# Picking Robot Arm Trajectory Planning Method 

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#### Abstract

The picking robot arm is scheduled to complete picking tasks in the working space, to overcome the shaking vibration to improve the picking stability, its movement should follow specific consistence trajectory points. Usually we should give definite multiple feature picking points, map their inverse kinematics to the joint space, establish motion equation for the corresponding point in the joint space, then follow these equations motion for the interpolation on the joint so that we can meet the movement requirements. Trajectory planning is decisive significance for accuracy and stability of controlling robot arm. The key issue that picking arm complete picking task will be come true by trajectory planning, namely, robot arm track the desired trajectory. which based on kinematics and statics picking analysis in a joint space according to the requirements of picking tasks, and obtain the position and orientation for picking robot arm, study and calculate the theory of trajectory parameters timely. Copyright © 2014 IFSA Publishing, S. L.


Keywords: Picking robot arm, Trajectory planning, Posture, Interpolation.

## 1. Introduction

Trajectory planning can be in the joint space and in Cartesian space. Carried out in the joint space. It's joint variables is expressed as time domain function to describe the trajectory. Study on the joint trajectory interpolation and the moment constraints and dynamic force range have been Conducted [1]. These joints are concentrated in only continuity and smoothness of tracking interpolation trajectory with little consideration of the drive torque and the dynamics on the robot arm constraints [2]. The little proportion of time spent picking trajectory planning method to determine the feasibility. The movements of picking robot arm reach the target position with the homogeneous transformation matrix, with each joint displacement, velocity, acceleration planning will fit to Cartesian position and orientation, through
joint space quadratic polynomial interpolation procedures to ensure two adjacent tracks smooth transition, the realization of Cartesian space using the straight line segments constitute picking trajectory optimization method; Wicaksono, Handy et al. propose quintic polynomial interpolation function trajectory to avoid Jacobian by solving the inverse matrix from a Cartesian space to the joint space trajectory planning results involving the conversion of matrix inversion and other complex computing problems, and ensure joint angle and angular velocity continuity and stability [3]. As for the shortest joint space trajectory optimization problems, Monta M. et al. apply a uniform nonperiodic fourth-dimension, BSpline curves based on chaos optimization theory to ensure that the approximation error is near to zero [4], running time on the joint space trajectory was optimized. The aboving various planning methods
were used joint interpolation function, so the position of each control instantaneous of picking the arm were determined by joint interpolation polynomial [5].

## 2. Picking Arm Trajectory Planning

### 2.1. Description of Trajectory Planning

The trajectory planning is to control picking arm move along certain target trajectory, to guarantee track target trajectory with high accuracy control has been the primary content of trajectory control of picking arm, general method of assigning the target trajectory the is the demonstration reappearance, namely to tell picking arm how to do by timing computation, then to remember this running process, the following control may be the needed to duplicate this movement at the right moment while picking, all
spots in picking space need to remember trajectory by the demonstration, but that is unfeasible in fact regarding complex the picking environment [6], moreover the more the spots demonstrated and higher precision, the lower efficiency is instead. We can demonstrates several characteristic points with regard to the orderly trajectory path whiling the picking, attain the point the coordinates using the interpolation algorithm, 2 spots must be demonstrated regarding the straight path, 3 spots must demonstrated for the circular arc path, then transforms these point coordinates through the reversion kinematics algorithm to the arm various joints position and the angle $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$. The following angular position closed-loop control system shown as Fig. 1 enforces expected path planning. The whole picking plan may achieved completely by repetition above continuous interpolation.


Fig. 1. Picking-arm trajectory planning control process.

Picking the path planning was carried out in Cartesian coordinate space, picking arm's shape position information, such as the posture of initial position, the raising spot, the releasing and the end point position, the speed and the acceleration of 3 joints must be gain, and guarantee picking arm run on fast, accurate and the steady condition, then carries out the joint interpolation, then ensure the joint position, the speed, the acceleration on the entire time-gap continuity and so on. The interpolation method is to select different type joint interpolating function and produces the different path. The task of path planning are contained the inverse kinematics solution, the interpolation calculation transformation equation.

### 2.2. Description of Picking Objects

The description of picking object can be possible to draw a series of position posture $P_{i}(\mathrm{i}=1,2, \ldots, \mathrm{n})$ in the picking coordinate system. This description not only conforms to real man-picking the apple, but also advantage to describe and produce the way of picking path. As for the description of picking movement, it advantage to separate picking the way, concrete picking arm and the picking manipulator, which forms the description method modulation [7]. In this way, it is possible to transform the object from the original state movement to the final state, regarding the specific transformation of coordinate system from initial point $\left\{\mathrm{T}_{0}\right\}$ to terminating position $\left\{\mathrm{T}_{\mathrm{f}}\right\}$.

In the path planning, picking point condition is indicated with its spot position posture in coordinate system, such as the picking outset position posture, the termination position posture uses the initial station, the end point condition expression respectively. When anywhere position the picking movement is described, not only needs to plot out picking initial point and end point, but also should provide certain node between the two points, which also called the path node [7]. The trajectory not only needs to study the position posture restraint, moreover study time assignment problem on various path node. Namely, movement time between two path node should be known on condition of stipulation way. In addition to this, the movement of picking and harvesting should be smooth stable, nonsmooth stable movement not only aggravates joint spot attrition, moreover causes various arm vibration and impact, this kind of vibration also increased obviously fluttering [8]. Therefore the description equation of picking path of motion should be continuous, even its first derivative and the second time derivative also should be continuous.

To sum up, the position posture in relative reference system of random picking apple can be described the coordinate system fixed firmly.

The picking apple, relative to any point in picking the coordinate system, may be indicated with the corresponding position vector P , any direction may be expressed with the direction cosine. After achieving the geometry shape of picking apple and the picking coordinate system, if the picking
coordinate system the position posture is stipulated, then the posture in the spatial position may be restructured. The picking robot arm work process is available to provide from picking manipulator position posture point sequence, position of each node in arm coordinate system is available to describe the homogeneous transformation in the picking coordinate system [9]. Corresponding joint variable may be computed by inverse kinematics solution. Fig. 2 shows the picking robot manipulator
capture apple work, requesting to take out the apple from the tree and put into bucket of robot, the symbolic representation various points position posture along the path movement, make picking arm move and complete the picking task along the dashed line. Establishes $P_{i}(\mathrm{i}=0,1,2,3,4,5)$ to picking rectangular coordinates system node pass through. Refer to these points, the position posture is described to series of picking the movement and action of the picking robot arm shown as Table 1.


Fig. 2. Picking trajectory of picking-arm for apple harvest.

Table 1. The process of picking and putting back for apple harvest.

| Knot <br> point | $\mathbf{P}_{\mathbf{0}}$ | $\mathbf{P}_{\mathbf{1}}$ | $\mathbf{P}_{\mathbf{2}}$ | $\mathbf{P}_{\mathbf{2}}$ | $\mathbf{P}_{\mathbf{3}}$ | $\mathbf{P}_{\mathbf{4}}$ | $\mathbf{P}_{\mathbf{5}}$ | $\mathbf{P}_{\mathbf{6}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Moving <br> object | Initialization | Moving <br> apple | Moved | Grasp | Lifting | Moving <br> Fruit Box | Putting <br> Into box | Release |

One node $P_{i}$ corresponds a transformation equation, thus solves correspondingly picking robot arm transformation ${ }^{0} \mathrm{~T}_{6}$. From this obtaining the basic structure of the picking description: The picking node $P_{i}$ correspond the picking arm transform matrix ${ }^{0} \mathrm{~T}_{6}$, transforming from one to another transformation is to achieve by picking arm joints movement.

Picking arm's working can be represented by a series of knot points indicating it's position posture. How to defines and produces a series of points between the beginning point $P_{i}$ and the end point $P_{i+l}$, is the most important question on path planning in the rectangular coordinates system space [10]. Between two points the simple way is a spatial translational motion and rotating around the fix axis. After the movement time is assigned, may produce movement controlled the linear velocity and the angular velocity. Movement from the node $P_{0}$ to the close picking apple node $P_{1}$ or from any point $P_{2}$ to the next point $P_{i+1}$ may be expressed from ${ }^{0} \mathrm{~T}_{3}$ ${ }^{3} \mathrm{~T}_{\mathrm{T}}={ }^{0} \mathrm{~T}_{\mathrm{B}}{ }^{B} \mathrm{P}_{\mathrm{i}}$, namely move from

$$
\begin{equation*}
{ }^{0} T_{3}={ }^{0} T_{B}{ }^{B} P_{i}^{3} T_{T}^{-1} \tag{1}
\end{equation*}
$$

To

$$
\begin{equation*}
{ }^{0} T_{3}={ }^{0} T_{B}{ }^{B} P_{i+1}{ }^{3} T_{T}^{-1}, \tag{2}
\end{equation*}
$$

where ${ }^{3} T_{T}$ is the conversion to robot arm coordinate system $\{\mathrm{T}\}$ relative to picking manipulator; ${ }^{B} P_{i}$ and ${ }^{B} P_{i+1}$ is respectively homogeneous transformation between the node $P_{i}$ and $P_{i+1}$ relative to the coordinate system $\{B\}$.

### 2.3. Picking Space Circular Interpolation

Space arc is arbitrary three points determined the spatial arc, related theoretical research has carried out, it is difficult to solve the traditional matrix or Euler angle rotation spatial interpolation algorithm on linear interpolation theory, Hagras Hani designed quaternion five-axis circular interpolation algorithm to achieve a smooth movement from one point to another point smooth continuous interpolation; Pervozvanski Anatoli A planned multi-channel multiarc trajectory algorithm by 3 points teaching
programming [10]; to gain an accurate interpolation trajectory Sakai Satoru conducted timing non-linear error compensation in coordinate transformation principle of circular interpolation [11]. Summarizing the above literature research methods, they will convert generally the joint space issues to the flat circular arc issue, to calculate picking circular interpolation space generally "three-step" method shown as Fig. 3. Firstly, identify arc plane that is, convert three-dimensional issue to two-dimensional; secondly calculate the interpolation point coordinates according to two-dimensional interpolation algorithm; finally convert the coordinate values calculated to the value of the fundamental coordinate $\begin{array}{lllllll}s & y & s & t & e & m\end{array}$


Fig. 3. Interpolation of joint space circle arc plane.

Through three node non-collinear in space $\mathrm{P}_{1}, \mathrm{P}_{2}$, $\mathrm{P}_{3}$ can be determined the three-point-arcs and circles, the line of intersection the plane were $\mathrm{AB}, \mathrm{BC}, \mathrm{CA}$ between basic coordinate plane and three-point arc. Constructing circular interpolation plane coordinate system, look the $\mathrm{P}_{1}$ as the original point of the spatial coordinate $\mathrm{O}_{\mathrm{R}}, \mathrm{P}_{1} \mathrm{P}_{3}$ as the $\mathrm{Z}_{\mathrm{R}}$ axis, $\mathrm{P}_{1} \mathrm{P}_{3} \times \mathrm{P}_{1} \mathrm{P}_{3}$ as $\mathrm{Y}_{\mathrm{R}}$ axis, $\mathrm{X}_{\mathrm{R}}$ axis determined by the right-hand rule, so the arc will be fixed in $O_{R} X_{R} Y_{R} Z_{R}$ plane. The points $P_{1}, P_{2}, P_{3}$ in the coordinate system represented in $O_{R} X_{R} Y_{R} Z_{R}$. The homogeneous transformation is given in Step 2.

1) To solve interpolation point coordinates according to plane circular interpolation algorithm;
2) To convert the coordinates value in $O_{R} X_{R} Y_{R} Z_{R}$ by the calculation from first step to the coordinate values in $\mathrm{OX}_{0} \mathrm{Y}_{0} \mathrm{Z}_{0}$.

To achieve the second step, we must first solve the coordinate axes direction cosine from the coordinate system $\mathrm{O}_{\mathrm{R}} \mathrm{X}_{\mathrm{R}} \mathrm{Y}_{\mathrm{R}} \mathrm{Z}_{\mathrm{R}}$ to $\mathrm{OX}_{0} \mathrm{Y}_{0} \mathrm{Z}_{0}$ and the homogeneous transformation determined from the origin of the coordinate translation. $\mathrm{Z}_{\mathrm{R}}$ axis and $\mathrm{P}_{1} \mathrm{P}_{3}$ vectors are in the same direction: $\mathrm{P}_{1} \mathrm{P}_{3}=\left\{\mathrm{x}_{3}-\mathrm{x}_{1}, \mathrm{y}_{3}-\mathrm{y}_{1}\right.$, $\left.\mathrm{Z}_{3}-\mathrm{Z}_{1}\right\} ; \mathrm{Y}_{\mathrm{R}}$ axis same as $\mathrm{P}_{1} \mathrm{P}_{3} \times \mathrm{P}_{1} \mathrm{P}_{2}$ vectors: $\mathrm{P}_{1} \mathrm{P}_{2}=\left\{\mathrm{x}_{2}-\right.$ $\left.\mathrm{x}_{1}, \mathrm{y}_{2}-\mathrm{y}_{1}, \mathrm{z}_{2}-\mathrm{z}_{1}\right\}$ thus $\mathrm{P}_{1} \mathrm{P}_{3} \times \mathrm{P}_{1} \mathrm{P}_{2}$ can be obtained in this way; $X_{R}$ axis determined by $Z_{R} \times Y_{R}$; while unitising unit vectors $\left\{\mathrm{n}_{\mathrm{x}}, \mathrm{n}_{\mathrm{yl}}, \mathrm{n}_{\mathrm{z}}\right\},\left\{\mathrm{o}_{\mathrm{x}}, \mathrm{o}_{\mathrm{yl}}, \mathrm{o}_{\mathrm{z}}\right\}\left\{\mathrm{a}_{\mathrm{x}}\right.$,
$\left.a_{y 1}, a_{z}\right\}$ belong to $X_{R} Y_{R} Z_{R}$ vector can be obtained. Therefore, the homogeneous coordinate transformation can be expressed from $\mathrm{OX}_{0} \mathrm{Y}_{0} \mathrm{Z}_{0}$ to $O_{R} X_{R} Y_{R} Z_{R}$ coordinates.

$$
{ }_{o_{R}}^{o} T=\left[\begin{array}{cccc}
n_{x} & o_{x} & a_{x} & x_{1}  \tag{4}\\
n_{y} & o_{y} & a_{y} & y_{1} \\
n_{z} & o_{z} & a_{z} & z_{1} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

3) Homogeneous coordinate is ${ }^{\circ R}{ }_{o}$ inverse Transformation from coordinate system $O_{R} X_{R} Y_{R} Z_{R}$ to $X_{R} Y_{R} Z_{R}$.

$$
{ }_{o}^{o_{R}} T=\left[\begin{array}{cccc}
n_{x} & o_{x} & a_{x} & -\left(n_{x} \cdot x_{1}+n_{y} \cdot y_{1}+n_{x} \cdot z_{1}\right)  \tag{5}\\
n_{y} & o_{y} & a_{y} & -\left(o_{x} \cdot x_{1}+o_{y} \cdot y_{1}+o_{x} \cdot z_{1}\right) \\
n_{z} & o_{z} & a_{z} & -\left(z_{x} \cdot x_{1}+z_{y} \cdot y_{1}+z_{x} \cdot z_{1}\right) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

From the above analysis, the process of spatial circular interpolation process can be designed shown as Fig. 4.


Fig. 4. The flowchart of interpolation for space circle arc plane.

### 2.4. Picking Robot Joint Spatial Interpolation Algorithm

As for joint space trajectory planning of picking robot arm, a series of positioning points such as the starting point and end point given posture may be selected in the joint coordinate system. It should select parametric trajectory according to joint interpolation and the functions satisfied interpolation points constraint, The constraints (functions) can conclude picking points position and orientation, velocity and acceleration constraints of movement
direction (initial point), leaving direction (lifting points), putting down and back direction of picking manipulator while gripping apple, etc. Also be the continuity constraints of the arm joint displacement, velocity and acceleration at the time interval. These constraints ensure smooth and stable picking planning. To meet the above constraints, you can apply different types of smooth interpolation function to represent different joints trajectories planning.

### 2.4.1. Cubic Polynomial Interpolation

On the process of picking arm spatial interpolation, the corresponding posture of the arm joint angles to the starting point and the termination point can be obtained by inverse kinematics. So the picking trajectory description can be represented for smoothing interpolation function $\theta(\mathrm{t})$ to the termination and starting joint angle. To achieve higher stability picking movement, the trajectory of each joint function $\theta(\mathrm{t})$ must be satisfied with velocity and position constraint of two end points four timely cases. Endpoints position constraint $\theta(\mathrm{t})$ represents joint angle of termination posture and starting posture, assume the terminate joint angle $\theta(\mathrm{t})$ represents value on the terminal time $t_{f}$ when the value of the moment, with the staring joint angle $\theta_{0}$ represents value on the starting time $t=0$, namely

$$
\left\{\begin{array}{c}
\theta(0)=\theta_{0}  \tag{6}\\
\theta\left(t_{f}\right)=\theta_{f}
\end{array}\right.
$$

to ensure smooth and stable joint velocity, joint velocity of terminating point and the starting point can be considered close to zero, namely

$$
\left\{\begin{array}{c}
\dot{\theta}(0) \approx 0  \tag{7}\\
\dot{\theta}\left(t_{f}\right) \approx 0
\end{array}\right.
$$

According to (6) and (7), the constraints of the cubic polynomial only be confirmed

$$
\begin{equation*}
\theta(t)=a_{0}+a_{1} t+a_{2} t^{2}+a_{3} t^{3} \tag{8}
\end{equation*}
$$

the movement equation for velocity and acceleration of joint arm is solved as following

$$
\left\{\begin{array}{l}
\dot{\theta}(t)=a_{1}+2 a_{2} t+3 a_{3} t^{2}  \tag{9}\\
\ddot{\theta}(t)=2 a_{2}+6 a_{3} t
\end{array}\right.
$$

to obtain cubic polynomial the coefficients we substitute $a_{0}, a_{1}, a_{2}, a_{3}$ into the (5) and (6) the given constraints equations

$$
\left\{\begin{array}{l}
\theta_{0}=a_{0}  \tag{10}\\
\theta_{f}=a_{0}+a_{1} t_{f}+a_{2} t_{f}^{2}+a_{3} t_{f}^{2} \\
0=a_{1} \\
0=a_{1}+2 a_{2} t_{f}+3 a_{3} t_{f}^{2}
\end{array}\right.
$$

The equations can be solved as following

$$
\left\{\begin{array}{l}
a_{0}=\theta_{0}  \tag{11}\\
a_{1}=0 \\
a_{2}=\frac{3}{t_{\mathrm{f}}^{2}}\left(\theta_{f}-\theta_{0}\right) \\
a_{3}=-\frac{2}{t_{f}^{3}}\left(\theta_{f}-\theta_{0}\right)
\end{array}\right.
$$

As for the each arm joint motion, when initial and the termination velocity are both zero, cubic polynomial interpolation functions satisfy the requirements of smooth and stable movement is be expressed as

$$
\begin{equation*}
\theta(t)=\theta_{0}+\frac{3}{t_{f}^{2}}\left(\theta_{f}-\theta_{0}\right) t^{2}-\frac{2}{t_{f}^{3}}\left(\theta_{f}-\theta_{0}\right) t^{3} \tag{12}
\end{equation*}
$$

According to Equation (12), equation expression of the joint angular velocity and angular acceleration can be obtained, therefore

$$
\left\{\begin{array}{l}
\dot{\theta}(t)=\frac{6}{t_{f}^{2}}\left(\theta_{f}-\theta_{0}\right) t-\frac{6}{t_{f}^{2}}\left(\theta_{f}-\theta_{0}\right) t^{2}  \tag{13}\\
\ddot{\theta}(t)=\frac{6}{t_{f}^{2}}\left(\theta_{f}-\theta_{0}\right)-\frac{12}{t_{f}^{3}}\left(\theta_{f}-\theta_{0}\right) t
\end{array}\right.
$$

As for the picking arm with rotating joints, the implementation picking operation of joint movement last 3 s by the experiment. According to timely fieldcontrol, the movement of shoulder joint must be smooth namely, picking robot arm can provide with following states: Initially, wrist joint can be on stationary position $\theta_{0}=0^{\circ}$; while movement is over, $\theta_{\mathrm{f}}=90^{\circ}$, the joint velocity is 0 . According to the above requirements, wrist joint movement may be planned. meeting the requirements, the cubic polynomial interpolation function can used to plan movement of wrist joint. given, substituting $\theta_{0}=0^{\circ}, \theta_{\mathrm{f}}=90^{\circ}$ into (11), obtain cubic polynomial coefficients

$$
a_{0}=0.0, a_{1}=0.0, a_{2}=2.35, a_{3}=-1.63
$$

According to (12) and (13), trajectory of the wrist joint can be obtained, namely

$$
\left\{\begin{array}{l}
\theta(t)=2.35 t^{2}-1.63 t^{3}  \tag{14}\\
\dot{\theta}(t)=4.7 t-4.9 t^{2} \\
\ddot{\theta}(t)=4.7-9.8 t
\end{array}\right.
$$

The curve simulation of cubic polynomial interpolation joint trajectory is shown as Fig. 5. It can be seen that angular displacement curve, an angular velocity and acceleration curves were hyperbolic sine curve line, parabola and straight lines, respectively.


Fig. 5. Cubic polynomial interpolation of joint trajectory for picking-arm.

### 2.4.2. Linear Interpolation with Parabolic Transition

In the cubic polynomial trajectory planning, the linear interpolation function is chosen for a given start and end points shown as Fig.6. On the case of actual picking trajectory control, sometimes joint movement velocity of starting and end points is not smooth and non-continuous, even acceleration and the emergence of the rigid impact of the two end points are so large that picking arms "jitter" occurs. Naveen Kuma studied a simple linear interpolation may cause vibrations [12]. So it is necessary to revise linear function interpolation scheme, the specific method as following: Set a parabola as two interpolated end points buffer. Acceleration of the buffer region will be constant due to the second derivative is constant. So may ensure the moving speed a smooth transition between the start and end points, that the position and velocity of whole picking trajectory is continuous track. Parabolic function and the original linear function is smoothly connected to picking trajectory shown as Fig. 6.


Fig. 6. Linear interpolation trajectory between two picking points.


Fig. 7. Linear trajectory with parabolic transition domain.

Now given 2 parabolic interpolation trajectory with the same duration time $t_{a}$ between two end points, it's constant acceleration $\ddot{\theta}$ which is equal and opposite. This kind of trajectory planning shown as Fig. 8 exist several solutions, so its trajectory is not unique, but all trajectory are symmetrical around the position midpoint $\theta_{\mathrm{h}}$ and the time midpoint $\mathrm{t}_{\mathrm{h}}$.


Fig. 8. Multiple solutions and symmetry for picking trajectory.

In accordance with control requirement, picking trajectory ensure smooth, continuous, i.e. average angular velocity of linear parabolic trajectory segment must be equal to its velocity of destination time, so the following formula

$$
\begin{equation*}
\ddot{\theta} t_{a}=\frac{\theta_{h}-\theta_{a}}{t_{h}-t_{a}} \tag{15}
\end{equation*}
$$

where $\theta_{\mathrm{a}}-\mathrm{t}_{\mathrm{a}}$ corresponds to the joint angle of the parabolic duration time $t_{\mathrm{a}} . \theta_{\mathrm{a}}$ can be obtained according to (15)

$$
\begin{equation*}
\theta_{a}=\theta_{0}+\frac{1}{2} \ddot{\theta} t_{a} \tag{16}
\end{equation*}
$$

We can obtain the total joint exercise time $t_{f}$ from the starting point to the end point, namely, $\mathrm{t}_{\mathrm{f}}=2 \mathrm{t}_{\mathrm{h}}$

$$
\begin{equation*}
\theta_{h}=\frac{1}{2}\left(\theta_{0}+\theta_{f}\right) \tag{17}
\end{equation*}
$$

From (15) to (16)

$$
\begin{equation*}
\ddot{\theta} t_{a}^{2}-\ddot{\theta} t_{f} t_{a}+\left(\theta_{f}-\theta_{0}\right)=0 \tag{18}
\end{equation*}
$$

where $\theta_{0}, \theta_{\mathrm{f}}, \mathrm{t}_{\mathrm{f}}$ are generally known to determine the appropriate $\ddot{\theta}$ and $\mathrm{t}_{\mathrm{a}}$ according to (13), may obtain the corresponding trajectory. Using identified the acceleration value $\ddot{\theta}$ and (18) calculate the corresponding $\mathrm{t}_{\mathrm{a}}$

$$
\begin{equation*}
t_{a}=\frac{t_{\mathrm{f}}}{2}-\frac{\sqrt{\ddot{\theta} t_{\mathrm{f}}^{2}-4 \ddot{\theta}\left(\theta_{\mathrm{f}}-\theta_{0}\right)}}{2 \ddot{\theta}} \tag{19}
\end{equation*}
$$

According to (18), ensure $t_{a}$ solvable, the acceleration value $\ddot{\theta}$ may be satisfied

$$
\begin{equation*}
\ddot{\theta} \geq \frac{4\left(\theta_{\mathrm{f}}-\theta_{0}\right)}{t_{f}^{2}} \tag{20}
\end{equation*}
$$

The (20) gives the new proof by obtaining for equality, then the linear length of the tracking segment is reduced to zero, two transitional domain is the entire trajectory of all, their joint velocity (slope) in convergence point is equal; the larger acceleration $\ddot{\theta}$ value is, the shorter the length of the transition region is, if the acceleration is infinite, the tracking returns to the case of a simple linear interpolation. The starting angle of shoulder joint $\theta_{0}$ is $15^{\circ}$, the end angle $\theta_{\mathrm{f}}$ is $75^{\circ}$, the total control time $t_{\mathrm{f}}=3 \mathrm{~s}$ in the picking trajectory by measuring experimentally. Then design on linear trajectory with parabolic transition can be obtained. According to (19), we can set up the range of the acceleration. If the known conditions meet (20), there exist $\ddot{\theta} \geq 26.67^{\circ} / \mathrm{s}^{2}$, then

1) As for the $t$ picking trajectory of elbow join, setting $\ddot{\theta}_{1}=42^{\circ} / \mathrm{s}^{2}$, calculate transition time $\mathrm{t}_{\mathrm{a} 1}$ according to (20)
$t_{a 1}=\left(\frac{3}{2}-\frac{\sqrt{42^{2} \times 3^{2}-4 \times 42(75-15)}}{2 \times 42}\right) \mathrm{s}=0.59 s$

According to (18) and (19), compute joints positions $\theta_{a 1}$ and joint velocity $\dot{\theta}_{1}$ in the transition end time, exist

$$
\left\{\begin{array}{l}
\theta_{a 1}=15+\left(\frac{1}{2} \times 42 \times 0.59^{2}\right)^{\circ}=22.3^{\circ}  \tag{22}\\
\dot{\theta}_{1}=\theta \ddot{\theta}_{1} t_{a 1}=(42 \times 0.59)^{\circ} / \mathrm{s}
\end{array}\right.
$$

2) For the picking trajectory of wrist joint, joint velocity $\dot{\theta}_{2}=27^{\circ} / \mathrm{s}^{2}$ by measuring experimentally, calculate
$\left\{\begin{array}{l}t_{a 2}=\left(\frac{3}{2}-\frac{\sqrt{27^{2} \times 3^{2}-4 \times 27(75-15)}}{2 \times 27}\right) \mathrm{s}=1.33 \mathrm{~s} \\ \theta_{a 2}=15+\left(\frac{1}{2} \times 27 \times 1.33^{2}\right)^{\circ}=38.88^{\circ}\end{array}\right.$
So, $\dot{\theta}_{2}=\ddot{\theta}_{2} t_{a 2}=(27 \times 1.33)^{\circ} / s=35.91^{\circ} / s$
According to the above calculated values, Fig. 9, Fig. 10, Fig. 11 can be plotted as transition linear interpolation with parabolic trajectory curve.

The linear function with a parabolic interpolation transition trajectory planning shows the corresponding acceleration, constant speed and deceleration movement law of arm joint motors. Wherein the curve of joint position, velocity and acceleration versus time shown as Fig. 9 - Fig. 11. The movement of three joint in picking trajectory all go through a set of points, scheme of transition region with a linear parabolic trajectory algorithm shown as Fig. 12 can be adopt. Joint movement alone picking path go through these points, by the joint angle $\theta_{i}, \theta_{j}, \theta_{k}$ indicate the three adjacent trajectory points, and every two adjacent points in picking path connected in the form of a linear function, and all the points are taken advantage of the smooth parabola transition.


Fig. 9. The displacement trajectory curves with greater variance of acceleration.


Fig. 10. The velocity trajectory curves with greater variance of acceleration.


Fig. 11. The acceleration trajectory curves with greater variance of acceleration.

Fig. 12 shows that the parabolic trajectory segments are used to plan the transition domain linear function, if the acceleration is sufficiently large, arm movement joints can not really reach those picking path points, but the actual trajectory is also very close to the ideal trajectory points.


Fig. 12. Line trajectory of multiple line with parabolic transition domain.

## 3. Experimental Simulations and Analysis

### 3.1. Experimental Technique and Steps

1) Picking robot arm started from relative to the base coordinates initial station $\mathrm{P}_{1}(10,3,4)$, through the circular point $P_{2}(11,3,5)$ then arrived the end
point $\mathrm{P}_{3}(10,3,6)$. In the experiment, set $\mathrm{N}=0-45$, namely 45 insertion points is planned between $\mathrm{P}_{1}, \mathrm{P}_{3}$. According to the circular interpolation algorithm, the Matlab simulation experiment showed space circular interpolation curve shown as Fig. 13.


Fig. 13. The simulation of space arc interpolation.
2)The picking arm in the apple-picking work mainly is continuous operation movement shown as Table 1, achieving an operating cycle, including the picking realization and picking playback, needs to complete the path planning bypassing 6 points, the planning method is cubic curve interpolation with the parabola, its point value may be solved through (15),
(17) and (19) interpolation computation, presently using the Matlab simulation, the wrist joint angle was carried on cubic interpolation path planning with parabola, the record was tested normalization average value of three longest time at the minimum speed of wrist joint, and timely the displacement, the speed, and acceleration curve on the performance wrist joint picking shown as Fig. 12 according to following step.
A. Planning preparation: Suppose the initial picking position coordinates $\mathrm{X}_{0}=(0,0, \quad 0)$, $\dot{X}_{0}=(0,0,0), \quad \ddot{X}_{0}=(0,0,0)$.
B. Planning start: input endpoint parameter vector.
C. Confirms whether $\mathrm{X}_{\mathrm{f}}$ lives in the planning picking space or not, otherwise returns to the starting point phase, discarding parameter on picking the endpoint, rebuilt picking planning space.
D. The partition reaches from $\mathrm{X}_{0}$ to $\mathrm{X}_{\mathrm{f} 0}$ through $\mathrm{X}_{\mathrm{f} 2}$, again returns to $\mathrm{X}_{0}$, travelling each section last 2 s , the use physics parameter of the picking robot arm and formula of cubic linear interpolation with parabola transition satisfy step A to calculate

$$
\begin{gathered}
\left\{\begin{array}{l}
X_{f 1}=(-11.2,9.4,-109.6) \\
X_{f 2}=(13.2,-10.8,-99.3)
\end{array}\right. \\
\left\{\begin{array}{l}
\dot{X}_{f 1}=(-9.3,13.4,134.1) \\
\dot{X}_{f 2}=(-15.4,16.2,164.2)
\end{array}\right. \\
\left\{\begin{array}{l}
\ddot{X}_{f 1}=(-13.5,27.4,104.3) \\
\ddot{X}_{f 2}=(-17.2,21.1,114.7)
\end{array}\right.
\end{gathered}
$$

E. Plan picking trajectory in method of cubic linear interpolation with parabola transition, compute displacement, velocity and acceleration curve at the interval of 0.01 s .


Fig. 14. Trajectory planning curves of picking manipulator.

### 3.2. Experimental Results and Analysis

As a whole, the picking robot arm achieve space trajectory curve and the circular arc by discrete
trajectory interpolation, if picking arm run trajectory err great, mean interval gas of discrete point deep. Therefore it is necessary to reduce the interpolation discrete distance, to cause to picking the arm approach expected the trajectory in the small precision. The interpolation experiment simulation also indicated that, the denser is interpolation spot, the more really approaches trajectory curve.

In the very short time, the picking arm trajectory is generally regarded as the movement velocity as the millimeter level of straight line. Seen from Fig. 13, a coordinate of picking path point is interpolated, the arm controller drive the joint motor to complete this interpolation, to guarantee the stability movement, the time interval cannot be long, the time can be considered clock cycle of picking arm controller. At the millisecond level of interpolation time interval, the distance between 2 interpolation points is approximately proportional to the velocity of planning path movement. So long as interpolation interval is enough small, the precision on trajectory movement can be guaranteed, also the stable request guaranteed correspondingly. If the picking path planning require high accuracy to ensure the picking stability, likewise seen from Fig. 13, in order to guarantee the trajectory moving stability to increases the precision, 2 characteristic interpolation distance is enough small, requiring the time interval of interpolation change evenly, Exactly, the picking path planning curve reveal the smooth- stability with the parabola transition.

Seen from Fig. 13, using the cubic curve trajectory interpolation with parabola may guarantee the picking manipulator the continuity of speed and the displacement in to pick on the condition of field picking, namely may complete the apple-picking the smooth movement of path planning, specially, acceleration continuity reveal the picking manipulator not to the vibrate and guarantee the stability of coming picking localization and capture.

## 4. Conclusions

This paper aims at the picking arm running along specific path sequence in advance in view of the most super time, according to path planning request and the restraint of picking robot arm, on the base of position posture and object description, proposed cubic curve interpolation with the parabola transition, completed trajectory planning of the picking arm. The simulation experiment result indicated that the algorithm may realize its smooth path tracking under on the condition of field picking, and that picking the manipulator may not have the big vibration. This algorithm provided the reliable guarantee on the following apple picking including the localization and the capture stability.

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