(t)
\end{align*}
\tag{17}
\]

where \( x \) and \( y \) are the system solution when there is no noise. \( \Delta x, \Delta y \) represent the fluctuation caused by noise.

In order to quantitatively analyze the impact of noise on the discriminate method based on the difference distribution, supposed the parameters of the detection formula (17), \( \xi = 0.6, f = 0.983, \omega = 1 \), the measured signal \( acos(\omega t + \theta) \) has two phases \( \theta = \pi \) and \( \theta = 0 \) that can make the duffing oscillator enter chaotic state and large scale state respectively. After adding different intensity of noise, the difference value \( Z \)’s standard deviation \( \sigma(Z) \) in chaotic state and the large scale periodic state are under different SNR conditions as shown in Table 2.

The data of Table 2 shows: in the chaotic state, there is no big impact to the difference value \( Z \) with the characteristics of dispersed distribution whether there is noise or not. In large scale periodic state, the difference value \( Z \) has fluctuations for the influence of noise and then the distribution spreading. Therefore, only when we ensure the dispersion in large scale periodic state not surpasses the dispersion of the difference value \( Z \) in chaotic state, can we be able to judge the two states accurately.

Table 2. The average standard deviation \( \sigma(Z) \) with difference SNR.

<table>
<thead>
<tr>
<th>SNR /dB</th>
<th>( \sigma(Z) ) in chaotic state</th>
<th>( \sigma(Z) ) in large scale periodic state</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>0.4171</td>
<td>0.0216</td>
</tr>
<tr>
<td>-20</td>
<td>0.4265</td>
<td>0.0641</td>
</tr>
<tr>
<td>-40</td>
<td>0.4314</td>
<td>0.1029</td>
</tr>
<tr>
<td>-50</td>
<td>0.4425</td>
<td>0.1216</td>
</tr>
<tr>
<td>-60</td>
<td>0.4524</td>
<td>0.1573</td>
</tr>
</tbody>
</table>

From the data in Table 2 we can see, when the SNR is -60 dB, the average standard deviation \( \sigma(Z) \) in chaotic state is 0.4524, the average standard deviation
\( \sigma(Z) \) in large scale periodic state is 0.1573. Although the average standard deviation \( \sigma(Z) \) in large scale periodic state have been greatly increased, but there is obvious difference with the average standard deviation \( \sigma(Z) \) in chaotic state. Therefore, the system state can be discriminate accurately.

Besides, \( \Delta x, \Delta y \) can be decomposed:

\[
\begin{align*}
\Delta y &= \Delta g + \Delta f_y, \\
\Delta x &= \Delta g + \Delta f_x,
\end{align*}
\]

where \( \Delta g \) is the common-mode interference and \( \Delta f_x, \Delta f_y \) are the non common-mode interference, then the expression of difference value \( Z \) after the noise added is:

\[
Z = [x(nT + \theta) + \Delta g + \Delta f_y] - [y(nT + \theta) + \Delta g + \Delta f_y]
\]

And the simplified expression of equation (19) is:

\[
Z = x(nT + \theta) - y(nT + \theta) + \Delta f_x - \Delta f_y
\]

From the formula (20), to obtain the difference value \( Z \) of the system solution, the common mode interference \( \Delta g \) is mutually offset. Therefore, the common-mode interference can be effectively suppressed in discriminate the state with this method.

4. Experimental Research

In order to verify the actual detection capability of this method, the noise in the power line is classified as the background noise and impulse noise, injected the background noise and impulse noise generated by signal generator Agilent 33220A in the receiving end of the power line carrier communication. Used the double duffing oscillator detection system to detect the QPSK carrier signals that has been added noise. The model of the power line communication is as shown in Fig. 5.

4.1. Background Noise

The parameter of the equation (8) of double duffing oscillator detection is set: \( \xi=0.6, \omega=1, f=0.983 \), the initial value \( x=0.6, y=0.5 \), the step size is \( 2\pi/100 \). The AD data acquisition card sampled the carrier signals with the center frequency of 120 \( k \) at the sampling rate 12 \( M \). Each period has sampled 100 times.

In this way, the carrier signal is just like the period driving force, its angular frequency is converted to \( \omega=1 \), which is the same as the frequency of periodic driving force.

The process of the actual detection is:

1) Firstly, adjust the amplitude of the periodic driving force in the detection equation (8) and make the system enter into the transition state from chaotic state to large scale periodic state. At this moment, the standard deviation \( \sigma(Z) \) is greater than the threshold.

2) Carrier signal is consisted by one start bit and eight data bits. Once the carrier signal reached, the phase of the start bit is \( \tau=0 \) and at this moment, since the standard deviation \( \sigma(Z) \) changes from greater than the threshold value to less than the threshold value, then the system confirmed the carrier signal has reached and then has one discrimination every other period.

Fig. 5. Power line communication model.

(a) QPSK carrier signal (b)The \( \sigma(Z) \) of the first oscillator (c)The \( \sigma(Z) \) of the second oscillator

Fig. 6. Detection process.
4.2 Impulse Noise

The impulse noise in grid has strong time variation and high power spectral density. And the bits (bit) errors and burst errors are very easily to be caused in transmission and have relatively serious interference to the carrier signals.

By adding different impulse noises in different position of the QPSK carrier signal for different types of impulse noise experiments, we can get the experiment results: In chaotic state, the impulse noise has little disturbance on the difference value Z while in the large scale periodic state, some impulse noise has relatively great influence on the difference value Z, however, if we can chose the appropriate threshold, we still can determine the system’s state.

4.3. The Comparison of Algorithms of the Difference Distribution

In order to compare the performance of different discrimination algorithms after white noise with different intensity are added in the double duffing oscillator used different discrimination algorithms to determine the system state. The result is as shown in Table 3.

From the Table 3 we can know that, when we are using the diagram method, if the SNR is -40 dB, the noise will cause the serious fluctuation of phase-plane trajectory and then lead to the misjudgment of the system’s state. Besides, the misjudgment is easily occurred if the interval of the state transition is short. When we are using the time domain waveform envelope method, if the SNR is relatively high, we can determine the system’s state accurately. Besides, this method used time domain discriminate with good real-time performance. But, when the SNR is -50 dB, the fluctuation of the periodic state is very serious. The misjudgment rate increased dramatically. When we are using the Lyapunov index method, the discrimination is accurate though the SNR is -50 dB. But it needs to be estimated with long sample data and the transition zone of the state is hard to determine and misjudgment is easily caused.

<table>
<thead>
<tr>
<th>SNR /dB</th>
<th>Envelope method</th>
<th>Image method</th>
<th>Lyapunov Index method</th>
<th>Difference distribution method</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>0.0 %</td>
<td>0.2 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
</tr>
<tr>
<td>-20</td>
<td>0.4 %</td>
<td>0.9 %</td>
<td>0.0 %</td>
<td>0.0 %</td>
</tr>
<tr>
<td>-40</td>
<td>0.9 %</td>
<td>3.0 %</td>
<td>0.3 %</td>
<td>0.1 %</td>
</tr>
<tr>
<td>-50</td>
<td>2.3 %</td>
<td>4.1 %</td>
<td>1.1 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>-60</td>
<td>9.1 %</td>
<td>6.7 %</td>
<td>2.8 %</td>
<td>1.0 %</td>
</tr>
</tbody>
</table>

Table 3. The error rate of different discrimination methods.

The difference distribution discrimination method proposed in this paper has strong noise suppression; this method can have an accurate judgment of the phase though the phase trajectory of the system fluctuated seriously. When the SNR is -60 dB, the misjudgment rate is lower than 2 %. The discrimination is not only related to the difference value of that moment but also the previous N-1 difference values, it has strong anti-noise performance.

5. Conclusions

1) For the problems of big attenuation of QPSK carrier signals in low voltage power line communication, the paper proposed the detection method of QPSK carrier signal based on the double duffing oscillators. This method is very sensitive to the phase of weak QPSK carrier signal and can suppress the disturbance of background noise and impulse noise. Compared to traditional filtering, spectral method, it can detect the phase information of the QPSK signal at the condition when SNR is -20 dB.

2) Analyzed change rule of system solution x and y of the duffing oscillator system, proposed the difference distribution algorithm and compared it with other discrimination methods as image method, envelope method etc. This algorithm has the features as less calculation work, low misjudgment rate and it is more suitable for the engineering application.

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References


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