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Research on the Periodic Solutions of the Rotor-ABMs System

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Abstract: Active Magnetic Bearings (AMBs) have been widely used in industry, aeronautics and astronautics for some significant advantages. The sensor is one of the important parts of the electromagnetic bearing system, the features of the sensor can affect the performance of the whole system. The nonlinear electromagnetic force may cause the considerable oscillations of the rotor with some parametric excitation. Thus, the research on characters of the nonlinear dynamics and the stability for the rotor-ABMs system has practical implication. The works in this current study focus on the study of the existence of the periodic solution, the numerical simulation of the solution and the stability of the periodic solution. Firstly, we present the motion equations of the rotor-ABMs system, by applying the multiple method of scale to the equations, we have the average equations and we get the sufficient condition of the existence of the periodic solution through using transformations, the Poincare mapping and the Melnikov function. Then, we have the phase diagrams by using the Matlab calculation software; we also analyze the phase diagrams which were under different parameters. The simulation results demonstrate the theory of the paper is correct. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Viscoelastic belt, Sensor, Periodic solution, Numerical simulation.

1. Introduction

The history of the development for the magnetic technology can be traced back to ancient times; humans began to pay attention to the electromagnetic phenomenon from the thunder and lightning, the lodestones of nature. The compass was invented by Chinese people before 1086 AD; this is the first time that human begun to use the magnetic technology.

With the development of the control theory and the electronic technology, the research on the maglev supporting technology has entered a brand new period since 1960s.

In the 21st century, Active Magnetic Bearings (AMBs) have been widely used in industry, aeronautics and astronautics for some significant advantages. The sensor is one of the important parts of the electromagnetic bearing system, the features of the sensor can affect the performance of the whole system.

Many scholars have discussed the behaviors of the Rotor-ABMs system. In 2000, Ji [1] investigated the non-linear dynamics of a rigid rotor levitated by active magnetic bearings. The vibrations in the horizontal and vertical directions are analyzed on the center manifold near the double-zero degenerate point by using normal-form method; it is shown that the horizontal and vertical directions are different due to the effect of rotor weight; the vibratory in the vertical direction can be reduced on the center manifold to the B-T form. In this paper, the authors analyzed the autonomous case and the non-

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autonomous case respectively. In 2008, Li [2] considered the bifurcations of multiple limit cycles for a rotor-active magnetic bearings (AMB) system with the time-varying stiffness. By using the method of multiple scales, the governing nonlinear equation of motion is first transformed to the averaged equation. Four groups of parametric controlling conditions are given to obtain the configurations of compound eyes. It is found that there exist respectively at least 17, 19, 21 and 22 limit cycles in the rotor-AMB system with the time-varying stiffness under the different controlling conditions. In 2010, the nonlinear dynamic behavior of a rigid disc-rotor supported by active magnetic bearings (AMB) is investigated. The vibration of the rotor is modeled by a coupled second order nonlinear ordinary differential equations with quadratic and cubic nonlinearities. Their approximate solutions are sought applying the method of multiple scales in the case of primary resonance. The N-R method and the pseudo-arclength path-following algorithm are used to obtain the frequency response curves. By choosing the Hopf bifurcations as the initial points and applying the shooting method and the pseudo-arclength path-following algorithm, the periodic solution branches are obtained. At the same time, the Floquet theory is used to determine the stability of the periodic solutions. A detailed bifurcation analysis of the dynamic (periodic and chaotic) solutions of the averaged equations is presented. The three types of primary Hopf bifurcations are found for the first time in the rotor-AMB system. In 2009, Han [3] investigated the periodic solution of a four dimensional system of autonomous ordinary differential equations depending on a small parameter. In 2010, the nonlinear dynamic behavior of a rigid disc-rotor supported by active magnetic bearings (AMB) is investigated [4]. The vibration of the rotor is modeled by a coupled second order nonlinear ordinary differential equations with quadratic and cubic nonlinearities. In 2012, Chu [5] presented the formula of the first twist coefficient for a nonlinear damped equation based on the third order approximation developed by Ortega. As an example, they considered the Lyapunov stability of a superlinear damped differential equation. In 2013, Li [6] established the complete CAD/CAE model of the electric furnace roof with finite element software based on the theory of transferring heating subject, and respectively calculated the stable temperature and stress field of the firebrick roof and the prefabricate block roof in last melting stage. In 2013, Wei [7] presented three methods to improve the performance of a high frequency aluminum nitride (AlN) ultrasonic transducer.

From the above description, it is important to note that, the most previous works are confined to the controlled systems. This current study aimed at the main difficulties of the system, such as its strong nonlinear feature and the distribution of parameters. Firstly, through using the multiple-scale method, the average equations of the rotor-ABMs system are given; the linear transformation, Melnikov function and the Poincare mapping are used to obtain the sufficient condition of the existence of the periodic solution. Then, numerical simulations are performed to verify the analytical predictions. When $\varepsilon = 0$, we have the results that, the first two equations are Hamiltonian system, the next two equations are 1-order weak focus of planar autonomous system; when $M(r) \neq 0$, it is shown that the system of (3) has no periodic orbit; on the other side, when $M(r) = 0$, $M'(r) = 2\pi b_{0100} \neq 0$, the system of (3) has a unique periodic orbit with periodic near 2π . Finally we get the phase diagrams of the system under different parameters.

2. Average Equations of the Rotor-ABMs System with two Degrees of Freedom

Through using the multiple-scale method and eliminating the secular term, by turning the equations from the polar form into a rectangular form, we have the average equations of the Rotor-ABMs system:

$$
\begin{aligned}
\frac{dx_1}{dT_1} &= -\frac{1}{2}\mu x_1 - \frac{1}{2}(\sigma_1 - f_1)x_2 - \frac{3}{2}\alpha_1 x_2 (x_1^2 + x_2^2) \\
&\quad - \frac{1}{2}\alpha_2 x_2 (x_3^2 + 3x_4^2) - \alpha_2 x_1 x_3 x_4 , \\
\frac{dx_2}{dT_1} &= \frac{1}{2}(\sigma_1 + f_1)x_1 - \frac{1}{2}\mu x_2 + \frac{3}{2}\alpha_1 x_1 (x_1^2 + x_2^2) \\
&\quad + \frac{1}{2}\alpha_2 x_1 (3x_3^2 + x_4^2) + \alpha_2 x_2 x_3 x_4 , \\
\frac{dx_3}{dT_1} &= -\frac{1}{2}(\sigma_2 - f_2)x_4 - \frac{1}{2}\mu x_3 - \frac{3}{2}\beta_1 x_4 (x_3^2 + x_4^2) \\
&\quad - \frac{1}{2}\beta_2 x_4 (3x_2^2 + x_1^2) - \beta_2 x_1 x_2 x_3 , \\
\frac{dx_4}{dT_1} &= \frac{1}{2}(\sigma_2 + f_2)x_3 - \frac{1}{2}\mu x_4 + \frac{3}{2}\beta_1 x_3 (x_3^2 + x_4^2) \\
&\quad + \frac{1}{2}\beta_2 x_3 (3x_1^2 + x_2^2) + \beta_2 x_1 x_2 x_4 .
\end{aligned} \tag{1}
$$

In order to make the system hold: the first two equations is Hamiltonian system, the following two equations is a 1-order weak focus of planar autonomous system, we assume,

$$u_1 = \frac{(\sigma_1 + f_1)x_1 + \mu x_2}{2}, u_2 = x_2,$$

$$v_1 = \frac{(\sigma_2 + f_2)x_3 + \mu x_4}{2N}, \ v_2 = x_4 .$$

By using the linear transformation into (1), then the system of (1) can be transformed into the system (2) as follows:

$$\frac{du_1}{dT_1} = -u_2 + M_{1a}(u_1, u_2, v_1, v_2)\ ,$$
$$\frac{du_2}{dT_1} = u_1 - b_{0100}u_2 + M_{1b}(u_1, u_2, v_1, v_2)\ , \quad (2)$$
$$\frac{dv_1}{dT_1} = -c_{0001}v_2 + M_{2c}(u_1, u_2, v_1, v_2)\ ,$$
$$\frac{dv_2}{dT_1} = d_{0010}v_1 - d_{0001}v_2 + M_{2d}(u_1, u_2, v_1, v_2)\ .$$

Considering the perturbed system of the system (2)

$$\frac{du}{dT_1} = f(u) + \varepsilon P(u, v)\ , \quad (3A)$$

$$\frac{dv}{dT_1} = g(v) + \varepsilon Q(u, v) \quad (3B)$$

where $f(u) = (-u_2, u_1)^T$, $u = (u_1, u_2)^T$, $g(v) = (-c_{0001}v_2 + M_{3c}(u, v), d_{0010}v_1 + M_{3d}(u, v))^T$, $P(u, v) = (M_{1a}(u, v), -b_{0100}u_2 + M_{1b}(u, v))^T$ $Q(u, v) = (M_{4c}(u, v), -d_{0001}v_2 + M_{4d}(u, v))^T$, $v = (v_1, v_2)^T$, M_{ij} are higher polynomials of u and v , $i = 1, 3, 4$, $j = a, b, c, d$.

When $\varepsilon = 0$, the system of (3) holds:

(A) Planar autonomous system

$$\frac{du}{dT_1} = f(u)\ , \quad (4)$$

is a Hamiltonian system and there exists an open interval J and the system (4) has a family of periodic orbits $L_h = \{(u_1, u_2): u_1^2 + u_2^2 = 2h\}$.

(B) When $\beta_1 \neq 0$, then $v = 0$ is a 1-order weak focus of planar autonomous system (see [8])

$$\frac{dv}{dT_1} = g(v)\ . \quad (5)$$

3. Sufficient Conditions for the Existence of Periodic Solution

In order to simplify the expressions, let us introduce $\partial_i^{m,1}(a) = (0, \cdots, a, \cdots, 0)_{m\times 1}^T$, which is a $m \times 1$ block matrix and the i-th element is a , the others are zeros. Assume that $v = v$, $0 \le \theta \le 2\pi$, $u = G(\theta, h) = (\sqrt{2h}\cos\theta, \sqrt{2h}\sin\theta)^T$, then $G(\theta, h)$ is 2π periodic in θ .

The mentioned periodic transformation could transform the system (3) into the following system:

$$\frac{dh}{d\theta} = -\varepsilon \frac{f(G(\theta, h))\Lambda P(G(\theta, h), v)}{1 + \varepsilon G_h(\theta, h)\Lambda P(G(\theta, h), v)}\ ,$$
$$\frac{dv}{d\theta} = \frac{g(G(\theta, h), v) + \varepsilon Q(G(\theta, h), v)}{1 + \varepsilon G_h(\theta, h)\Lambda P(G(\theta, h), v)}\ . \quad (6)$$

Suppose that $(h(0, r, v_0, \varepsilon), v(0, r, v_0, \varepsilon))$ is a solution of system (6) and satisfy $(h(0, r, v_0, \varepsilon), v(0, r, v_0, \varepsilon)) = (r, v_0)$, which has an expansion of the following form:

$$h(\theta, r, v_0, \varepsilon) = r + \varepsilon(h_1(\theta, r) + h_2(\theta, r)v_0 + h_3(\theta, r)\varepsilon + O(|v_0, \varepsilon|^2))\ , \quad (7A)$$

$$v(\theta, r, v_0, \varepsilon) = v_1(\theta, r)v_0 + v_2(\theta, r)\varepsilon + O(|v_0, \varepsilon|^2)\ . \quad (7B)$$

$h_i(0, r) = 0$, $(i = 1, 2, 3)$, $v_1(0, r) = I_2$, $v_2(0, r) = 0$, where I_2 is the identity matrix of order 2. Substitute (7) into (6), and then compare the coefficients of ε on both sides, we obtain:

$$\frac{dh_1(\theta, r)}{d\theta} = -f(G(\theta, r))\Lambda P(G(\theta, r), 0)$$
$$\frac{dh_2(\theta, r)}{d\theta} = 0\ , \quad \frac{dh_3(\theta, r)}{d\theta} = -\frac{1}{2\pi}H(\theta, r)\ ,$$
$$\frac{dv_2(\theta, r)}{d\theta} = Bv_2(\theta, r) + Q(G(\theta, r), 0)\ ,$$
$$\frac{dv_1(\theta, r)}{d\theta} = Bv_1(\theta, r)\ ,$$

where $B = \partial_{1,2}^{2,2}(-c_{0001}) + \partial_{2,1}^{2,2}(d_{0010})$.

For sufficiently small $\varepsilon \neq 0$, there generates a periodic orbit of the system (3) in the neighbor of $L_r \equiv \{(u, v): H(u) = r, v = 0\}$, if and only if the following equations have a solution in (r, v_0) . That means:

$$\pi r(M_1 M_1^1 + rM_1) - r\varepsilon + O(|v_0, \varepsilon|^2) = 0$$
$$(\exp(2\pi B) - I_2)v_0 + \exp(2\pi B)\varepsilon + O(|v_0, \varepsilon|^2) = 0\ ,$$

where $M_1 = 3b_{0300} + b_{2100} + 3a_{3000} + a_{1200}$,

$$M_1^1 = \frac{2b_{0100}}{3b_{0300} + b_{2100} + 3a_{3000} + a_{1200}}$$

Here, we denote $h_1(2\pi, r)$ to be the Melnikov function of (3).

That is

$$M(r) = \int_0^{2\pi} f(G(\theta, r))\Lambda P((G(\theta, r), 0)d\theta\ .$$

Lemma Considering the theory of successor function, it is easy to see that, if $M(r) \neq 0$, there does not exist any periodic orbit with period near 2π in the neighborhood of L_r of system (3). Hence, if there has a periodic orbit for system of (3), then $M(r) = 0$.

Theorem (Sufficient Condition for the Existence of the Periodic Solution) For $0 < |\varepsilon| << 1$, if $M(h_0) = 0$, $M'(h_0) \neq 0$, and $c_{0001} \neq k$ for a certain $h_0 \in J$, then the system (3) has a unique periodic orbit with period near 2π in the neighbor of L_{h_0} .

4. The Numerical Simulations of the Periodic Orbit

In this section, numerical simulations are performed to verify the analytical predictions. In order to be closer to the system of (1), we fix the coefficients of the most primitive parameters of (1).

(A) When $\varepsilon = 0$, then the (3A) is a Hamiltonian system. When $\mu\beta_1 \neq 0$, then $v = 0$ is a 1-order weak focus of planar autonomous system (3B). By using the Matlab, we obtain the following Fig. 1.1-Fig.1.4

(B) We have known that, if $M(r) \neq 0$, then system of (3) has no periodic orbits with period near 2π in a neighborhood of L_r . When $\sigma_1 = \sigma_2 = \beta_1 = \alpha_1 = 1$, $\mu = 2$, $r = 1$, he Melnikov function $M(r) \neq 0$. We have Fig. 2.1-Fig.2.4

(C) Hypothesis $r = M_1^1$, then $M(r) = 0$, $M'(r) = 2\pi b_{0100} \neq 0$ and $c_{0001} \neq k$, then there has a unique periodic orbit with period near 2π . When $\alpha_2 = 0$, $\beta_2 = 0$, $\sigma_1 = \sigma_2 = \beta_1 = \alpha_1 = 1$, $\mu = 2$, $r = \frac{2}{9}$. Then in this condition, the requirements in the theorem are satisfied we get the following Fig. 3.1-Fig.3.4

5. Conclusions

Active magnetic bearings (AMBs) are an enabling technology that has facilitated the development of innovative rotating equipment applications. Rotors supported by active magnetic bearings (AMBs) can spin at high surface speeds with relatively low power losses. This makes them particularly attractive for use in flywheels for energy storage in applications such as electric vehicles and uninterruptible power supplies. Periodic system and periodic solution exist in many natural and man-made systems. In this paper, we first present the sufficient conditions for the existence of periodic solution by using nonsingular linear transformation, Melnikov function. We get the results that when $M(r) \neq 0$, then the system of (3) does not exist any periodic solution. If $M(h_0) = 0$, $M'(h_0) \neq 0$, then the system of (3) has a unique periodic solution. It is conformed to the theorem. Numerical simulations obtained in this paper indicate that there exist different forms of the periodic responses in the viscoelastic belt under certain parametric excitation, parameters and initial conditions. And the solutions are completely different.

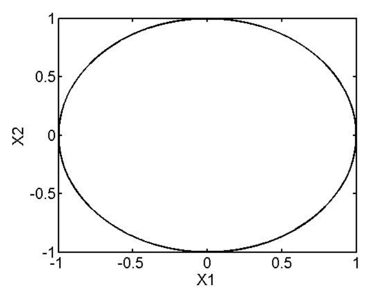

Fig. 1.1

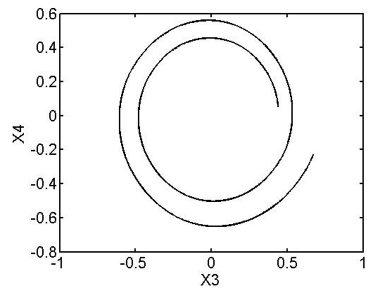

Fig. 1.2

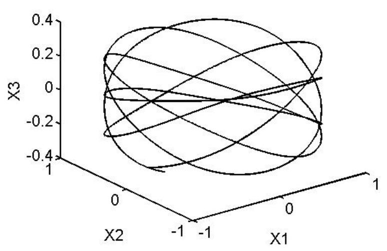

Fig. 1.3

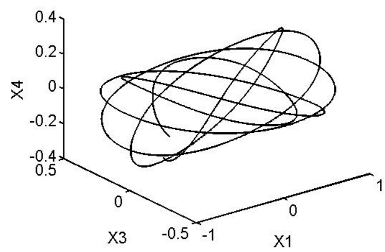

Fig. 1.4

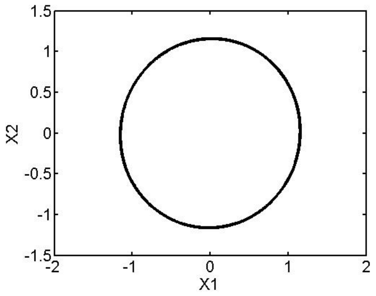

Fig. 2.1

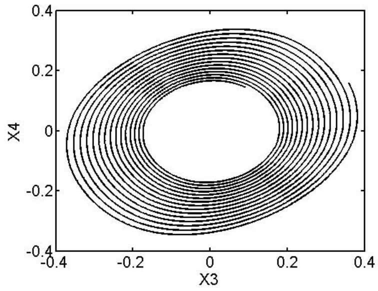

Fig. 2.2

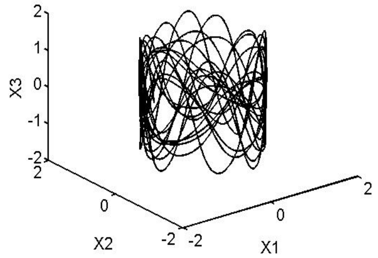

Fig. 2.3

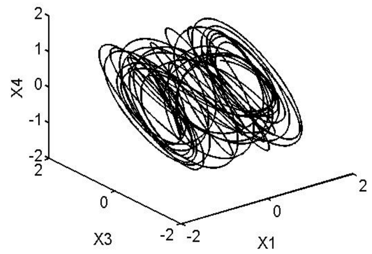

Fig. 2.4

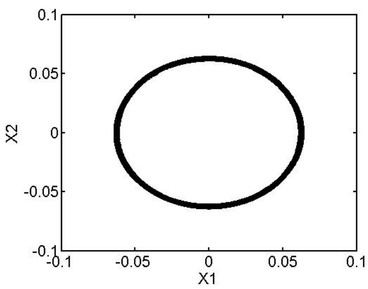

Fig. 3.1

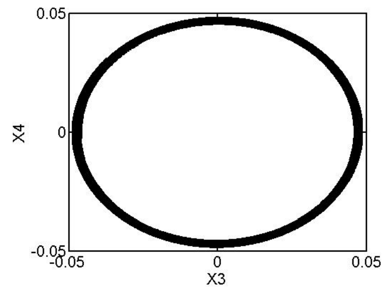

Fig. 3.2

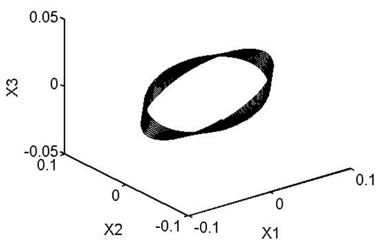

Fig. 3.3

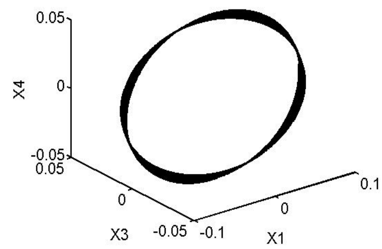

Fig. 3.4

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