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Optimal Waveform Design with Constant Modulus Constraint for Rank-One Target Detection

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Abstract: In radar systems, many researchers focused on the waveform design to improve the performance. And many algorithms just only consider the energy of the transmitted waveform. However, in practice, only the energy constraint is insufficient to guarantee that the signal will satisfy common envelope requirements, such as constant modulus that is important to radar transmitter. In this paper, we propose a constant modulus waveform design method for rank-one target detection. Firstly the optimal waveform under the energy constraint is obtained. Then the optimal waveform owns the constant modulus property in time domain from its Fourier transform magnitude is obtained finally. Finally, simulation results show the effectiveness of the proposed algorithm. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Waveform design, Cognitive radar, Relative entropy, Constant modulus constraint, Target detection.

1. Introduction

Cognitive radar is proposed as a new generation radar system by Haykin [1, 2], which is not only dependent on the adaptive signal processing in the radar receiver, but also through an adaptive radar transmitter in order to improve the performance. Thus the optimal waveform design problem is the key technology of the cognitive radar.

Lots of research efforts have been focused on the radar waveform optimization. Bell proposed to maximize the mutual information (MI) between the received signal and the target impulse response (TIR) to optimize the transmitted waveform [3]. In [4], the authors extended the MI method to the multiple-input multiple-output (MIMO) radar and proposed another method that adopted the mean-square error (MSE) in estimating the TIR as the cost function and minimized MSE to optimize the waveform for target identification and classification. The proposed two methods have the same solution. Pillai et al. developed an eigensolution for optimal signal/filter pairs for target detection when the target and clutter can be seen as time invariant random processes [5, 6], which used the signal to clutter-plus-noise ratio (SCNR) as the cost function and maximized the SCNR to optimize the waveform. Goodman et al. published a series of papers about the waveform design based on the sequential hypothesis testing that controls when hard decision may be made with adequate confidence to design waveform. The detailed work can be found in [7, 8]. In [9], Kay derived the optimal NP detector firstly, which shows that the NP detection performance does not immediately lead to an obvious signal design criterion so that a divergence criterion is proposed for signal design, also based on the relative entropy in signal input multiple output (SIMO) radar scenario. However, the Kay's method just only considers the energy constraint in the transmitter side. However the constant modulus is also important. Since the radar systems commonly require constant modulus waveforms in order to utilize nonlinear power amplifiers, otherwise the target detection performance will be degraded. In this paper, we solve the optimal waveform design with constant modulus problem for target detection in rank-one target model.

In next section, the signal model and problem formulation is given. The optimal waveform design considering the constant modulus constraint is solved in section 3. Simulation results are shown in section 4. The conclusion is summarized in section 5.

2. Signal Model and Problem Formulation

When the transmitted signal band B is sufficiently large so that B is comparable to $c/2d$, where c is the speed of the light and d is the range span of the target, the target is not within a signal range cell. In this case, it should be modeled as extended target whose impulse response function can be expressed as [10]:

$$h_t(t) = \sum_{l=1}^{L} a_{t,l}\delta(t-\tau_l), \tag{1}$$

where $a_{t,l}, l = 1,2,\cdots L$ are the complex reflection coefficients of the target. The clutter impulse response function can be expressed as

$$h_c(t) = \sum_{m=1}^{M} a_{c,m}\delta(t-\tau_m), \tag{2}$$

where $a_{c,m}, m = 1,2,\cdots M$ are the complex reflection coefficients of the clutter. So the received signal in additive Gaussian noise condition can be expressed as

$$x(t) = s(t) * h_t(t) + s(t) * h_c(t) + n(t), \tag{3}$$

where $s(t)$ is the transmitted signal and $n(t)$ is the complex additive Gaussian noise. $*$ denotes convolution operator. According to [11], formula (3) can be expressed in frequency domain, that is

$$\begin{aligned} \boldsymbol{X} &= \boldsymbol{S}\boldsymbol{H}_t + \boldsymbol{S}\boldsymbol{H}_c + \boldsymbol{N} \\ &= diag(\boldsymbol{H}_t)\boldsymbol{s} + diag(\boldsymbol{H}_c)\boldsymbol{s} + \boldsymbol{N} \end{aligned}, \tag{4}$$

where $\boldsymbol{s} = \left(S(F_{-N/2}),\ldots,S(F_{N/2-1})\right)^T$, $\boldsymbol{S} = diag(\boldsymbol{s})$

$$\boldsymbol{H}_t = \begin{pmatrix} \exp(j2\pi F_{-N/2}\tau_1) & \cdots & \exp(j2\pi F_{-N/2}\tau_L) \\ \vdots & \ddots & \vdots \\ \exp(j2\pi F_{N/2-1}\tau_1) & \cdots & \exp(j2\pi F_{N/2-1}\tau_L) \end{pmatrix}, \times\left(a_{t,1},\cdots,a_{t,M}\right)^T \tag{5}$$

$$\boldsymbol{H}_c = \begin{pmatrix} \exp(j2\pi F_{-N/2}\tau_1) & \cdots & \exp(j2\pi F_{-N/2}\tau_M) \\ \vdots & \ddots & \vdots \\ \exp(j2\pi F_{N/2-1}\tau_1) & \cdots & \exp(j2\pi F_{N/2-1}\tau_M) \end{pmatrix}, \times\left(a_{c,1},\cdots,a_{c,M}\right)^T \tag{6}$$

$\boldsymbol{H}_t$ is a zero mean random Gaussian vector with covariance $\boldsymbol{R}_{ht}$. When $Rank(\boldsymbol{R}_{ht}) = 1$, the amplitude vector of the target scatters function $\boldsymbol{a}$ is fixed, the phase of the target scatters function $e^{j\phi}$ may fluctuate randomly due to changes in the distance between the target and the radar that is the scatters of a rigid target exit no relative Doppler. Some papers named this kind of target after rank-one target. The covariance $\boldsymbol{R}_{ht}$ can be decomposed $\boldsymbol{R}_{ht} = \boldsymbol{V}_{ht}\boldsymbol{V}_{ht}^H$ by the low rank decomposition, the $\boldsymbol{V}_{ht}$ is defined by the target signature vector. The clutter is also complex random Gaussian vector with zero mean and covariance $\boldsymbol{R}_{hc}$. And the $\boldsymbol{R}_{hc}$ is also can be decomposed as $\boldsymbol{R}_{hc} = \boldsymbol{V}_{hc}\boldsymbol{V}_{hc}^H$.

When a target exists in radar environment, the detection procedure is given by decision on one of the two possible hypotheses as

$$\begin{aligned} &H_0 : \boldsymbol{X} = \boldsymbol{S}\boldsymbol{H}_c + \boldsymbol{N} \\ &H_1 : \boldsymbol{X} = \boldsymbol{S}\boldsymbol{H} + \boldsymbol{S}\boldsymbol{H}_c + \boldsymbol{N} \end{aligned}, \tag{7}$$

Our aim is to design the optimal waveform to maximize the probability of detection. In next section, we will elaborate how to optimize the waveform design.

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3. Waveform Design with Constant Modulus Constraint

According to model (5), the probability density function (PDF) under H_0 can be written as

$$p(\boldsymbol{X} \mid H_0) = \frac{1}{\pi^{2N}\det(\boldsymbol{R}_{cn})}\exp\left[-\boldsymbol{X}^H(\boldsymbol{R}_{cn})^{-1}\boldsymbol{X}\right], \tag{8}$$

The PDF under H_1 can be written as

$$p(\boldsymbol{X} \mid H_1) = \frac{1}{\pi^{2N} \det(\boldsymbol{R}_s + \boldsymbol{R}_{cn})} \times \exp\left[-\boldsymbol{X}^H (\boldsymbol{R}_s + \boldsymbol{R}_{cn})^{-1} \boldsymbol{X}\right], \quad (9)$$

where $\boldsymbol{R}_{cn}$ is the covariance matrix of $\boldsymbol{SH}_c + \boldsymbol{N}$, $\boldsymbol{R}_s$ is the covariance matrix of $\boldsymbol{SH}$ and $\boldsymbol{R}_s = \boldsymbol{V}_s \boldsymbol{V}_s^H$. Thus, the likelihood radio test can be expressed as

$$\begin{aligned} l(\boldsymbol{X}) &= \ln \frac{p(\boldsymbol{X} \mid H_1)}{p(\boldsymbol{X} \mid H_0)} \\ &= c + \boldsymbol{X}^H \left[\boldsymbol{R}_{cn}^{-1} - (\boldsymbol{R}_s + \boldsymbol{R}_{cn})^{-1}\right] \boldsymbol{X} \end{aligned}, \quad (10)$$

where $c = \ln\left[\frac{\det\left((\boldsymbol{R}_s + \boldsymbol{R}_{cn})^{-1}\right)}{\det\left(\boldsymbol{R}_{cn}^{-1}\right)}\right]$. The first item in the right of equal sign in (10) is a constant. According to the Woodbury's identity, (10) can be expressed as

$$l(\boldsymbol{X}) = c + \frac{\boldsymbol{V}_s^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s}{1 + \boldsymbol{V}_s^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s} \frac{\left|\boldsymbol{X}^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s\right|^2}{\boldsymbol{V}_s^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s}, \quad (11)$$

The relative entropy is defined as the exponent in probability of error in a hypothesis test between distribution p_0 and p_1 [12]. There exists the probability of error in each hypothesis can be expressed as

$$\begin{aligned} D &= D(p_0 \| p_1) + D(p_1 \| p_0) \\ &= \int p(\boldsymbol{X} \mid H_0) \ln \frac{p(\boldsymbol{X} \mid H_0)}{p(\boldsymbol{X} \mid H_1)} d\boldsymbol{X} \\ &+ \int p(\boldsymbol{X} \mid H_1) \ln \frac{p(\boldsymbol{X} \mid H_1)}{p(\boldsymbol{X} \mid H_0)} d\boldsymbol{X} \\ &= E_0\left[-\ln \frac{p(\boldsymbol{X} \mid H_1)}{p(\boldsymbol{X} \mid H_0)}\right] + E_1\left[\ln \frac{p(\boldsymbol{X} \mid H_1)}{p(\boldsymbol{X} \mid H_0)}\right] \end{aligned}, \quad (12)$$

where $D(p_i \| p_j)$ denotes the relative entropy in favor of H_i against H_j. In radar systems, the target detection problem has two probabilities of error, false alarm probability P_{fa} and miss probability P_m , $\exp\left[-N \bullet D(p_0 \| p_1)\right]$ and $\exp\left[-N \bullet D(p_1 \| p_0)\right]$ are equal to P_{fa} and P_m, respectively. It can be seen that maximizing the relative entropy could result in minimizing the P_{fa} and P_m . Assuming $\boldsymbol{V}_s^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s \gg 1$ [9], Substitute (11) into (12), we have

$$\begin{aligned} D &= P_{fa} + P_m \approx \boldsymbol{V}_s^H \boldsymbol{R}_{cn}^{-1} \boldsymbol{V}_s \\ &\approx \sum_{n=-N/2}^{N/2-1} \frac{\left|V_s(F_n)\right|^2}{P_c(F_n) + P_n(F_n)} \\ &\approx \sum_{n=-N/2}^{N/2-1} \frac{\left|S(F_n)\right|^2 \left|V_{ht}(F_n)\right|^2}{\left|S(F_n)\right|^2 P_{hc}(F_n) + P_n(F_n)} \end{aligned}, \quad (13)$$

Note that $P_m + P_d = 1$, where P_d is the target detection probability. We hope the target detection probability P_d as large as possible while the false alarm probability P_{fa} as small as possible. Thus, the optimization criterion can be designed as

$$\max D = \max\left[D(p_0 \| p_1) + D(p_1 \| p_0)\right], \quad (14)$$

Considering the total energy limit, the optimization model is

$$\begin{aligned} &\max D \approx \int_{-B/2}^{B/2} \frac{\left|S(F)\right|^2 \left|V(F)\right|^2}{\left|S(F)\right|^2 P_{hc}(F) + P_n(F)} dF \\ &s.t. \quad \int_{-B/2}^{B/2} \left|S(F)\right|^2 dF \le E_s \end{aligned}, \quad (15)$$

After using Lagrange multiplier, the optimal waveform is

$$\left|S(F)\right|^2 = \max\left(\frac{\left|V(F)\right|\sqrt{P_n(F)/\lambda} - P_n(F)}{P_{hc}(F)}, 0\right), \quad (16)$$

The parameter λ is found by solving

$$\int_{-B/2}^{B/2} \max\left(\frac{\left|V_{ht}(F)\right|\sqrt{P_n(F)/\lambda} - P_n(F)}{P_{hc}(F)}, 0\right) dF = E_s, \quad (17)$$

The optimal waveform is the same as the result of [12]. However, the optimal waveform of equation (16) just only considers the energy constraint. In modern radar systems, the constant modulus waveforms are commonly required in practice. Since the nonlinear power amplifiers in radar systems require the input signal to be constant modulus. Thus the optimal waveform with the constant modulus constraint could be considered.

The method to the constant modulus is based on the technique in [13].the constant modulus signal is generated in time domain after given its Fourier transform magnitude. Let C_M denotes the set of function $\{g(t)\}$ that have the same Fourier transform magnitude $M(\omega)$ over the frequency interval Ω . A magnitude projection operator P_M defined can make the arbitrary function $f(t)$ project to a "nearest neighbor" $P_M f(t)$ that belongs to C_M . For arbitrary function $f(t)$, its Fourier transform can be representation by $F(\omega) = \left|F(\omega)\right| e^{j\psi(\omega)}$. The magnitude projection of $x(t)$ can be defined as

$$P_M f(t) \leftrightarrow \begin{cases} M(\omega) e^{j\psi(\omega)}, & \omega \in \Omega \\ F(\omega), & \omega \in \Omega' \end{cases}, \quad (18)$$

Equation (18) guarantees that radar systems maintain the transmit signal to own the optimal waveform in frequency domain. Continue the constant modulus element. Let C_A denote the set of functions $\{g(t)\}$ which have constant envelope level A , There exists a amplitude projection operator P_A to assign an arbitrary function $f(t)$ a" nearest neighbor" $P_A f(t)$ on C_A so that there is no element $g \in C_A$ to satisfy $\|x-g\| < \|x-P_A x\|$. Given an arbitrary function $f(t)=a(t)e^{j\theta(t)}$, the projection procedure is

$$P_A f(t)=\begin{cases} Ae^{j\theta(t)}, t \in T \\ a(t)e^{j\theta(t)}, otherwise \end{cases}, \quad (19)$$

The magnitude and amplitude projection are combined according to

$$f_{k+1}(t)=P_A P_M f_k(t), \quad (20)$$

The equation (20) has the error reduction property [13]:

$$d_{k+1}=\|f_{k+1}(t)-P_A P_M f_{k+1}(t)\| \le \|f_k(t)-P_A P_M f_k(t)\|=d_k, \quad (21)$$

where $f_k(t)$ is the k^{th} projection iterated function.

When the optimal waveform without considering the constant modulus $s(t)$ that owns the magnitude $M(\omega)=|S(2\pi f)|$ is obtained, the optimal waveform $g(t)$ with the constant modulus could be obtained by the equation (18), (19) and (20). The equation (21) guarantee the error reduce when the $k \to \infty$. After a number of iteration, the signal $s(t)$ has the constant modulus envelope with the prescribed Fourier transform magnitude.

4. Simulations Results

In this section, we will give the simulation to demonstrate the effectiveness of the proposed method. Suppose the frequency of the signal is normalized to be $(0,1)$. The length of the target impulse response is 31. The sampling frequency is 2. Fig. 1 shows the target spectral signature and the clutter power spectral density (PSD). The energy is constrained to be 50 (energy unit). The noise PSD is 0.15.

Fig. 2 shows the optimal waveform without constant modulus constraint and the Fig. 3 shows the optimal waveform with constant modulus constraint. The two figures show that the optimal waveform with/without constant modulus constraint allocates the transmitted energy among most of the spectral peaks of the target signature and the two waveforms are similar, which demonstrates the optimal waveform considering the constant modulus constraint still guarantees the detection performance. The magnitudes of the constant modulus waveform are smaller than the one of the non-constant modulus waveform. However, the optimal waveform with constant modulus allocates the transmitted energy into additional frequency bands. Fig. 4 is the complex constellation of constant modulus constrained waveform. It can be seen that the optimal waveform has the constant envelope in time domain after considering the constant modulus constraint, which ensures the modulus of the transmitted signal could not exceed the maximum input value of DAC in practice.

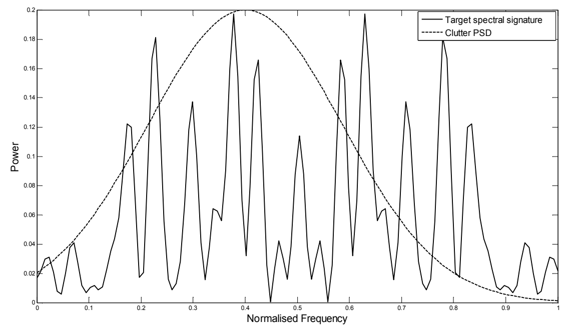

Fig. 1. The target spectral signature and the clutter PSD.

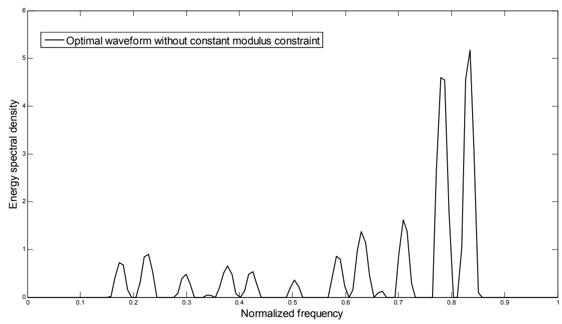

Fig. 2. The optimal waveform without constant modulus constraint.

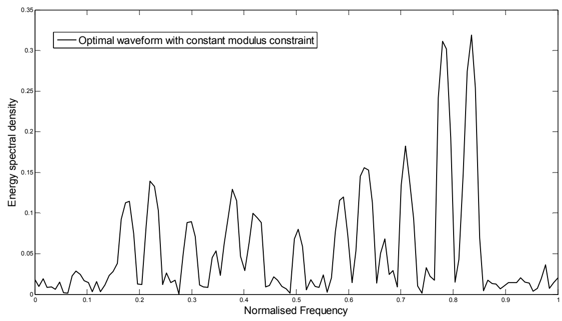

Fig. 3. The optimal waveform with constant modulus constraint.

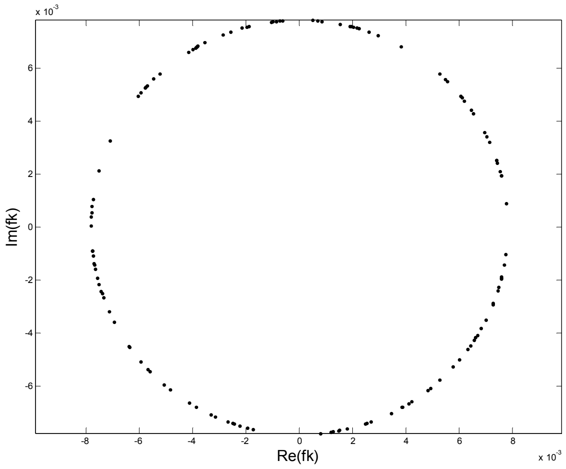

Fig. 4. Complex Constellation of optimal waveform with constant waveform.

5. Conclusions

This paper focuses on the waveform design for target detection considering the constant modulus constraint. Maximizing the relative entropy as the optimization criterion designs the optimal waveform with the transmitted energy constraint for the rank-one target. In addition, considering the constant modulus in practice, the constant modulus constraint is applied to design the optimal waveform. The energy spectral density of the optimal waveform without constant modulus is regarded as he prescribed Fourier transform magnitude. Then the constant modulus waveform could be obtained in time domain. It is similar to the optimal waveform that only considers the total energy constraints. The simulation results present the proposed method validation.

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