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The Aerosol Mass Measurement Based on the Two-parameter Mathematical Model in Optical Sensor

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Abstract: On the basis of the analysis of random pulse signal, the mathematical model of the two-parameter random signal is established. Based on which, it is found that the random signal has a non-integer dimension fractal characteristics, and the fractal dimension is the ratio of the count distribution and the number of discrete degree $\sigma_{\ln}$. Besides, the mass distribution of the particle group is also a typical random distribution, and satisfies the lognormal distribution. Thereby, the functional relationship between the average quality and average voltage of the voltage is given, and a theoretical basis of the quality measurement by the light scattering counting particle is established. Finally, the physical meaning of the fractal dimension is proposed. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Two-parameter, Mass subset, Lognormal distribution, Fractal dimension.

1. Introduction

Aerosol mass concentration measurement methods include [1-4]: membrane weighing, β ray absorption, piezoelectric crystal, light scattering, the blackness of law and so on. Among these, light scattering method with the advantages of high accuracy, speed, and can be used for online measurement has been widely used. In fact, the light scattering method is divided into particle group method [5] and single-particle counting method. At present, many domestic and foreign companies produced the particle counter based on the single particle counting method. However, these instruments are merely used to purify the indoor particle detection, but not for the measurement of aerosol mass concentration.

A new calculation method for measuring the aerosol mass by optical scattering counting is reported in the literature [5, 6], based on which the average single particle quality $\overline{m}_i$ can be obtained. For aerosol, the experiments show that it is in good agreement with the measurement results by the standard instrument with the concentration is 5 mg/m³. However, the theoretical basis of this calculation method should be given from the perspective of the physical model and the theory of measurement.

In this paper, on the basis of the analysis on random pulse signal, the two-parameter mathematical model is established. Based on this model, the experimental results show that the counting distribution of the randomness pulse signal amplitude and width of the subset match well with the logarithmic normal distribution with a natural number as the independent variable with high accuracy. Besides, the relation between the two characteristic parameters of the random pulse signal is the nonlinear transformation, which means the two parameters statistical distribution owns the non-integer dimension fractal characteristics. The mass distribution of the particle group is also the typically random distribution, satisfies the lognormal distribution. Thereby, the relationship between the average quality and average voltage is given, and a theoretical basis of the quality measurement of the light scattering counting particle is established, as well as the physical meaning of the fractal dimension is proposed.

2. The Two-parameter Mathematical Model and Basic Characteristics of Random Pulse Signal

The particle counting photoelectric sensor's output signal corresponds to a time sequence $\{u_i\}$, the maximum and minimum values of them are limited, and they are recorded as u_{Max} and $u_{\min}$ respectively. Obviously, we should have $u_{Max} \geq u_i \geq u_{\min}$. Supposing that the total number of elements of $\{u_i\}$ is N, the timing difference of the adjacent elements is $\Delta\tau_i$, and the corresponding amplitude difference of them is $\Delta\phi_i$. As a result, the corresponding duration time of $\{u_i\}$ is $t = \sum_{i=1}^{N} \Delta\tau_i$. For the amplitude u_i which is experimentally measured, it contains not only the stable part of the random measurement process u_c, but also contains the description of the stochastic fluctuation part Δu_i. Hereby, the experimental results should be expressed as $u_i = u_c + \Delta u_i$. Taking δu as the minimal value of amplitude, the measured value should be the integer multiple $(l_i, l_i \geq 1)$ of the measurement accuracy. So, the removing the stable part of $\{u_i\}$, it can be mapped to the mathematics model $\{l_i\}$:

$$\frac{u_i - u_c + \delta u}{\delta u} \equiv l_i \tag{1}$$

and the total gears $L \equiv \frac{u_{Max} - u_{\min}}{\delta u} + 1$, while $l_{\min} \equiv 1$.

Supposing that $\Delta\phi_i \to 0$ and $\Delta\tau_i \to 0$, the model of signal tends to the continuous format like u(t). At this time, $\{u(t)\}$ describes a continuous curve in $l \sim i$ flat. The two-parameter model of function $l(i)$ can be expressed as:

$$\lim_{\substack{\delta u, \delta\tau \to 0 \\ N \to \infty}} \frac{u_i - u_{min}}{\delta u} + 1 \equiv l(i) \quad i \in [1, \infty) \tag{2}$$

According to Eq. (1), supposing N_{l_i} represents the total number of the signals which meet $u_i = l_i \cdot \delta u + (u_{\min} - \delta u)$. Hence, the basic probability distribution function of the random noise signal set is $\tilde{p}(l_i) \equiv \frac{N_{l_i}}{N}$. With the increase of the total sample number, the distributing form of the function $\tilde{p}(l_i)$ tends to stable. And the same time, in a limited observation range $[u_{\min}, u_{Max}]$, when the total number of binning $\lim_{\substack{\delta u, \delta\tau \to 0 \\ N \to \infty}} L \to \infty$, the histogram distribution $\tilde{p}(l_i)$ corresponding to the continuous probability density function form $p(l_i)$.

3. The Characteristic Parameters' Statistical Distribution of the Random Signal Generated By the Optical Sensor

The laser airborne particle counting system, whose sampling rate is 28.3 L/min and is shown in Fig. 1, is applied as the experimental measuring equipment. In this equipment, the output signals of optical sensor (photomultiplier tube used in experiment) are digitized by the high-speed data acquisition card PCI-9812. And then, they are inputted to the computer to implement the multi-channel counting statistics.

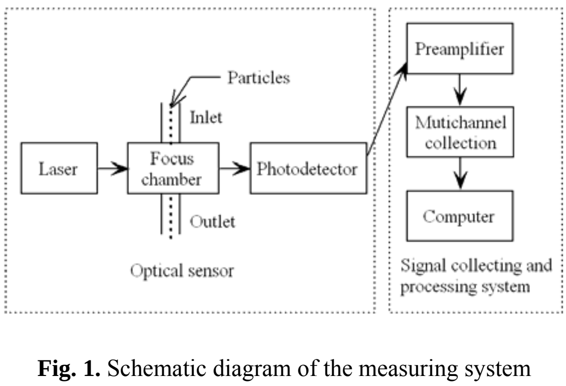


Fig. 1. Schematic diagram of the measuring system of optical particle counter.

The main technical parameters are as follow: the maximum signal sampling rate f_e is 20 MHz (corresponding the time-precision $\Delta\tau$ is 0.05 μs), the average width of pulse signal $\overline{\tau}$ is 5 μs ; the range of voltage is 5 V, the division precision ΔV of 2048 counting channels is $5V / 2048 \sim 2.44\ mV$. By using this device, taking the serial width scope $\tau_k \sim \tau_k + \Delta\tau$, the measured counting distribution $p_{\tau_k}(m)$ of random pulse signal amplitude can be shown as the discrete points in Fig. 2.

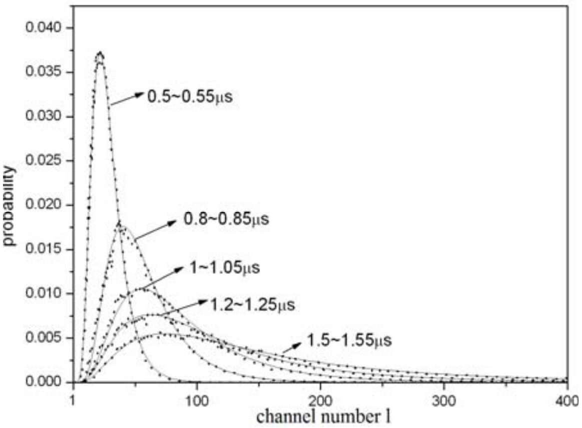


Fig.2. Scattering signal amplitude distributions under different signal width.

As shown, the function $p_{\tau_k}(m)$ has center asymmetry obviously. Besides, the nonlinearly similar characteristic between the different function sequences is quite well. Considering the signal amplitude distribution orderliness generated by standard particle group [7] and the statistical distribution law of particle size obtained by high-precision measurements [8], we choose a statistical function with the form of the log-normal distribution to fit the measured data, which the fitting results can be seen as the sequence of solid lines in Fig.2. The equation corresponding to the solid line in Fig. 2 satisfies:

$$p_{\tau_k}(m) \equiv \frac{1}{\Omega\sigma_{\ln} m} e^{-\frac{(\ln m - \mu_{\ln})^2}{2\sigma_{\ln}^2}}, \tag{3}$$

where $m = \frac{(V - V_{0k})}{\Delta V}$, $\mu_{\ln} = \overline{\ln m}$, $\sigma_{\ln} = \sqrt{\overline{\left(\ln m - \overline{\ln m}\right)^2}}$, Ω is normalization coefficient. Formally, this equation corresponds to the lognormal distribution function $p(l)$, but the independent variable is the natural number and normalized coefficient is not $\sqrt{2\pi}$.

The large experimental results show that the pulse signal group's counting distribution of the relative amplitude, width and different signal subset's corresponding characteristic parameter can stably obey the lognormal distribution with natural number as the independent variable [9], which indicates that the counting distribution of pulse signal group and different signal subset's relative characteristic parameter with the statistical self-similarity. At the same time, the calculated result indicates that the corresponding parameter in the definition of a subset of the pulse within the nonlinear transformation relations, when using amplitude u and width τ to statistically describes random pulse signal. In other words, the random signal's different parameter statistical distributions owns the non-integer fractal dimension characteristic, and the fractal dimension corresponds to the logarithmic dispersion ratio $\sigma_{\ln}$ of the feature parameter's counting distribution [9].

4. Aerosol Mass Subset

Mass is one of the basic characteristics for suspended particulates. Any particle group N can be classified according to the size of the particle's mass, based on which the particles can be divided into "mass subset" $N(m)$ with different levels. The particles generated by random process with the same quality but the geometry can vary widely, so, there is no convergence concept of shape generally [10-11]. Aerosol in air is typically a random group, and the aerosol' irregular geometry is the main random feature of mass subset. And, the mass distribution of the particles is the random distribution which is marked by the quality parameter.

Thanks to the morphology of the suspended particles is irregular, it is difficult to measure its degree of geometric lines. In the field of particle measurement, the line degree of particle is commonly described by "equivalent particle size". In this study, the maximum diameter D of the particle projection is chosen as the geometric lines. According to Non-Euclidean geometry, the volume V of particle is measured by D^{α} which is the higher dimensional scale of *D*. For regular geometry, the line dimension $\alpha = 3$, and for irregular geometry, $3 > \alpha > 2$. That is to say, line dimension is non-integer number under normal circumstances. For the same quality subset, sorting the particle size by the equivalent diameter, based on which it can be divided into L_D intervals, and each interval has N_i particles with the corresponding equivalent diameter $\overline{D_i}$.

5. The Mass Measurement of the Aerosol Based on the Two-parameter Mathematical Model

Due to the irregular morphology of particles is completely random for the quality subset, the equivalent particle size distribution is also random. So, the morphology meets the lognormal distribution [9] with a large number of particles in the statistical distribution. Besides, the experimental results also show that the particle group's signal characteristic parameter counting distribution is also meets the lognormal distribution. Therefore, the relation between the signal amplitude u and the equivalent particle size can be described as [9]:

$$\overline{D}=\left(k'u\right)^{\beta}, \tag{4}$$

where $\beta=\sigma_{\ln D}/\sigma_{\ln u}$ [9].

As a result of the particle volume $u \propto D^{\alpha}$, we can obtain the relation between the particle mass m and signal amplitude u, as:

$$m=\rho V=k\overline{D}^{\alpha}=k\left(k'u\right)^{\beta\alpha}=Ku^{\alpha'}, \tag{5}$$

where α' is the fractal fractal dimension which corresponds to the ratio between the equivalent particle counting distribution and the logarithmic dispersion $\sigma_{\ln}$ of amplitude counting distribution.

Supposing that a particle subset can be divided into L_M intervals, and it has N_i particles in each interval. Hereby, the relation between the amplitude u and the equivalent mass can be expressed as:

$$\overline{m}=\left(k''u\right)^{\beta'}, \tag{6}$$

where β' is the fractal dimension which is the ratio between the equivalent mass counting distribution and the logarithmic dispersion $\sigma_{\ln}$ of amplitude counting distribution.

Since the above distributions are all the counting distributions of characteristic parameters, Eq. (6) can be feasible to the particle quality measurement based on counting method.

6. Experimental Verification

6.1. The Experimental Setup

Fig. 3 shows the experimental flow chart. The sample stored in a container flowed into a mixed container, a test container in turn, and at last it was detected by an aerosol monitor (TSI, SIDEPAK AM510) and an OPC. The SIDEPAK AM510 was used as a norm-referenced instrument, and it has been calibrated by the gravimetric method. This instrument is a light scattering laser photometer that measures the light scattered at 901 and measures concentrations ranging from 0.001 to 20 mg/m^3. It indicates mass concentration CTSI and stores the data in real time. The OPC displays the pulse height distribution of the sample, which is collected by a multichannel pulse height analyzer (ADlink Tech Inc., PCI9812). The analyzer has 2048 count channels, and the lower limit of particle size of the OPC is 0.3 mm.

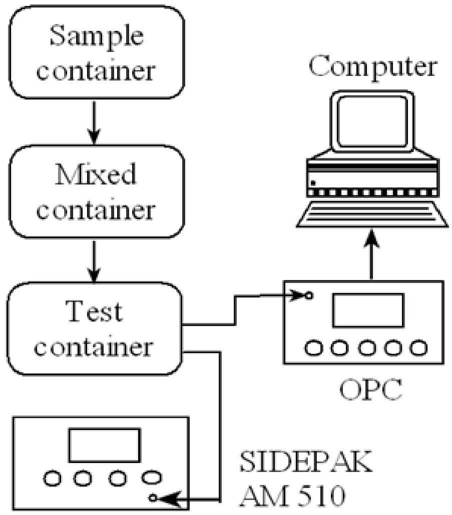


Fig. 3. Experiment setup for measurement of the signal height distribution of aerosols and the aerosol mass concentration.

Before experimental measurements, the flow rates of the SIDEPAK AM510 and the OPC were both adjusted to about 1.5 l/min, their zero responses were checked in filtered air, and their sampling cycles were selected to 1 min. At last, the two instruments were applied to detect a sample automatically and continuously. In addition, the electrical stability of the OPC was also checked. When filtered air was detected by the OPC, an oscillograph was simultaneously utilized to record the root mean square (RMS) amplitudes of background noises of the OPC. The test was performed every 10 h and run four times. The time variations of RMS amplitudes were plotted in Fig. 4, and it was found that RMS amplitudes are basically consistent.

6.2. The Experimental Results

In [12], we have experimentally verified the mass calculating equation as:

$$M = k_A \sum_{j=1}^{q} n\left[j(u)\right] u_j^{\alpha} = \sum_{j=1}^{q} n\left[j(u)\right] \overline{m_j}\,, \quad (7)$$

where u represents the voltage channel's relative threshold voltage, $n\left[j(u)\right]$ is the number of signal pulse in the j^{th} voltage channel, $\overline{m_j} = k_A u_j^{\alpha}$ denotes the average quality of the j^{th} voltage channel.

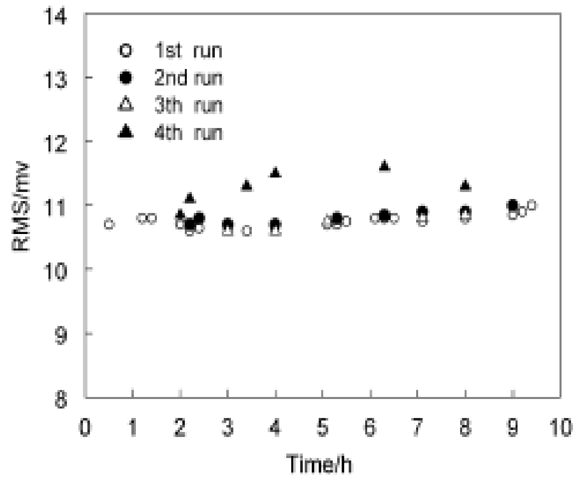
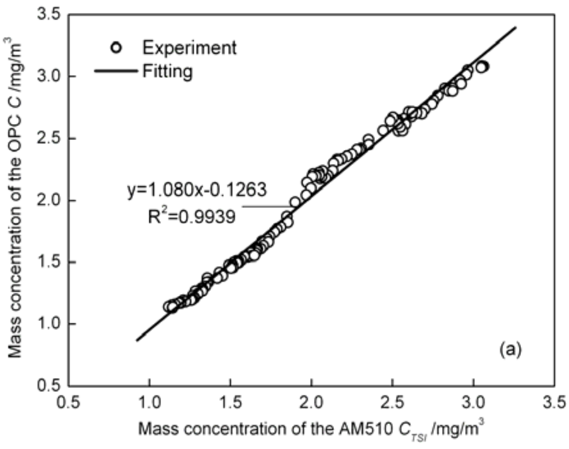


Fig. 4. Time-stability of the optical particle counter.

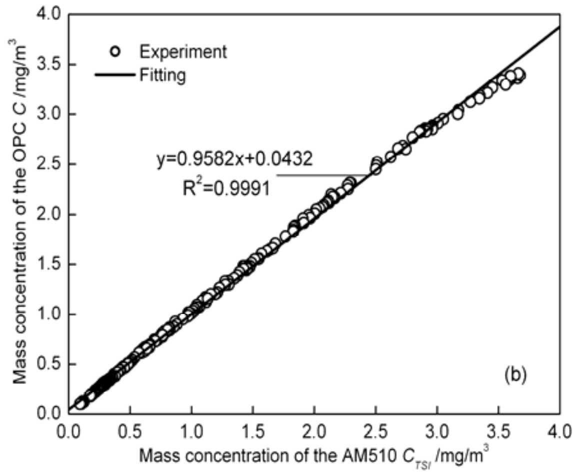
Substituting α=0.435 and $k_A = 3.014\times10^{-5}\,mg/m^3$ [12] into Eq. (7), the mass M can be calculated. And then, M is compared with the standard reference value M_{TSI} which is obtained by U.S. TSI intelligence dust explosion-proof instrument SIDEPAK AM510. The relation between the measurement values of OPC and SIDEPAK is shown in Fig. 5. The results manifest that the measured soot particle quality value M is consistent with the standard reference quality M_{TSI}, which reflects the application value of the concept of the fractal dimension and the calibration of α.

7. Conclusions

On the basis of the analysis of random pulse signal, the mathematical model of the two-parameter random signal is established. Based on which, it is found that the random signal has a non-integer dimension fractal characteristics, and the fractal dimension is the ratio of the count distribution and the number of discrete degree $\sigma_{\ln}$. Besides, the mass distribution of the particle group is also a typical random distribution, and satisfies the lognormal distribution. Thereby, the functional relationship between the average quality and average voltage of the voltage is given, and a theoretical basis of the mass measurement by the light scattering counting particle is established. Finally, the physical meaning of the fractal dimension is proposed.



(a)



(b)

Fig. 5. The relation between the measurement values of OPC and SIDEPAK AM510 (a) smoke (b) soot.

Acknowledgements

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