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Research on Mapping and Reconstitution of Non-manifold Polyhedral for Automated Design Computing

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Abstract: In this paper, a class of non-manifold polyhedral is introduced from the function-to-form mapping perspective. Modelling of non-manifold polyhedral and the situated reconstitution is introduced to develop a "top-down" assembly-oriented computational design tool, design examples are introduced to testify the methodology. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Function-to-form mapping, Non-manifold polyhedral, Reconstitution.

1. Introduction

The design process is frequently thought of in terms of several sequential phases: conceptual design, preliminary or embodiment design (layout), and detailed design. The recent researches are extending from functional design [1] (Toimiyama, 1993) to requirement management [2] (McKay, 2001), the mapping of requirement-to-function [3] (Feng, 2000), and function-to-form [4] (Roy, 2001) etc. So far, computer-based design tools support design routine activities, such as documentation, storage, and translation of the design results.

The attention of contemporary design applications is on the later phases of the design process, while the early phases are still poorly automated and receive little information support [5] (Dietz, 1998). Just as [6] (Yazdani, 1994) and [7] (Váncza, 1999) have shown that, up till now no principal breakthrough has come about in applying computer-aided design tools in conceptual design phase. There are reasons to think about that the absence of a design theory, which would coherently explicate the whole design process in a scientific way, is the main predicament for the development of more sophisticated computer tools capable of assisting human designers in their non-routine activities [8] (Cavallucci, 2000).

There are many decomposition and reconstitution (D&R) models been proposed till now. Paul and Beiz, 1996 [9] firstly put forward the D&R model. Similar works was found in the Function- Behavior-State models [1] (Tomiyama, 1993). However there are several major differences between our approaches with that of Paul's etc. Our approach is a computational design management model, with automatic mapping schemes based on artificial intelligence; it is also an integrated methodology integrating conceptual and embodiment design.

2. Decomposition Product Model

A computational product model is decomposed, involving four major levels. Each level as associated with three domains, i.e. function domain, meta-object domain and form domain. The design domains associated with requirement, function and form etc. are decomposed into specific granularity and in different graph styles, like in a parallel net-work structure in function domain or a hybrid model in the form domain, and the decomposition method helps to set up domain specific knowledge base, mapping rules and criteria etc. The following sections give the details.

2.1. Function Domain

Function domain is classified into three groups, quantitative, abstract, and assembly. We believe that functional definition is correlated to the problem concerned; product-level, assembly-level or component-level functions are to be taken into considerations as in Fig. 1. As long as the assembly design is concerned, the assembly level function should be summarized, classified and analyzed.

Five levels of functional definitions are arranged in each parallel row, where the assembly-level function is summarized as a connection link model, each node of which is an information unit to express the prerequisite assembly requirement, like positioning, transmission, supporting and lubrication etc in a network structure. The information unit could be correlated with many alternatives of functional carriers, what-ever it might be, a conceptual face, functional feature, component or mechanism etc.

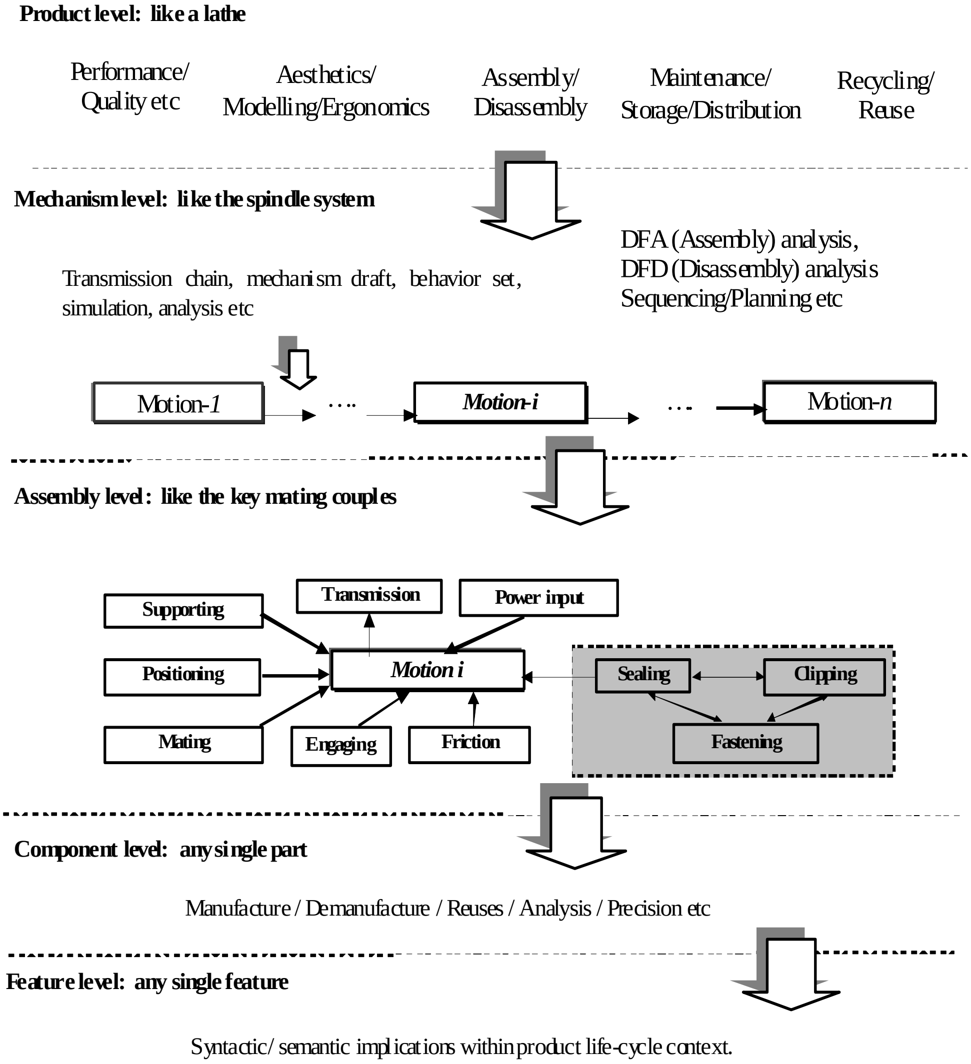


Fig. 1. Decomposition of the product model.

2.2. Meta-object Domain

One temporary design domain called meta-object domain is introduced in the ***F-F*** mapping model. The reason of the proposition of this domain is due to the gap between function and form. Actually, function and form belongs to different design domains, there are no natural connections between function connotations and geometric forms if no engineering implications are added to the physical forms.

In order to fill up the information gap, an interim domain, called the meta-object domain is introduced.

The concept of meta-object is similar to the object defined in the meta-modal proposed by [10] Yoshikawa (1994), which is usually a functional carrier, like an abstract feature corresponding to the kinematics requirement in F-F mapping. The meta-object could be a bridge of function and form domains.

2.3. Form Domain

In order to cope with information management of conceptual design, several conceptions are introduced, like conceptual face, abstract feature, conceptual solid, zero solid and conceptual part etc (in Fig. 2).

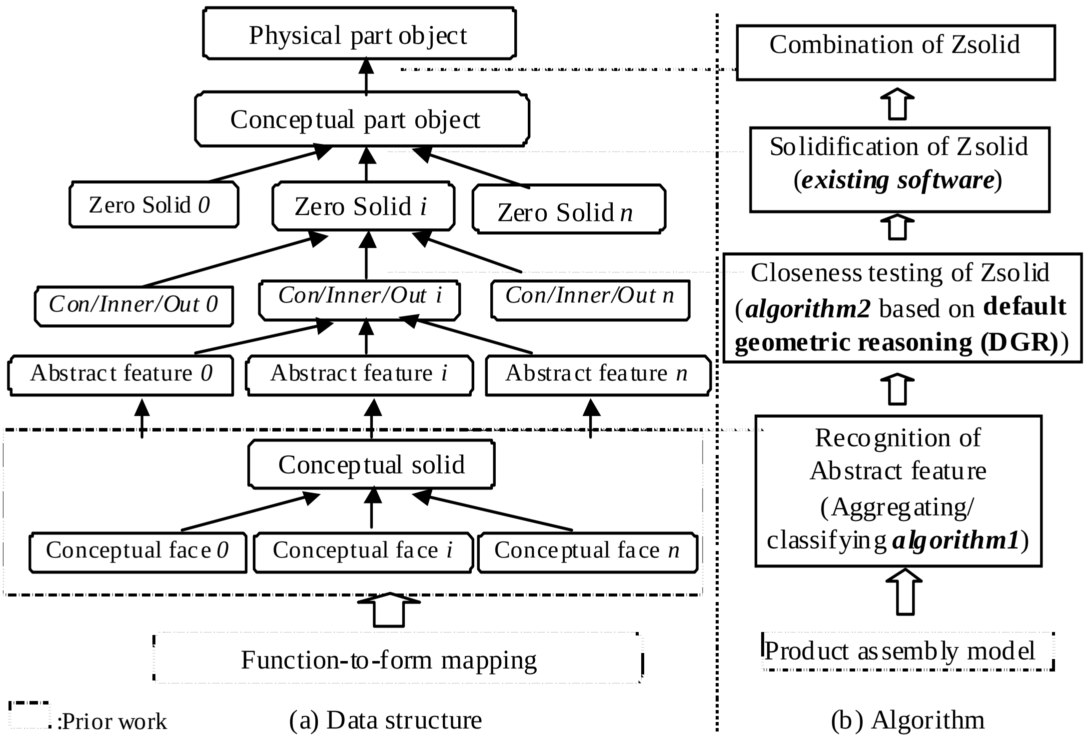


Fig. 2. Definition of the form domain.

Theoretically, everything could be decomposed infinitely; the same is true for the form domain, while the decomposed granularity is the key issue; it will correlate with such considerations as the levels of automation, reconstitution difficulties and potential of creativity.

Obviously, conceptual face and abstract feature are better options for high level support of creative design, however, the reconstitution algorithm is getting complicated, so some novel conceptions, mathematical fundamentals and algorithms are introduced with the help of traditional algorithms with manifold polyhedral.

Conceptual face: According to varied levels of development, conceptual face could be hierarchically defined in Table 1.

Table 1. Hierarchical definitions of conceptual face.

Conceptual face: *Fi.type*	**Concept** and **definition**
Directional face ***DC***	Described by outer vector (or axis of symmetry), a kind of limited face
Oriented face ***OC***	Described by outer vector (or axis of symmetry) and a positioning point, a kind of limited faces
Qualitative face ***QC***	Described by outer vector (or axis of symmetry), a positioning point and a functional area.
Boundary face BC	Has the geometric information of boundary, vertex, edge, loop etc
Closed face CC	Traditional displaying face

Conceptual Solid: A solid is composed of a group of conceptual faces Fi, i=1,2,..n, and/or position correlations, denoted by S.

Closeness of Conceptual Solid: For the Conceptual Solid S, if the stretched Fi, i=0,1,..n, under a certain positioning condition, could constitute a 3D closed space, then S is called closed. A closed solid is denoted by G, which is called a complete bond graph.

If sub-graph, Gi is the graph of related conceptual faces belonging to G, $\tilde{G}_i$ is a set of unrelated functional faces, then S (Gi, $\tilde{G}_i$). Symbol S: Fi.Type means, S is composed of the functional faces of the type Fi.Type, Fi.Type (DC, OC, QC, BC, CC) (from Table 1). For example, Gi:DC means the conceptual solid is composed of correlated DCs, while $\tilde{G}_{i:QC}$ means the solid being composed of un-related QCs.

Abstract Feature: A special region belonging to a component to express functional information and geometric implication. It behold either abstract information group composed of a number of faces and/or their relationships belonging to the domain of a conceptual solid, or accurate engineering semantics within design, manufacture, assembly and disassembly domains.

2.4. Proposition of Non-manifold Polyhedral

A graphic database, called Zero Solid (Zsolid) is firstly proposed based on the following considerations:

1) Zero height: The functional face is regarded as an object with zero height, and it could be variation in size and shape;

2) Information minimization: The initial information contained in a conceptual part is minimized based on the direct and simple mapping;

3) Information independence: There is no function overlapping between Zsolids, and no redundant information;

Zero Solid (Zsolid), means variation in solid size, boundary, shape and topological information within the conceptual solid domain.

Conceptual part is a kind of non-manifold polyhedron, which is a network data structure of zero solids, denoted by ZsolidNet.

3. Design Process Based on Non-manifold Polyhedral

The design process involves two procedures, i.e. functional design for 3D transmission chain, function-to-form mapping for non-manifold polyhedral and reconstitution of the non-manifold polyhedral to finish the design. Detailed approaches are introduced, where creativity are testified.

Functional design is to generate product concepts from two major approaches: 1. Novel design concepts could be generated based on kinematics reasoning from the initial design requirement. And the frequently used functional elements are categorized, classified, analyzed and arranged in the functional element library (FEL). The FEL is open to accept new elements and to be editable to fit for different applications. According to the kinematics requirement, elements in FEL could be automatically selected and connected (ASC) to form arbitrary mechanism layout. Novel design ideas are to be explored based on the existing mechanism rules, like the dual-vector algebra [11] (Moon, 2002). Symbolic and geometric fuzzy reasoning are employed to form the transmission chain, i.e., the computational mechanism topological model in a highly automatic way.

Case based reasoning (CBR) is an effective innovative approach to generate design ideas from previous design cases. Some successful design cases are stored in case libraries with appropriate indexing methodologies [12] (El-Mehalawi, 2003). Old design ideas could be retrieved and modified to fit for novel design requirements.

The interactive/intelligent mechanism generation based on CBR and ASC could be quick conceptualization tools, it served as the first-level creative support, which could be called the functional creative support.

The second-level creative support came from the function-to-form (F-F) mapping, which could be called structural creative support, embedded in the following two mapping steps.

The first step function-to-form mapping is called g1 mapping: g1 mapping is to find suitable features for the corresponding kinematics requirement. It is the mapping process from the assembly-level function (Fig. 3) to the functional features depending on the kinematics positioning requirement of the component. The results could be dynamically stored in a list format. There could exist many alternatives for a single component in one mechanism, so for a mechanism having a large number of components, the combinational alternatives could be intensively multiplied, and will have two results: the first one is the potential to explore in a larger solution space, which is crucial for creative support; the other is the problem to cause combinational explosion and perturbations, suitable knowledge base is therefore necessary to be introduced.

The second step function-to-form mapping is called g2 mapping: g2 mapping is the mapping process from functional features to product form. And the functional features are actually the meta-objects. It should be mapped to two corresponding elements belonging to parti and partj respectively in a mechanism. And the g2 mapping process depends on the content and nature of such a function link model, each node of which is a specific motion node indicating two correlated parts and the assembly properties, like positioning, transmission, supporting and lubrication etc in the network structure etc. And the mapping steps are correlated closely to the requirement, function and form domains.

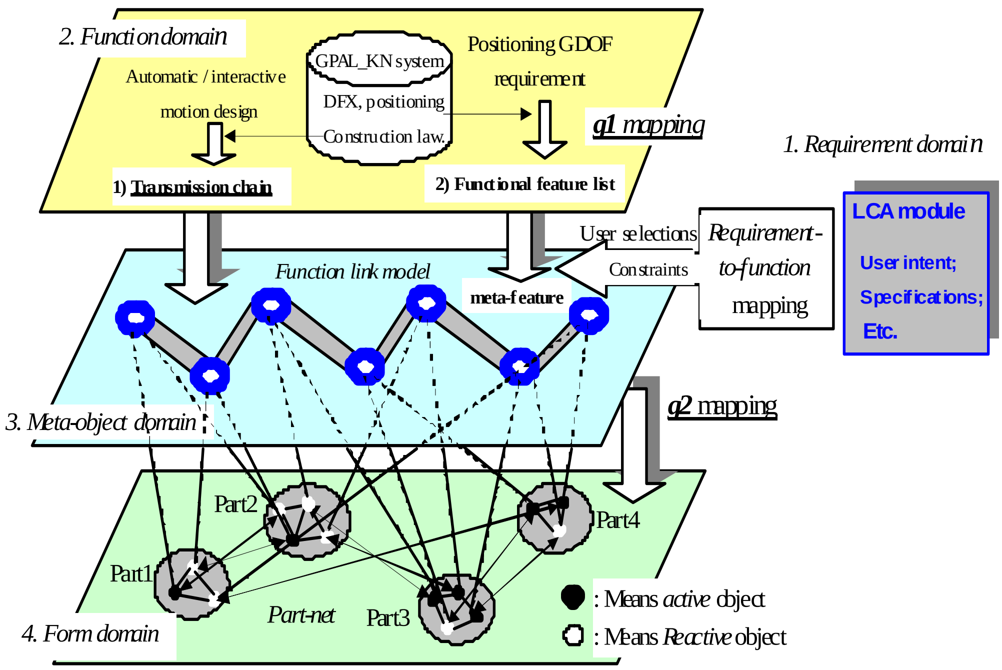


Fig. 3. Design process based on the D-M-R model.

4. Situated Reconstitution of Non-manifold Polyhedral

Situated reconstitution is a specific process of feature recognition, geometric reasoning and shape synthesis, which served as the process of shape creation based on the D-M-R (decomposition, Function-to-Form mapping and reconstitution) process. As the information obtained from the Function-to-Form mapping processes are usually provisional, inconsistent and the geometric information is always in a high degree of uncertainty, it is a specific class of Non-manifold geometry [13] (Crocker, 1991). The proposed synthesis scheme is described in the following situated reconstitution process (in Fig. 4).

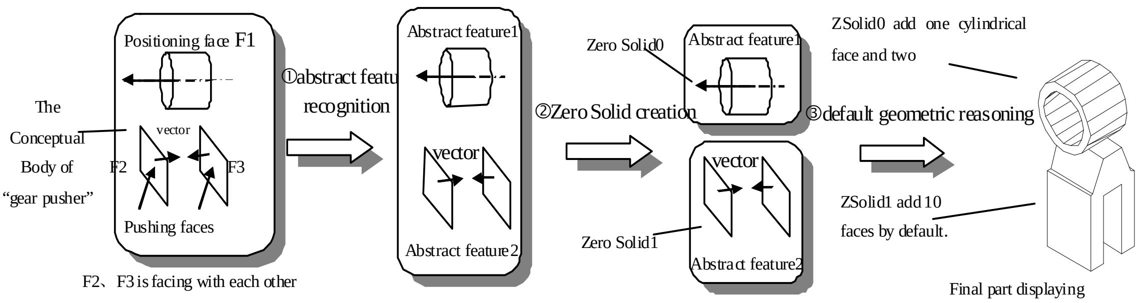


Fig. 4. Process of situated reconstitution.

Thus the proposed situated reconstitution process falls into three major steps, i.e. recognition of abstract feature; creation of zero solid; default geometric reasoning.

The proposed reconstitution algorithms evolving default logic is to cope with ambiguous information occurred in conceptual design stage, which is effective exploration tool for concept creation and visualization, called convergent exploitation. There is intrinsic relationship between decomposition and reconstitution in form domain, the relationship could be complicated when it is correlated with levels of automation, reconstitution difficulties and potential of creativity etc.

4.1. Proposal of Default Geometric Reasoning

Fig. 5(a) is the traditional definition of manifold polyhedral. The body is composed of limited faces;

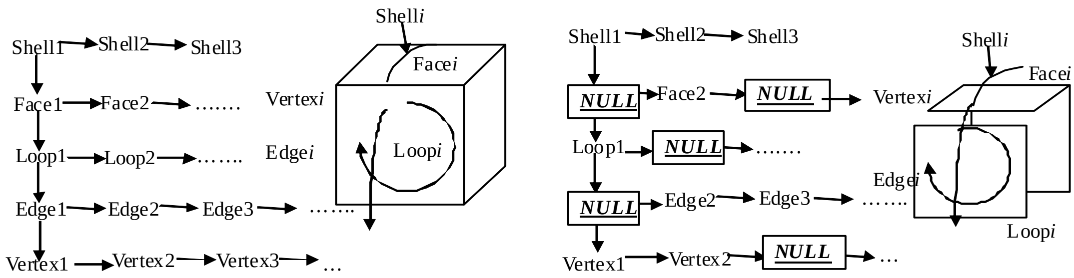
every edge of the manifold polyhedral has only two connected faces, where each face can be visited by traversing from one face to another. From a topological point of view, face is a closed loop, edge is shared with two faces, and there is no isolated vertex in the body. The information in a solid is unambiguous and consistent, and the number of vertex, edge and face meet the Decart-Euler equation:

$$V-E+F=2, \tag{1}$$

where V means number of vertexes; E means the number of edges; F means the number of faces. However, according to the previous conceptions of conceptual solid, there are many missing geometric elements in the "solid" topological structure, such as the missing elements (NULL) in the loop etc in Fig. 5(b). The object in Fig. 5(b) could be classified into a kind of non-manifold polyhedral. And in order to cope with this conceptual solid, default geometric reasoning is introduced.

Default Geometric Reasoning (DGR) means, when any of the geometric elements in a 'conceptual solid' such as: vertex, edge, face, loop etc are incomplete or indefinite; the missing geometric elements and the relationship among the geometric elements are to be reconstructed to form an unambiguous and informational consistent solid under default logic.

As functional features are stored in each part in a random order after the two step F-F mapping, and the information of size or shape etc is in a high degree of uncertainty. So methodology for ambiguous geometric information modeling, vague geometric information reasoning and algorithms on part reconstitution are put forward.



(a) manifold 3D object

(b) non-manifold 3D object

Fig. 5. Definition of manifold and non-manifold object.

Some definitions and proofs in the following sections could support the qualitative as well as quantitative analysis, and some key observations are high lightened.

4.2. Theoretical Foundations

A Path can be a line or a curve that is continuous. When the start and end of a path is connected, then it is called a Closed-Path.

Total turning, is defined as the angle the oriented tangent vector turns around as the path stepped along.

Theorem 1 (Closed-Path): A total turning is a topological invariant for a closed path. For any closed path the total turning is an integer multiple of 2 (Jablokow, 1996)[14].

Theorem 2 (Simple Closed-Path): The total turning for a simple (non- intersecting) closed path is 2 (clockwise or counter-clockwise).

From the above theories and definitions, the following theories are deduced.

Theorem 3 If conceptual solid CS can form one or more simple closed paths in every three orthogonal sections under a certain positioning condition, it is called closed.

Theorem 4 (Whitney-Graustein): Two co-planar closed paths can deform between each other, if and only if they have the same total turnings (Jablokow, 1996).

On the basis of Theorem 4, deduction1 can be got.

Deduction1: Adding or deleting features on a conceptual solid did not affect its closeness.

4.3. Algorithm s on Situated Reconstitution

Situated reconstitution is a process of adding supplementary faces and their topological relationships to a "conceptual solid" to form a geometrically and topologically consistent object.

In order to describe the intersecting tendency between faces, the concept of "aggregating and classifying coefficient" is put forward, denoted by coe(fi,fj), the definition is as the follows.

$$coe(f_i, f_j) = \frac{\overline{\theta}(\vec{v}_{f_i}, \vec{v}_{f_j})}{90°}, \quad where\,\theta = \begin{cases} \theta, & 0° \le \theta \le 90° \\ 180-\theta, & 90° < \theta \le 180° \end{cases}, \tag{2}$$

$coe(f_i,f_j)$ means the extending of intersecting correlation between f_i, and f_j, it is easier to get that: $0 \le coe(f_i, f_j) \le 1$.

The closer coe(fi,fj) is to 1, the stronger the intersecting tendency of these two faces has. Prior to the "aggregating" process, grouping is necessary, where the correlated faces are identified and put into corresponding sets. It is obvious that when coe(fi,fj) is near to 1, f1, f2 are more likely to intersect. Considering the geometric complexity, the coefficient is assigned a parametric range, say 0.8~1. Here is the definition of correlated faces.

Correlated face, if f_i meets the following requirements:

i) f_i, f_j have the correlation of concave/convex, and coe(fi,fj)>0.8;

ii) fi, fj are correlated by other face. "Correlated by other face" means: if there exists a face fm, that fi,fm meet the criteria i), in the mean while fm,fj also meet the criteria i).

Then fi is called the correlated face of fj, denoted by fi.RelatedFace,

The algorithm based on default geometric reasoning is called DGR algorithm, which is consisted of two steps:

Step 1: Classifying the existing functional faces according to the out normal:

Given that the normal vector V11 on F1 and V12 on F2 could form a primary normal plane called main_plane1, and there are two other orthogonal normal planes, i.e. main_plane2 being formed by V21, V22; and main_plane3 being formed by V31, V32 .

Then decide which of the main plane, i.e. main_planei, i=1,2,3, the normal V3 of a third plane F3 belongs to. The scheme is to calculate their mixed product of Vi,Vj,V3, the smallest abstract value is the main plane, to which V3 belongs.

1. Rationalize Vi,Vj,V3, and calculate their mixed product, Vi (V11 V21 V31) ,Vi (V12 V22 V32)

$M_{(Vi,Vj,V3)}$ = mixed-product(V_i,V_j,V_3)

$$=(V_i \times V_j) \times V_3 = \begin{vmatrix} v_{i,x} & v_{i,y} & v_{i,z} \\ v_{j,x} & v_{j,y} & v_{j,z} \\ v_{3,x} & v_{3,y} & v_{3,z} \end{vmatrix}$$

2. Then the main_plane with the smallest value of M(V_i,V_j,V_3) is what V_3 belongs to (Fig. 6).

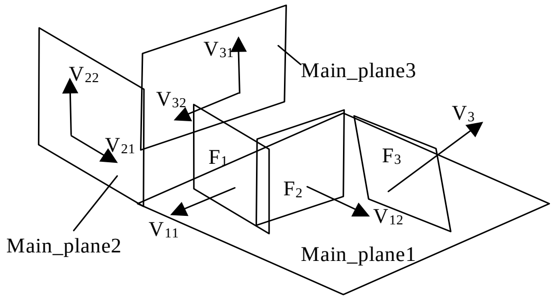


Fig. 6. Main vector discrimination.

Step 2: Closeness testing and supplementary faces addition:

As shown in Fig. 7, three orthogonal planes are used to "cut" the Conceptual Solid (CS for short), and get at least three directional 2D section profiles (Fig. 7(b));

Decide which one of the 2D section profiles is not a simple path;

If it is not a simple path, then add one complementary face at the middle position of the two faces, then go back to the top of step 2;

If it is a simple path and, if all the planes in the CS have been searched, then stop.

- If not, go back to step1.

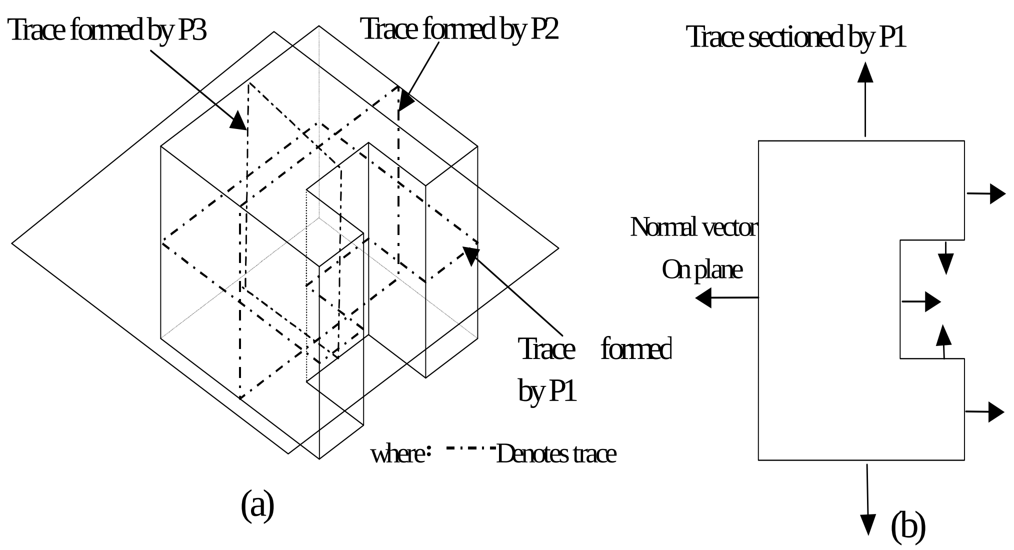


Fig. 7. Closeness of conceptual solid.

In order to make the problem simple, three orthogonal planes are introduced to cut the CS, and three 2D section profiles are obtained, then DGR algorithm is utilized to cope with it. For example, three orthogonal planes P1, P2, P3 are used to cut the CS; three 2D section profiles could be obtained. Taking any one of the profiles as example, calculate the total turning of the 2D profile, the scheme is like this:

$\theta_{total} = \sum_{i,j=1,i\neq j}^{n} \theta(v_i, v_j),$ where $\theta(v_i, v_j)$ is the inclination of the vector Vi and Vj.

According to **theorem 2** and **theorem 3**, any total turning of the 2D simple path should be: $\vartheta_{tatol} = \pm 2\pi$,

If it is not the case, a supplementary face should be added between them. For instance, if the inclination (denoted by seta) of two normal vectors:

seta (*face1.normal*, *face2.normal*) = 180. Then a supplementary *face3* should be added between *face1* and *face2*.

Let, ***seta*** (*face1. normal*, *face3.normal*) = ***seta*** (*face3.normal*, *face2.normal*)

= ***seta*** (*face1.normal,face2.normal*)/2.

The above algorithm for total turning calculation is implemented to test the closeness of the conceptual solid, and provide the deterministic evidence for adding supplementary faces to construct a consistent object.

5. Design Examples

A mechanical hand design example is to illustrate the design process and the creativity that involved. There are four major steps: 1) design requirement specification; 2) functional design; 3) F-F mapping for the embodiment design; 4) form reconstitution.

Step 1: Design requirement specification.

The main functional requirement of the mechanical hand is to clamp the object, and to move the object from position1 to position2, like in Fig. 8.

There are four movement requirements involved in the mechanism design (Table 2).

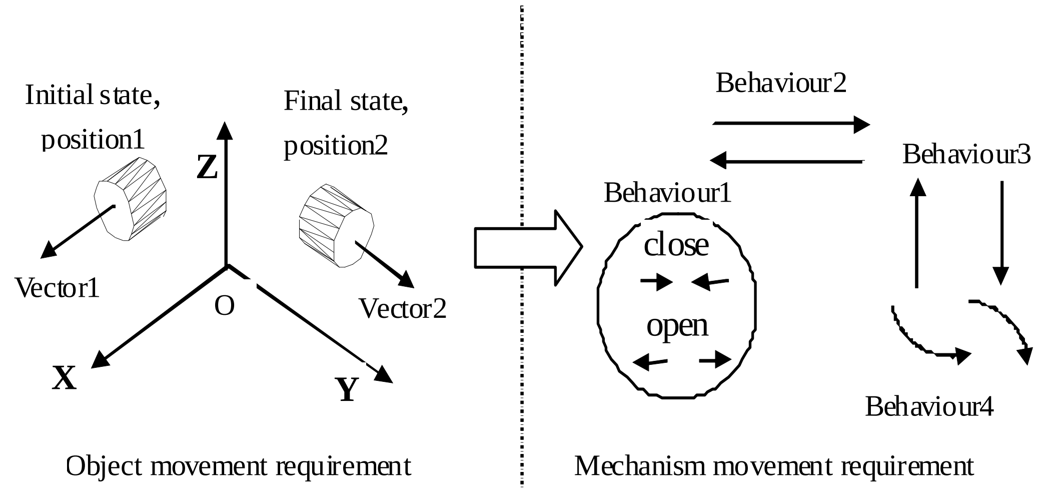


Fig. 8. Design requirement of the mechanism.

Table 2. Behavior set of the mechanical system.

Behaviour 1	**Behaviour 2**	**Behaviou r3**	**Behaviour 4**
Open and close movement to clamp the object	Left and right translation to move the object	Up and down movement to pick up or leave the object	clockwise counter-clockwise to change position of the object

Step 2: Functional design and creation of the computational transmission chain

The functional design falls into three steps:

1) the 3D orientation plane are firstly set up, three WCS (world coordinate system) work planes are created by default;

2) the functional elements library (FEL) is in a flexible position, and the functional elements could be automatically selected from the FEL dialog table and dropped on the 3D work planes, in the mean while the 3D navigation tools are implemented to facilitate the dropping procedure, the 3D physical connection model in the design area are established;

3) physical and symbolic reasoning is lastly utilized to deduce the computational 3D transmission chain of the mechanism, which is displayed in the left side of the system

Step 3: ***F-F*** mapping, The F-F mapping process falls into the following procedures.

From the transmission chain, each node has a GDOF requirement and two correlated parts. The g1 mapping process could be automatically carried out to generate a list of candidate functional carriers from the GDOF positioning requirement. And a specific knowledge base help to find the best candidate feature by different evaluation data and the user intent.

The g1 mapping could also be done interactively via an interface for the user to select appropriate features with the reference to different evaluation values from the specific knowledge base;

And the functional feature is displayed in the joint position in the physical connected model, and the orientation and the main vectors of the feature could be modified interactively;

The g2 mapping process (in Fig. 9) is to map the meta-object to two corresponding components at the joint position from the computational transmission chain; there are two steps, a) decide the concave/convex properties of the feature; b) add the feature to the feature list of each part.

After the two step mapping process, the ambiguous geometric information have been created in each part. So vague geometric information reasoning and part reconstitution based on default geometric reasoning could be utilized to generate the conceptual form quickly in a high automatic way, and displayed in Fig. 10.

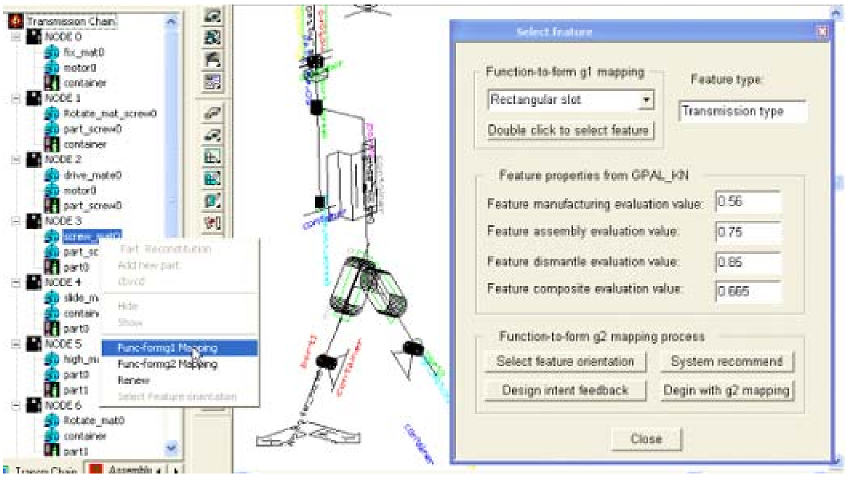


Fig. 9. F-F mapping in an automatic and interactive manner.

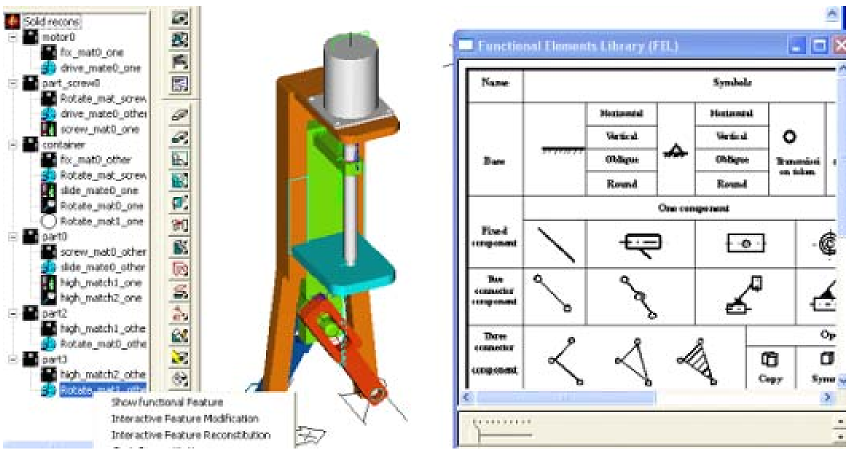


Fig. 10. The clip part of the mechanical hand.

6. Conclusions

From the perceive law, during the conceptual design stage, design engineers may pay attention only on several key faces. And a part usually has several key faces to express the brief functional information. F-F mapping algorithms on the evolution of a product are proposed during conceptual design stages, definitions on abstract feature, conceptual solid, zero solid and non-manifold polyhedron etc are studied to cope with the information generated from the conceptual design stages. Situated reconstitution is proposed in this paper. Several packages of design tools have been developed to testify the effectiveness of situated reconstitution practices.

Our research proved that modeling of physical processes in computational design technologies could facilitate the creative thought processes. And the key to managing physical processes like the software-based potential is to manage innovation processes effectively, and to develop and manage the software more effectively.

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