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Approximate Error Analysis of Acoustic Passive Directing with Cross Array Based on Piezoelectric Transducers

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Abstract: The four-element cross array is consisted of four piezoelectric transducers arranged in the form of cross. Passive directing with cross array is a very useful method for acoustic target detecting. This paper presented the algorithm of acoustic passive directing with cross array based on time delay estimation, and accurate formula and approximate formula for calculating the target's direction were proposed. The extent of application for approximate formula was suggested through analyzing the approximate error. The paper provided a basis for choosing the appropriate size of four-element cross array in practical engineering application. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Four-element cross array, Piezoelectric transducers, Acoustic passive directing, Time delay estimation, Approximate error.

1. Introduction

The four-element cross array is a common type of passive localization array in underwater acoustic engineering [1]. It is consisted of four piezoelectric transducers that can accomplish sound-electricity conversion, and the four transducers form a planar cross. The size of the cross array should be changed according to its application field.

Time delay estimation (TDE) method is an effective way for acoustic passive localization, and because the method shows high localization precision and good anti-interference, it is extensively adopted to locate underwater acoustic target [2].

The general principle of acoustic passive directing with four-element cross array based on TDE method is to estimate the target's direction using time delay between sound signals emanated from the remote target source and received by the four transducers that composed the cross array. This target detecting method can realize the function of acoustic target positioning both in plane and space, and because the array's redundancy is very low, its mathematic calculating procedure is easy to realize. Therefore, the four-element cross array can satisfy most underwater acoustic passive localization applications.

Generally, when we use the four-element cross array to locate acoustic target, in order to simplify mathematic calculating procedure, the approximate formula is usually adopted to estimate the target's direction (azimuth and depression). However, the approximate formula is educed while the target is at remote field, and if the formula can't satisfy its extent of application, i.e., when the cross array is adopted to locate near-field target, it always leads to erroneous localization results. So, the remote field condition for adopting approximate formula should be discussed.

This paper presented the algorithm of acoustic passive directing with four-element cross array based on time delay estimation, and accurate formula and

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approximate formula for calculating the target's direction were proposed. The localization error caused by approximate formula is analyzed specifically, and finally, the extent of application for approximate formula was suggested.

2. Algorithm of Acoustic Passive Directing

The structure of four-element cross array is shown in Fig. 1 [3]. The array is consisted of four piezoelectric transducers s_1, s_2, s_3 and s_4. The four transducers are arranged in the form of a planar cross, and their rectangular coordinates are respectively $s_1(d/2,0,0)$, $s_2(0,d/2,0)$, $s_3(-d/2,0,0)$ and $s_4(0,-d/2,0)$.

As shown in Fig. 1, the distance between target *T (x, y, z)* and origin *O(0,0,0)* is *r*, and target's azimuth is φ and its depression is θ.

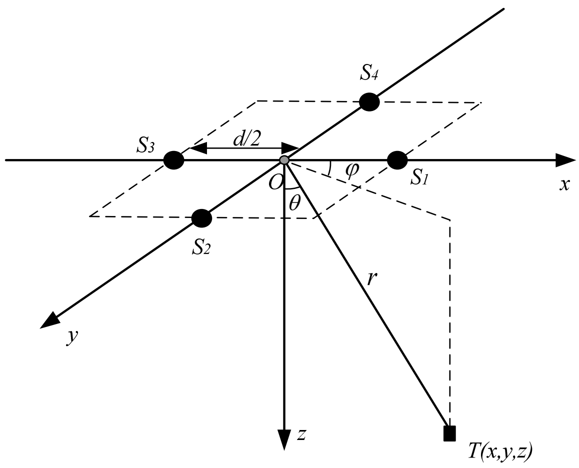

Fig. 1. Structure of four-element cross array.

The accurate formula for calculating target's azimuth is,

$$\varphi = arctg \frac{2(d_{14} - d_{12})r_1 + (d_{14}^2 - d_{12}^2)}{2d_{13}r_1 + d_{13}^2} \quad (1)$$

where $\begin{cases} d_{12} = \tau_{12} \cdot c \\ d_{13} = \tau_{13} \cdot c \\ d_{14} = \tau_{14} \cdot c \end{cases}$, $\begin{cases} \tau_{12} = t_1 - t_2 \\ \tau_{13} = t_1 - t_3 \\ \tau_{14} = t_1 - t_4 \end{cases}$, $r_1 = ct_1$.

The constant c is the velocity of sound underwater, and t_1, t_2, t_3, t_4 are respectively the propagation time of acoustic signal between target *T* and the four elements s_1, s_2, s_3, s_4. Generally, the range of target's azimuth is $0^\circ \le \varphi \le 360^\circ$.

Similarly, we also have the accurate formula for calculating target's depression as follows:

$$\theta = arcsin \frac{\sqrt{(d_{12} - d_{14})^2 + d_{13}^2}}{d} \cdot \Delta \quad (2)$$

where

$$\Delta = \sqrt{1 + \frac{4(d_{12} - d_{14})^2 (d_{12} + d_{14} - d_{13}) r_1 + O(d_{1i}, d)}{r_1^2 + d_{13} r + d_{13}^2 / 2 - d^2 / 4}}$$

$$O(d_{1i}, d) = d_{13}^2 + (d_{12}^2 - d_{14}^2)^2 + (d^2 - 2d_{13}^2)(d_{13}^2 + (d_{12} - d_{14})^2).$$

The meanings of the parameters above are as the same as that in *eq.* (1), and the range of target's depression is $0^\circ \le \theta \le 90^\circ$.

If the target *T* is at remote field, i.e., the distance (*r*) between sound signal source and array's center is much larger than the space (*d*) between two adjacent transducers in line, the approximate formulas for calculating target's azimuth and depression are [3],

$$\varphi \approx arctg \frac{\tau_{14} - \tau_{12}}{\tau_{13}} \quad (3)$$

$$\theta \approx \arcsin \frac{\sqrt{\tau_{13}{}^2 + (\tau_{14} - \tau_{12})^2} \cdot c}{d} \quad (4)$$

where $\begin{cases} \tau_{12} = t_1 - t_2 \\ \tau_{13} = t_1 - t_3 \\ \tau_{14} = t_1 - t_4 \end{cases}$.

According to *eq.* (3) and *eq.* (4), the remote-field target's direction is mainly related to the three time delays $\tau_{12}, \tau_{13}, \tau_{14}$ and the array's size.

3. Error Caused by Approximate Formula

3.1. Effect of Approximate Error

In practice, the target is generally supposed to be at remote field, and its azimuth and depression are usually estimated by approximate formulas (*eq.* (3) and *eq.* (4)), and there is a fixed rule while adopting approximate formula to locate remote-field target: The bigger the space (*d*) between two adjacent transducers is, the smaller the estimating error is. So, we need enlarge the array's size to obtain high detecting precision.

However, the size of cross array cannot enlarged unboundedly, that because the approximate formula's extent of application must meet the condition that the target is at remote field, i.e., the target's range is much larger compared with the array's size. If the above condition is not satisfied, on the contrary, the localization precision may decrease as the space (*d*) enlarges, as shown in Fig. 2. In that case, the value of *r/d* cannot satisfy the remote-field condition obviously, and we have an opposite result: The bigger the *d* is, the larger the estimating error is.

The conclusion of Fig. 2 demonstrates that while the approximate formula is used for detecting near-field target, the approximate error may be very large, and the conventional relationship between array's size and estimating error is changed, so, the discussion about the localization error caused by approximate formula is very necessary for practical engineering application.

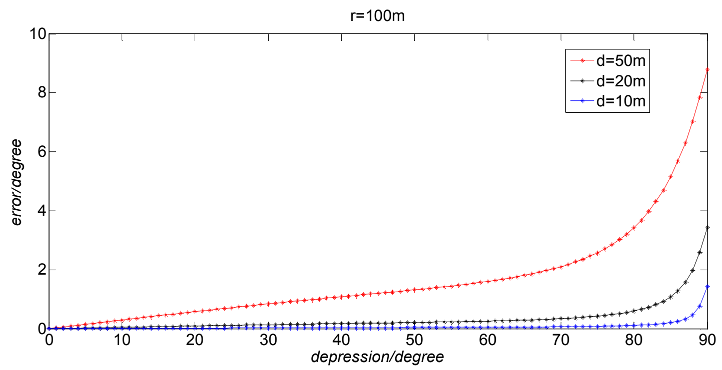

Fig. 2. Effect of approximate error.

3.2. Analysis of Approximate Error

Fig. 3 shows the relation between approximate error of depression and depression's value with different value of *r/d*, and in Fig. 3, the target's azimuth is set to 60° constantly. In Fig. 3(b), the above figure shows the simulation results when the range of target's depression is $0^{\circ} \le \theta \le 90^{\circ}$ and in the below figure, the depression's range is $0^{\circ} \le \theta \le 89^{\circ}$.

From the simulation results, we have the conclusion that when the range of target's depression is $0^{\circ} \le \theta \le 89^{\circ}$ and the value of *r/d* meets $40 \le r/d \le 60$, the approximate error of depression is less than 0.1°, and if $r/d \ge 60$, the error's range meets the condition : *error*≤0.05°. Otherwise, we also have the following conclusion: The closer the target's depression gets to 90°, the larger the depression's approximate error is.

The results of Fig. 3 show that the depression's approximate error has very close relationship with the value of *r/d*. As the ratio of *r/d* reducing, the approximate error is getting larger. Once the ratio is smaller than some numerical value, i.e., the remote-field condition is dissatisfied, the error is too large to satisfy practice application, and in that case, the approximate formula should not be adopted. The "some numerical value" can be called as "threshold value", and it depends on the requirement of positioning precision.

Although the value of *r/d* affects the depression's estimating error closely, the error does not relate to the range of *r*, and Fig. 4 demonstrates the above conclusion. If the value of *r/d* is the same, the depression's approximate error is changeless based on different *r*.

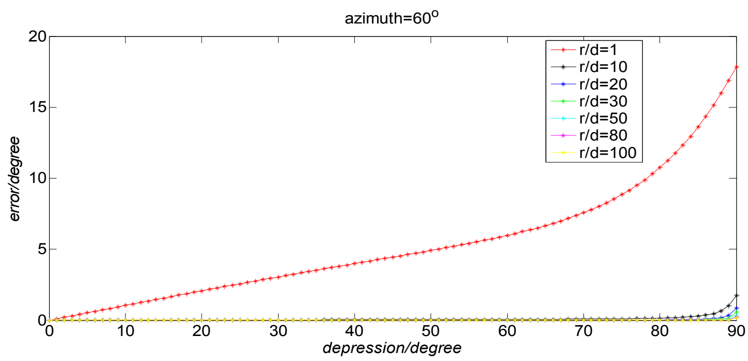

Fig. 3 (a). The relation between the approximate error of depression and depression's value (azimuth=60°) $1 \le r/d \le 100$.

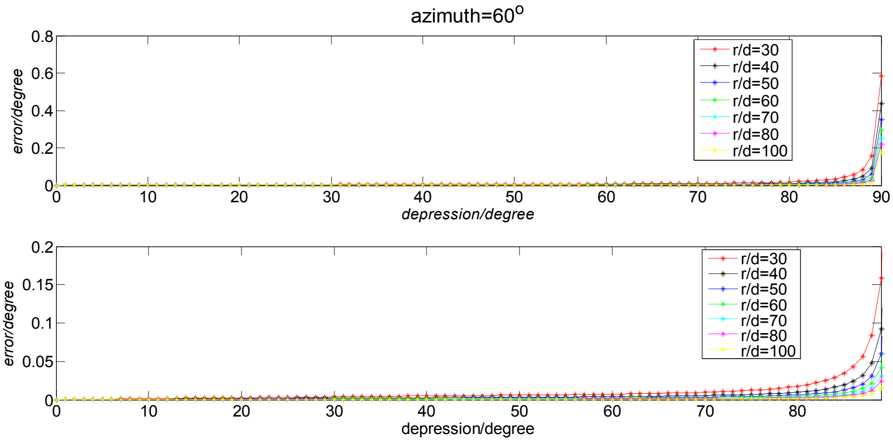

Fig. 3 (b). The relation between the approximate error of depression and depression's value (azimuth=60°) $30 \le r/d \le 100$.

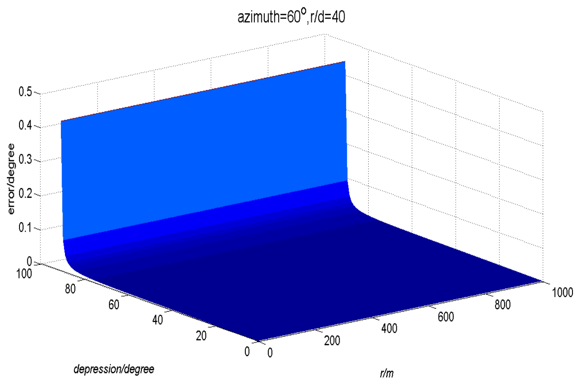

Fig. 4. The relation between approximate error of depression and target's range(r).

In addition, the effect on depression's approximate error from change of target's azimuth is little. In Fig. 5, when the depression's range is $0^{\circ} \le \theta \le 89^{\circ}$, the difference between the largest error and the smallest error is about 0.02°. Furthermore, the effect cannot be firmly established, as shown in Fig. 5.

Similar to Fig. 3(b), the above figure shows the simulation results when the range of depression is $0^{\circ} \le \theta \le 90^{\circ}$ and in the below figure, the depression's range is $0^{\circ} \le \theta \le 89^{\circ}$.

The changeful curves of azimuth's approximate error followed azimuth's change are illustrated in Fig. 6, and in this simulation, we set the target's depression to 60° constantly. The above figure shows the simulation results when the range of azimuth is $0^{\circ} \le \varphi \le 360^{\circ}$ and in the below figure, the azimuth's range is $0^{\circ} \le \varphi \le 359^{\circ}$.

By comparing the results of Fig. 3 and Fig. 6, we can see the azimuth's approximate error caused by approximate formula is much smaller than that of depression: when the range of azimuth is $0^{\circ} \le \varphi \le 359^{\circ}$ and the value of r/d meets $30 \le r/d \le 100$, the approximate error of azimuth is less than 0.0015°, and this error is so small that can be ignored. Besides, from Fig. 6, we can see that the change of azimuth's approximate error followed azimuth's value is periodic, that conclusion is also different from that of depression.

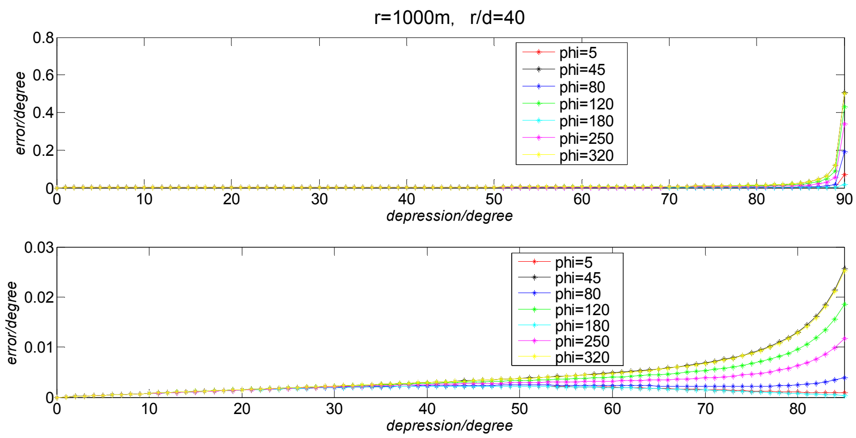

Fig. 5. The relation between approximate error of depression and azimuth's value.

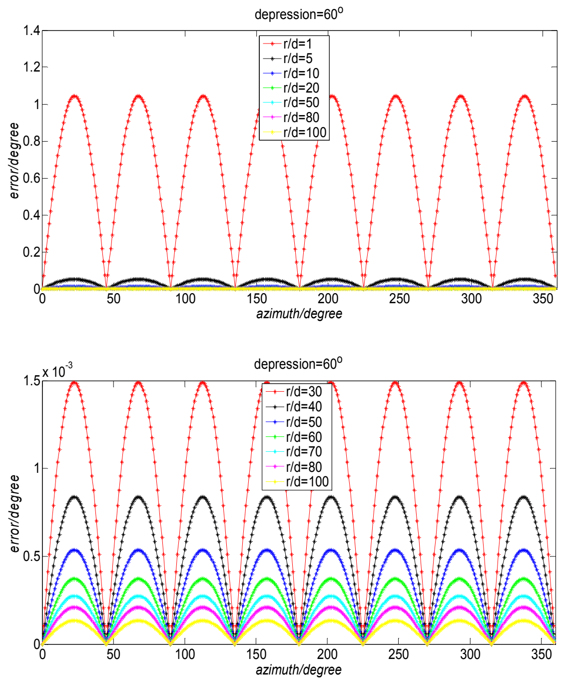

Fig. 6. The relation between approximate error of azimuth and azimuth's value.

Similar to depression, the approximate error of azimuth has no relation to target's range *r* (Fig. 7).

From the results of Fig. 4 and Fig. 7, we can see the positioning error caused by approximate formulas (*eq*. (3) and *eq*. (4)) mainly depends on the ratio of *r/d*, and the effect on approximate error from the size of array or the range of target is little. So, in practice, we should make the value of *r/d* satisfy the remote-field condition, in other words, the array's size and the target's range should not be considered separately.

The effect on azimuth's approximate error from depression's change is illustrated in Fig. 8, and the closer the target's depression gets to 90°, the larger the azimuth's approximate error is.

The above conclusion and the results of Fig. 3 both show that while the underwater target is vertically below the four-element cross array, the passive directing precision is too low to satisfy practice application.

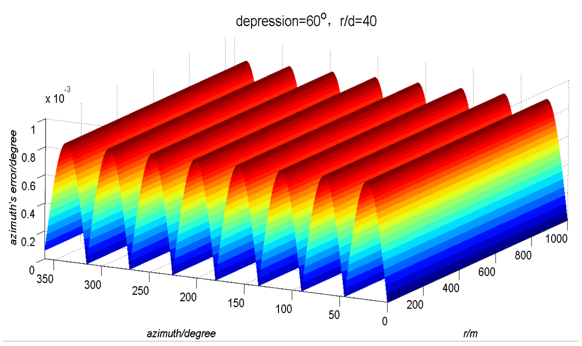

Fig. 7. The relation between the approximate error of azimuth and target's range(r).

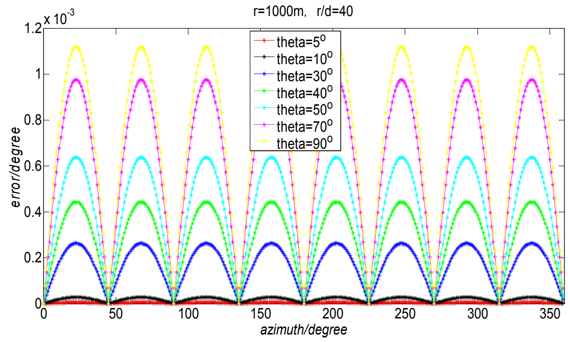

Fig. 8. The relation between the approximate error of azimuth and depression's value.

According to all the above simulation results, we have some conclusions about acoustic passive directing with four-element cross array based on time delay estimation as follows.

1) The approximate formula for underwater acoustic passive directing with four-element cross array is applicable to the condition that acoustic target is at remote field, i.e., the target's range is much larger compared with the array's size. It may bring about large error while the approximate formula is applied to detect the near-field target.

2) If the remote field condition is not satisfied, there would be estimating error using approximate formula to calculating the target's direction, and the approximate error of depression is much larger than that of azimuth. So, in practice, the effect on approximate error of depression from the value of *r/d* (the ratio of target's range and array's size) should be emphasized.

3) The approximate errors of depression and azimuth are both mainly affected by the value of *r/d*, and the errors don't relate to the target's range (*r*) or the size of array (*d*) individually. Besides, the effect on the two angles' estimating error from each other is just little.

4) In practical project application, the size of four-element cross array should be chosen according to localization precision and the range of target while using the approximate formula to estimate.

4. Conclusions

The four-element cross array is very useful for underwater acoustic passive localization, and the approximate formula is generally adopted to estimate the target's azimuth and depression. But the approximate formula is applicable to the condition that target is at remote field. It may bring about large error while detecting the near-field target. Therefore, the approximate positioning error must be taken into account while using approximate formula to estimate the target's direction.

In this paper, the algorithm of underwater acoustic passive directing with four-element cross array based on time delay estimation was presented.

And the extent of application for approximate formula was defined through analyzing the approximate error. From many simulation results, we have the conclusion that the approximate directing error is mainly affected by the value of *r/d*, where, *r* is the range of target and *d* represents the size of array. This paper provided a basis for choosing the four-element cross array's size in the engineering field.

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