Sensors & Transducers, Vol. 172, Issue 6, June 2014, pp. 301-307

Sensors & Transducers

The Application of Information Entropy Theory in Project Evaluation Based on Multiple Attribute Decision Making Context

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Received: 8 May 2014 /Accepted: 9 June 2014 /Published: 30 June 2014

Abstract: Taking the new product line project of company X for example, the approach based on information entropy theory is utilized to test the effectiveness, robustness and applicability of the method. Based on the computation results, the evaluation scores of alternative seven plans is 0.7468, 0.5444, 0.6266, 0.8059, 0.6274, 0.5546 and 0.5983. The optimal choice for mangers to choose is plan 4. The results indicate that all works can be done by simple computation, the effectiveness is proved; and the order based on the comprehensive scores is in line with the actual situations, which proves the robustness of the approach significantly; and then combining the two aspects above, the applicability of the applied approach is demonstrated significantly. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Entropy weighted approach, Project evaluation, Multiple attribute, Decision making.

1. Introduction

With the development of information era, more and more information is surged into management process of modern enterprises. The relevant information can be collected and captured with low costs and little time. However, all of the changes are not able to make the management and decision-making of modern managers be easier. Massive information makes the competition among companies be fiercer, and which also makes it more difficult for managers in modern enterprises to make decision [1]. There are two main issues to be hotly discussed in the areas of multiple attribute decision making (MADM), and one aspect is the classification of the information that is utilized to make decision and another aspect is how to determine the weight of the attributes the decision-making problem involved [2].

On the one hand, the classification of the information could be expressed as follows: quantitative and qualitative information [3], the exact and non-exact information [4], the positive and negative information [5] and so on. The effective methods to treat different types of information are the key point to improve the accuracy and robustness of decision-making approaches [6]. Given quantitative and qualitative information (namely mixed information), and the first task for managers is to change the qualitative information into quantitative information. The second stage is to choose suitable information conversion methods [7], for instance, the normalization is one of the most common used methods [8]. Moreover, the information the managers using to make decision is often mixed with exact and non-exact information. Moreover, the non-exact information is general called fuzzy, grey information and so on [9, 10]. For fuzzy or grey information, there are different types of methods to pre-treat the original information for managers to do further decision making. On the problem of multiple attribute decision making, the information is always negative or positive to the objectives of companies' managers [11]. The usual practice is to employ certain methods to change the negative influence factors into positive factors to the objectives of decision making and the effective methods such as minimum and maximum operators are able to be chosen.

The second main aspect issue is to determine the weight of each attribute in the multiple attribute decision-making problems [12]. The usual type of methods is to determine the weight in terms of the information preference [2], the experience of the managers and relevant experts and so on [13]. These methods make the work to collect relevant information more simple, but they also make the decision-making results with lower accuracy [14]. Another orientation on attribute weight determination is to compute the weights according to statistical characteristics of the given information [15], which is able to improve the accuracy and robustness of decision-making results comparatively to the usual methods [16].

To solve the two main problems mentioned above, the approach based on information entropy theory is introduced and a computation case of new product line project of company X is taken for example to test its effectiveness, robustness and applicability. The second section is to introduce the theoretical framework of information entropy theory and review the prior relevant literatures. In the third section, the relevant definitions about minimum and maximum standardization operator and entropy weight are introduced. In addition to the stages of entropy weighted approach, the computation case is also described in the section. The fourth section is about the study of computation case and the discussion of comprehensive evaluation results. Finally, the conclusions about the application of information entropy theory in evaluation are summarized and the suggestions about future research are also proposed out.

2. Theoretical Framework and Literature Review

2.1. Theoretical Framework

The concept of entropy firstly comes from physics science and then it is introduced to the wide applications of system theory, information theory and control theory. It is abstracted as the possibility of certain information occurrence. The concept of information entropy is firstly proposed out by Shannon and Weaver (1949), he utilized the concept to solve the uncertainty problem within the processes of information transmission and he thought that more information meant that the decrement of uncertainty and information entropy could measure this type of uncertainty [17]. In the study of Shannon and Weaver, they linked the information entropy with probability to measure the volume of information. The information entropy could reflect the transmission information when the event with specific probability occurred. Therefore, if the probability is higher, the information volume will be less, and vice versa [17]. Meanwhile, the concept of information entropy also reflects the order level of the specific system. In general, if the value of information entropy is lower, the specific system is in better order; otherwise if the system is more disordered, the corresponding value of information entropy is higher [18].

2.2. Literature Review

The theory of entropy and Fisher's information measures are both utilized to measure the efficiency of information transmission, but information entropy is of more advantage in making decision [19, 20]. With its particular advantage, information entropy can improve previous theoretical research in information measurement [19]. And Hall, Inoue et al. (2007) showed that the entropy of the limiting distribution of the GMM estimator could be written in terms of these long run canonical correlations [21]. Based on these theoretical promotion works, information entropy theory is developed and introduced in many application areas. To estimate the market share in each market segment, Saleh (2007) tried to formulate a generic framework based on the information entropy concept that utilizes such an attraction model to estimate competitors' sales in each market segment [22]. This work could provide helpful suggestions for the managers to make product production and market sales. Consideration the characteristics of mentioned mixed information, Wang and Cui (2007) investigated a hybrid multiple attribute decision making problem with precision number, interval number and fuzzy number and they finally proved the scientific nature[15]. To extent the application of information entropy, new non-probabilistic entropy of a vague set was proposed by means of the intersection and union of the membership degree and non-membership degree of the vague set [23]. Zhang and Jiang employed two numeric examples to illustrate the applications of vague cross-entropy to pattern recognition and medical diagnosis [23]. Jiang, Sui et al. (2010) proposed a novel definition of information entropy-based outliers in rough sets and they also introduced an algorithm to find such outliers, moreover they demonstrated the effectiveness of information entropy-based method for outlier detection on two publicly available data sets [24]. To improve the accuracy of information entropy-based approach, more researches are conducted along with combined models including other methods. Based on the research by Wang, Yang et al. (2010), they employed the selfish gene theory as well as mutual information and entropy based cluster model to optimize the probability distribution of the virtual population and according to their computation and simulation, they proved that the proposed methods performed better [25]. In addition, as an effective method, information entropy-based approach is also applied in the areas of rough sets. Jiang, Sui et al. (2010) proposed a novel definition of outliers-IE (information entropy)-based outliers in rough sets and the effectiveness of information entropy-based method for outlier detection was also demonstrated on two publicly available data sets [24].

In sum, information entropy-based approach has great advantages in solving problems with characteristics of mixed information and uncertain attribute weight from original information. Especially, it is also introduced to help to make decision in few areas. But its application in project evaluation and then making decision is serious insufficient. Therefore, the sample of new product line project in company X is taken to test the effectiveness, robustness and applicability of information entropy approach. Meanwhile, the minimum and maximum standardization operators are also utilized to solve the negative and positive information-mixed problem.

3. Methodology and Data

3.1. Relevant Definitions

During the comprehensive evaluation processes of project decision making, there are always two types of information. One type of information is that some indicators are negative to the evaluation objectives and another type of information is that some indicators are positive to the evaluation objectives.

Therefore, some measures are necessary to be carried out to overtake these types of problems. Additionally, taking the standardization requirements of pre-evaluation information into consideration, minimum and maximum standardization operators would be utilized to regulate the negative and positive aspect information respectively, whose definitions could be given as follows:

Definition 1: Suppose a_{ij} is the value of indicator i of plan j and $\min(a_{ij})$ is the minimum value in vector a_i. Therefore, the minimum standardization operator would be given as follows:

$$MinS = \min_i(a_{ij}) / a_{ij} , \quad (1)$$

where $i \in M$ and M is the number of evaluation influence indicators and $j \in N$, and N is the number of pre-existing plans.

Definition 2: Let a_{ij} and $\max(a_{ij})$ to be the maximum value in vector a_i. Hence, the maximum standardization operator would be defined as follows:

$$MaxS = a_{ij} / \max_i(a_{ij}) , \quad (2)$$

After the original data is preprocessed by minimum and maximum standardization operators in terms of the attributes of each variable to the evaluation objectives, the plans could be evaluated by employing entropy weighted approach. And the entropy weight could be given as following definition.

Definition 3: Let n is the number of pre-evaluation plans, and r_{ij} is the value of each attribute about all of the plans. And then $\dot{r}_{ij}$ is the element of the normalization matrix of attribute values, which is preprocessed by minimum or maximum standardization operator. Hence, the entropy weight E_j could be given as follows [26]:

$$E_j = -\frac{1}{\ln n}\sum_{i=1}^{n}\dot{r}_{ij} \bullet \ln \dot{r}_{ij} , \quad (3)$$

where $j \in M$, and when $\dot{r}_{ij}$ equals to 0, let $\dot{r}_{ij} \bullet \ln \dot{r}_{ij}$ =0.

3.2. The Decision-making Stages of Entropy Weighted Approach

The information entropy theory has provided an effective method to determine the weight of each attribute by the original data from the given project information. The stages of entropy weighted approach are constituted by the following steps: 1) construct the decision-making matrix; 2) compute the normalization matrix; 3) compute the entropy weight of each attribute; 4) compute the attribute weights; 5) compute the comprehensive evaluation scores; and 6) compare the alternative plans.

1) Construct the decision-making matrix. For certain multiple attribute problems, the matrix for making decision should be built firstly and the data is from the information of each attribute of various alternative plans. After the original data of decision-making matrix $A = (a_{ij})_{n \times m}$ being collected, it should be preprocessed by suitable methods, which could make the information reflect the positive effect to the decision-making objectives, such as the operators mentioned by definition 1 and definition 2, namely minimum and maximum standardization operators.

2) Compute the normalization matrix. The normalization matrix is converted from the preprocessed decision-making matrix A. And in the computation case study, the elements of normalization matrix could be computed by the following equation:

$$\dot{r}_{ij} = r_{ij} / \sum_{i}^{n} r_{ij} \ , \tag{4}$$

where $i \in N$ and $j \in M$. Additionally, i and j is the footnotes of plans and attributes respectively.

3) Compute the entropy weight. The weights of each attributes about the alternative plans could be computed based on the entropy weights of the attributes. And the computation method of entropy weight E_j is given by definition 3. It is the key steps in the evaluation and decision making processes by information entropy theory.

4) Compute the attribute weights. The weight vector of each attribute could be derived from entropy weight of each attribute E_i . And the attribute weight utilized to do comprehensive evaluation is given:

$$\omega_j = (1 - E_j) / \sum_{k=1}^{m} (1 - E_k) \ , \tag{5}$$

where ω_j is the weight of attribute j , and m equals to the number of all the attributes.

5) Compute the comprehensive evaluation scores. Based on the weights of all the attributes, the comprehensive evaluation score of each alternative plans could be computed by the following equation.

$$z_i(w) = \sum_{j=1}^{m} r_{ij} \bullet w_j \ , \tag{6}$$

6) Compare the computation results and make decision. The final step of general multiple attribute decision-making approach is to compute the comprehensive scores of each alternative plan by the preprocessed information and the attribute weights. In addition, the decision could be made according to the scores of various alternative plans. Generally, the score is higher, and the corresponding plan is more likely to be chosen and implemented. Otherwise, the low score plans would be given up to be carried out.

3.3. The Computation Case Description

To prove the effectiveness, robustness and applicability of the information entropy theory in the decision-making processes of multiple attribute problems, the sample of company X is chosen, which is confronting whether launching a new product line or not. In the decision-making problem, the managers of company X have to consider six aspects influence factors as follows: the R&D cost of new product, the increment market share bringing from new product, the marketing cost for new product, whether the new product could increase the brand loyalty or not, the changes of sales revenue and the market entry risk for new product. After the preliminary consideration, seven plans focusing on the six aspect influence factors are proposed out.

In the problem, the given influence factors could be divided into two groups, one of which is positive to the decision-making objective, such as the increment of market share, brand loyalty and sales revenue, and another of which is negative to the decision-making objective, such as the cost of R&D, marketing and entry risk. The information of R&D cost, marketing cost and sales revenue could be collected directly from the reports of company X, and the market share of new product could be computed according to the market capacity information. Moreover, the information of brand loyalty and market entry risk could be collected form market survey by certain questionnaires. Therefore, the original information about the decision-making problem is given as that in Table 1.

Table 1. The original data of new product line decision making problem in company X.

Influence Attribute	R&D Cost	Market Share	Marketing Cost	Brand Loyalty	Sales Revenue	Market Entry Risk
plan1	233	0.50	590	6	2240	4
plan2	570	0.55	540	7	3870	8
plan3	420	0.80	470	7	3000	7
plan4	210	0.30	400	4	1980	3
plan5	370	0.60	600	6	2540	5
plan6	600	0.70	700	7	4200	9
plan7	480	0.75	530	8	2890	7

Notes: The data is arranged from the reports of company X and the correlated questionnaires.

4. Computation Case Study

4.1. Description of Original Case

In the research, the computation case is chosen from a company which will launch a new product line and there are several alternative plans to be selected. There involves with two aspect influence indicators in the project: the positive and negative information to the decision-making objective. To eliminate the influence from negative indicators to evaluation, they need to be standardized. The minimum and maximum standardization operators are employed to preprocessed the original information according equation (1) and (2) (see Table 2). Based on the minimum standardization operator, the negative information about the influence indicators of R&D cost, marketing cost and the market entry risk are transferred to be consistent with the evaluation objectives. In addition to maximum standardization operator, all of the positive influence indicators are be preprocessed to eliminate the impact from the influence indicators with differential report standards.

Table 2. The standardization of original information.

Influence Attribute	R&D Cost	Market Share	Marketing Cost	Brand Loyalty	Sales Revenue	Market Entry Risk
plan1	0.9013	0.6250	0.6780	0.7500	0.5333	0.7500
plan2	0.3684	0.6875	0.7407	0.8750	0.9214	0.3750
plan3	0.5000	1.0000	0.8511	0.8750	0.7143	0.4286
plan4	1.0000	0.3750	1.0000	0.5000	0.4714	1.0000
plan5	0.5676	0.7500	0.6667	0.7500	0.6048	0.6000
plan6	0.3500	0.8750	0.5714	0.8750	1.0000	0.3333
plan7	0.4375	0.9375	0.7547	1.0000	0.6881	0.4286

4.2. Normalization of the Information from Original Case

The second stage of information entropy approach is to compute the normalization matrix from the preprocessed decision-making matrix A. Hence, the elements in normalized matrix are computed by those ones in matrix A according to equation (4). Therefore, the normalized matrix could be given (see Table 3). Although the standardization can eliminate the impact from the influence indicators with differential report standards, the varied influence indicators are not of unified scale between the alternative plans, which is to affect the results of evaluation significantly. Nevertheless, the influence indicators are normalized in terms of the corresponding data of varied alternative plans is to diminish the influence effects mentioned above.

Table 3. The normalization of the preprocessed information.

Influence Attribute	R&D Cost	Market Share	Marketing Cost	Brand Loyalty	Sales Revenue	Market Entry Risk
plan1	0.2185	0.1190	0.1288	0.1333	0.1081	0.1915
plan2	0.0893	0.1310	0.1408	0.1556	0.1868	0.0958
plan3	0.1212	0.1905	0.1617	0.1556	0.1448	0.1095
plan4	0.2424	0.0714	0.1900	0.0889	0.0956	0.2554
plan5	0.1376	0.1429	0.1267	0.1333	0.1226	0.1532
plan6	0.0849	0.1667	0.1086	0.1556	0.2027	0.0851
plan7	0.1061	0.1786	0.1434	0.1778	0.1395	0.1095

Notes: The data is arranged from the decision-making matrix according to equation (4).

4.3. The Computation of the Entropy Weight

After the normalization of preprocessed decision-making matrix, the next step is to compute the entropy weight of each attribute. And that is to compute the entropy weights of indicators according to equation (3). To make the computation processes be simple, the step is to be divided into two sub-phases. The first sub-phase is to compute the values of each attribute of all the alternative plans by

$$e_{ij} = \dot{r}_{ij} \bullet \ln \dot{r}_{ij} , \quad (7)$$

where e_{ij} stands for the medium conversion values of attribute j in plan i. Since the elements of normalization matrix $\dot{r}_{ij}$ that are less than 1, are processed by natural logarithm in terms of equation (7), the values of e_{ij} are negative (see Table 4). Moreover, the second sub-phase is to compute the entropy weight combining equation (3) and the data in Table 4. Just only by simple computation processes, the entropy weight vector of all the attributes is E = (0.9598, 0.9806, 0.9926, 0.9907, 0.9834, 0.9616).

Table 4. The medium conversion matrix.

Influence Attribute	R&D Cost	Market Share	Marketing Cost	Brand Loyalty	Sales Revenue	Market Entry Risk
plan1	-0.3323	-0.2534	-0.2640	-0.2687	-0.2405	-0.3166
plan2	-0.2158	-0.2662	-0.2760	-0.2895	-0.3134	-0.2247
plan3	-0.2558	-0.3159	-0.2946	-0.2895	-0.2798	-0.2421
plan4	-0.3435	-0.1885	-0.3156	-0.2151	-0.2244	-0.3486
plan5	-0.2729	-0.2780	-0.2617	-0.2687	-0.2573	-0.2874
plan6	-0.2093	-0.2986	-0.2411	-0.2895	-0.3235	-0.2097
plan7	-0.2380	-0.3076	-0.2785	-0.3071	-0.2748	-0.2421

Notes: The data is arranged by the author.

4.4. Comprehensive Evaluation by Entropy Weighted Approach

Based on the entropy weights of various attributes, the weight of each influence indicator could be computed according to equation (5), and that the vector of the attribute weight is w = (0.3066, 0.1478, 0.0565, 0.0707, 0.1264, 0.2924). Combining the normalization matrix of all the alternative plans with the attribute weight vector, the comprehensive evaluation scores of all the alternative plans could be got by equation (6) (see Table 5). The last column values in Table 5 indicate the comprehensive evaluation scores. Comparing the size of all the plan scores, the ranking order of the alternative plans is as follows:

$$\text{plan4}>\text{plan1}>\text{plan5}>\text{plan3}>\text{plan7}>\text{plan6}>\text{plan2}\,, \quad (8)$$

Based on the results, when the influence factors of R&D cost, market share, marketing cost, brand loyalty, sales revenue and market entry risk are to be taken into account according to the actual situations of company X, the order for the managers to choose the alternative plans is as that in equation (8).

Hence, the optimal choice to be implemented is plan 4, and then is plan 1, plan 5, plan 3, and so on.

Table 5. The results of comprehensive evaluation.

Influence Attribute	R&D Cost	Market Share	Marketing Cost	Brand Loyalty	Sales Revenue	Market Entry Risk	Comprehensive Score
plan1	0.2763	0.0924	0.0383	0.0530	0.0674	0.2193	0.7468
plan2	0.1130	0.1016	0.0419	0.0619	0.1165	0.1097	0.5444
plan3	0.1533	0.1478	0.0481	0.0619	0.0903	0.1253	0.6266
plan4	0.3066	0.0554	0.0565	0.0354	0.0596	0.2924	0.8059
plan5	0.1740	0.1109	0.0377	0.0530	0.0764	0.1754	0.6274
plan6	0.1073	0.1293	0.0323	0.0619	0.1264	0.0975	0.5546
plan7	0.1341	0.1386	0.0426	0.0707	0.0870	0.1253	0.5983

Notes: The data is arranged from the results of comprehensive evaluation.

5. Conclusion and Limitation

Taking the new product line project of company X as sample, the approach based on information entropy theory is employed to test corresponding effectiveness, robustness and applicability. From all of the computation processes, the works could be implemented not only by simple software but by simple hand computation, which proves the approach to be effective. Based on the attributes of each alternative plan, plan 4 states that it is lower R&D cost, marketing cost and market entry risk, and the market share, sales revenue and brand loyalty is also lower. According to the evaluation results, the optimal choice is plan 4, which is in line with the actual situations. Therefore, the robustness of information entropy approach is significant. Combining the aspects of effectiveness with robustness, it can conclude that applicability of the approach is demonstrated significantly.

However, the differentials about the comprehensive evaluation scores of plan 6 (0.5546) and plan 2 (0.5444) are extremely small. Meanwhile, if it occurs in similar evaluation process and there is little difference between the first two alternative plans, how can the managers make their decisions scientifically and efficiently? Therefore, the future work is to design and develop new approach to solve the evaluation difficulties when the comprehensive evaluation scores of the alternative plans are of little differentials. In addition, measures using for improve the evaluation accuracy are also needed to be developed and thereby to improve the effectiveness of decision making for modern enterprises.

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