One Hypersonic Aircraft Nonlinear Observer and Fault-tolerant Controller Design

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Abstract: A new nonlinear fault-tolerant controller is designed to solve the part of actuator failure problem, the hypersonic aircraft nonlinear observer and controller are designed synthetically. We develop the research model based on a SISO output feedback nonlinear unobservered minimum phase system under the condition of single channel actuator failure of the hypersonic aircraft, adopt K-filter to reconstruct state vectors, design the adaptive control law to guarantee the system boundedness. Dynamic surface control is employed strategy to eliminate the explosion of terms by introducing a series of first order filters to obtain the differentiation of the virtual control inputs. Lyapunov stability theorem guarantees the error uniformly bound. The pitch channel of hypersonic aircraft is studied as an example to validate above theory. Both theory analysis and simulation verification show the simpleness and effective of this method.

Keywords: Hypersonic aircraft, Observer, Dynamic surface adaptive backstepping, Fault-tolerant control.

1. Introduction

Compared with traditional aircraft, the near space hypersonic aircraft must cope with more complex flight environment, for example, limited flight corridor, more rigorous demand to flight attitude from kinetic system. The control system design must deal with the extra prominent dynamics characters such as highly coupled, strongly nonlinear, time variational and so on. thus, during the course of control system study and design for the hypersonic aircraft, the following factors that plant elastic deformation, multi-inputs multi-outputs, aerodynamic parameters uncertain, sensors and actuator failures must be considered, which are differ from the traditional aircrafts control system design. Actuator failures may have significant effect on the performance of control systems, will lead to instability and result in disasters if not be handed properly, which ask hypersonic aircraft own the performance of reconstructing state vectors and fault-tolerant control. Fault-tolerant control has attracted many researchers and is available in widely scattered publications. A feasible approach for such a control scheme is to predesign various controllers anticipating component failure and switching to an appropriate controller when failure occurs. Tang [1] studied adaptive actuator failure compensation for parametric-strict-feedback systems under different system structure conditions, and use adaptive state feedback control schemes to ensure asymptotic output tracking and closed-loop signal boundedness. Tao [2] developed a state feedback output tracking adaptive control scheme for plants with actuator
failures characterized by the failure pattern that some inputs are stuck at some unknown fixed values at unknown time instants, and design new controller parametrization and adaptive law under some relaxed system conditions. Reference [3] and [4] realized flight control system Fault-tolerant control by using switch and adjust control strategy to resolve failure self-regulation.

But these articles are research on the suppose that systems states are known. In this paper, we will study fault-tolerant control with states vector unmeasured, and will design a new adaptive observer and controller by K-filter, which not only can estimate states, but also can get unknown parameters, and we will use dynamic surface backstepping arithmetic to resolve the problem of terms explosion, which is the shortcoming of tradition backstepping control.

2. System Description

Considering the following nonlinear aircraft control system equation

\[ \dot{x} = f(x) + \sum_{i=1}^{m} \theta_i g_i(x) u_i, \]
\[ y = h(x) \]

where \( u_i \in R, i = 1,2,\ldots,m \) are the actuators inputs, \( x \in R^m \) are the vector of unmeasured states, \( y \in R \) is the measured output states, \( f(x) \in R^m, g_i(x) \in R^m, i = 1,2,\ldots;m \), \( h(x) \in R \) are smooth functions, \( \theta_i \in R, i = 1,2,\ldots,m \) are unknown constants.

The aircraft equation (1) can be transformed into the output-feedback form.

By one parameter-independent diffeomorphism \( z = \Phi(x) = [\Phi_1(x), \Phi_2(x), \ldots, \Phi_n(x)]^T \), which satisfies

\[ \Phi_i(\Phi(x)) = h(\Phi(x)) \]

\[ \frac{\partial \Phi_i}{\partial x} f(x) = \Phi_{ip}(\Phi(x)) + \phi_i(h(\Phi(x))) \]
\[ \frac{\partial \Phi_i}{\partial x} h(x) = 0 \]

\[ \frac{\partial \Phi_i}{\partial x} h(x) = \Phi_{ip}(\Phi(x)) + \phi_i(h(\Phi(x))) \]
\[ \frac{\partial \Phi_i}{\partial x} f(x) = \phi_i(h(\Phi(x))) \]

Considering actuator failure sufficiently, we can get

\[ \dot{x}_i = x_i + \varphi_i(y) \]
\[ \dot{x}_{i+1} = x_{i+1} + \kappa_i(y) \]

\[ \dot{x}_n = x_n + \varphi_n(y) + \sum_{i=1}^{n} b_{in} \sigma_i(y) \dot{\sigma}_i + b_n \sigma_n(y) u_n \]

\[ y = x_l \]

We suppose there are \( l = \sum_{i=1}^{k} l_i \) failed actuators, \( 0 \leq l \leq m-1 \), \( u \) is applied control signals.

Define \( a_i = \varphi_i, \sigma_i, r = 0,1,\ldots,m, i = 1,\ldots,l \), then rewrite (2) in a compact form as

\[ \dot{x} = A x + \varphi(y) + H(y) u + [0 \ b]^T \sigma_o(y) u \]

\[ y = C x \]

where \( \varphi(y) = [\varphi_1(y), \ldots, \varphi_n(y)]^T \),

\[ H(y) = \begin{bmatrix} \sigma_o(y) & \sigma_{o1}(y) & \ldots & \sigma_{on}(y) \\ \sigma_{1o}(y) & \sigma_{11}(y) & \ldots & \sigma_{1n}(y) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{lo}(y) & \sigma_{l1}(y) & \ldots & \sigma_{ln}(y) \end{bmatrix} \]

\[ A = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{bmatrix} \]

\[ C = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}, \quad r = 0,\ldots,m, j = \rho,\ldots,n \]

\[ a = [0, a_1, a_2, \ldots, a_l]^T \in R^l, \quad b = [b_m, \ldots, b_l]^T \in R^{m+1}, \quad x \in R^n \]

are unmeasured states vector, \( u \in R \) and \( y \in R \) are input and output of system, only output \( y \) can be measured. \( a \) and \( b \) are unknown constants vector. \( e \) are the \( i \)th coordinate
vector, $i=n-r$, $R^n$, $\sigma,y(0)$, $0\leq i \leq l$, $p \leq n$, and $\sigma_0(y)$ are known smooth function, and existing $\sigma_i > 0$ to get $\sigma(y)-\sigma_0$, $\forall y \in R$.

Aiming at system (5), we propose the following Assumption.

Assumption 1 system is minimum phase, $B(s)=b_0s^n+\cdots+b_Ms+b_0$ is Hurwitz polynomial.

Assumption 2 The sign of $b$ are known for $r=0,\cdots,m$, and existing known constant $b_m$, to make $0 \leq bm \leq BM$.

2.1. Adaptive Compensator Design

Using $K$-filter, we design the following state filter [5-7]

$$\eta = A_0\eta + K_0y + \varphi(y)$$

$$\xi = A_0\xi + \Theta(y)$$

$$\lambda = A_0\lambda + e_n\sigma_0(y)u$$

$$v_j = A_0\lambda \quad j = 0,\cdots,m,$$

where $A_0 = \begin{bmatrix} -k_0 & I_{n-I} \\ -k_s & 0 & \cdots & 0 \end{bmatrix}$, \ $\varphi(y)=[\varphi_1(y) \cdots \varphi_l(y)]^T$,

choose parameters vector $K_0=(k_1,k_2,\cdots,k_s)^T$ to make matrix $A_0$ is Hurwitz matrix, then $PA_0 + A_0^TP = -I$, $P=P^T>0$. $e_n$ are the ith coordinate vector of $R^n$.

Define $\Theta=(b_m^1,b_m^2,\cdots,b_m^r)^T$, $\Omega=(v_m^1,v_m^2,\cdots,v_m^r)$,

design state observer is $\hat{x} = \eta + \Omega^T\Theta$.

Design observer error $\delta = x - \hat{x}$, we can get

$$\delta = x - \eta - \Omega^T\Theta$$

Then we will design a new adaptive controller to ensure the closed-loop stability and output tracking.

2.2. Controller Design

Replacing system states $x$ by filter signal, we can get

$$x = \eta + \Omega^T\Theta + \delta$$

Define

$$z_1 = y - y_d, \ z_i = v_m^i - \alpha_{i-1}, \ i = 2,\cdots,n$$

Step 1, by equation (6) and (7), we can obtain

$$\dot{z}_1 = x_2 + \varphi_{i1} + \sum_{j=1}^q a_j\varphi_j - \dot{y}_d$$

$$= \eta_2 + \Omega(k_2)\theta + \delta_2 + \varphi_{i1} + H_{(i)}a - \dot{y}_d ,$$

$$= b_nv_m^2 + \eta_2 + \varphi_{i1} + \bar{w}^T\theta + \delta_2 - \dot{y}_d$$

where

$$\bar{w}^T=[v_{m-1,2},\cdots,v_{o,2},H_{(i)} + \xi_{(i)}],$$

$$w^T=[v_{m-1,2},\cdots,v_{o,2},H_{(i)} + \xi_{(i)}].$$

$H_{(i)}$ are the first row of matrix $\Phi$, $\Omega_{(i)}$, $\xi_{(i)}$ are the second row of matrix $\Omega$ and $\xi$, $\eta_2$ is the second vector of $\eta$. suppose

$$\begin{cases}
\dot{y}_{m,i} = v_{m,i+1} - k_iy_{m,i}, i = 2,\cdots,r - 1 \\
v_{m,i} = \sigma_0(y)u + v_{m,i+1} - k_iy_{m,i},\end{cases}$$

(6)

Design $z_2=v_{m,2}-a_1t$, where $a_1$ is the first virtual control input. Then $v_{m,2}=z_2+\alpha_1$

$$\dot{z}_1 = b_nz_2 + b_m\alpha_1 + \eta_2 + \varphi_{i1}(y) + \bar{w}^T\theta + \delta$$

Suppose $\kappa = \frac{1}{b_m}$, selecting $\alpha_1$ as

$$\begin{bmatrix}
\alpha_1 = \kappa \eta_2 \\
\kappa = -\mu_2 \text{sgn}(b_m)\xi_{(i)}z_1 \\
\sigma_1 = -((l_1z_1 + \eta_2 + \varphi_{i1}(y) + \bar{w}^T\theta - \dot{y}_d),
\end{bmatrix}$$

(7)

where $l_1, \mu_2$ are plus parameters, $\hat{\theta}, \kappa$ are the estimator of $\theta$, $K$, suppose $\hat{\theta} = \theta - \hat{\theta}, \hat{K} = K - \hat{K}$.

Defining the following Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\bar{w}^T\mu_1^{-1}\bar{w} + \frac{1}{2}\mu_2^{-1}\kappa^2 + \frac{1}{2d^2}\delta^T\delta$$

(8)

The time-derivative of $V_1$ is derived as

$$\dot{V}_1 = z_1 \cdot \dot{z}_1 - \bar{w}^T\mu_1^{-1}\bar{w} - \frac{1}{2}\mu_2^{-1}\kappa^2 - \frac{1}{2d^2}\delta^T\delta$$

$$\leq b_nv_m^2z_1 + \eta_2z_1 + \varphi_{i1}(y) + \bar{w}^T\theta z_1 + \delta z_1$$

$$- \dot{y}_d z_1 - \frac{1}{2}\mu_1^{-1}\bar{w}^T\mu_2^{-1}\bar{w} + \frac{1}{2d^2}\delta^T\delta$$

(9)

Select the first tuning function

$$\tau_1 = \dot{\theta} = \mu_1 \bar{w}z_1$$
Step 2. Differentiating $z_i$, we obtain
\[
\ddot{z}_i = v_{n,3} - k_i v_{n,1} - \frac{\partial \alpha_1}{\partial v} (\eta_2 + w^{\top} \theta + \delta_2 + \varphi_0(y)) + \varphi_0(y) - \frac{\partial \alpha_1}{\partial \theta} \dot{\theta} - \frac{\partial \alpha_1}{\partial \varphi} \dot{\varphi} + \frac{\partial \alpha_1}{\partial \eta} \dot{\eta} - \frac{\partial \alpha_1}{\partial \delta} \dot{\delta} + \frac{\partial \alpha_1}{\partial \varphi} \dot{\varphi} + \frac{\partial \alpha_1}{\partial \eta} \dot{\eta} + \frac{\partial \alpha_1}{\partial \delta} \dot{\delta} \cdot \ddot{y}_d.
\]

The calculation of $\alpha_1$ will bring out terms explosion, we adoptive dynamic surface to resolve it. Introduce a new state variable $\chi$, which can be obtained by the following first-order filter [8]
\[
\varepsilon_2 \dot{\chi}_2 + \chi_2 = \alpha_1, \chi_2(0) = \alpha_1(0)
\]
where $\varepsilon_2 > 0$ is the small constant.

Supposing $\dot{S}_2 = \chi_2 - \alpha_1$, then
\[
\dot{S}_2 = -\frac{S_2}{\varepsilon_2},
\]
the equation
\[
z_i \ddot{z}_i = z_i (b_m v_{n,2} + \eta_2 + \varphi_0(y) + w^{\top} \theta + \delta_2 - \dot{y}_d)
\]
will be rewritten.

By Young equation, we can obtain
\[
ab \leq a^2 + \frac{1}{4} b^2, \quad v_{n,2} = z_2 + \chi_2 = z_2 + S_2 + \alpha_1,
\]
then
\[
z_i \ddot{z}_i = b_m z_i + b_1 z_i + b_2 z_i + b_3 z_i + \eta_2 + \varphi_0(y) + w^{\top} \theta + \delta_2 - \dot{y}_d.
\]

Design virtual control input $\alpha_2$
\[
\alpha_2 = -b_m z_i - l_2 z_i + k_i v_{n,1} + \hat{x}_2,
\]
where $l_2$ is the plus parameter, $\hat{x}_2$ is the estimator of $b_m$. Introduce a new state variable $\chi_3$, which can be obtained by the following first-order filter
\[
\varepsilon_3 \dot{x}_3 + x_3 = \alpha_2, \quad x_3(0) = \alpha_1(0)
\]
where $\varepsilon_3 > 0$.

Suppose $\dot{S}_3 = x_3 - \alpha_1$, then
\[
\dot{S}_3 = -S_3 / \varepsilon_3
\]
Defining error differential equation
\[
\dot{S}_2 = -S_2 / \varepsilon_2 - \alpha_1, \quad \text{then}
\]
\[
\dot{S}_2 = \ddot{x}_2 - \dot{\alpha}_1 = -\frac{S_2}{\varepsilon_2} - \dot{x}_2 (l_2 z_i - \eta_2 + \varphi_0(y) - w^{\top} \theta - \dot{w}^{\top} \theta + \dot{\hat{y}}_d).
\]

We propose Assumption 3.

Assumption 3. Assume that the reference trajectory $y_d$ is sufficiently smooth functions and $\dot{y}_d, \ddot{y}_d$ are bounded functions in a compact set $D_3 \subset R^3$. Also, assume that the states in error dynamics $S_2, z_2, \cdots, z_n, \dot{\theta}, b_m$ stay in a compact set
\[
\left| \frac{\dot{S}_2 + S_2}{\varepsilon_2} \right| \leq b_3 (z_2, \cdots, z_n, S_2, \dot{\theta}, b_m, y_d, \dot{y}_d, \ddot{y}_d)
\]
By formula (7), suppose $\ddot{z}_1$ is bounded, $b_2 (z_1, \cdots, z_n, S_2, \dot{\theta}, b_m, y_d, \dot{y}_d, \ddot{y}_d)$ is bounded function. Then
\[
\left| \frac{\dot{S}_2 + S_2}{\varepsilon_2} \right| = -\frac{S_2}{\varepsilon_2} - \dot{S}_2 \leq -\frac{S_2}{\varepsilon_2} + \frac{1}{12} S_2
\]
By $z_2 = v_{n,2} - \chi_2$, get $\dot{z}_2 = v_{n,2} - \dot{\chi}_2 = v_{n,3} - k_i v_{n,1} - \dot{\chi}_1$.
Defining error differential equation \( \dot{S}_3 = -S_3/\varepsilon_3 - \alpha_z \), then
\[
\dot{S}_3 = \chi_3 - \alpha_z = -\frac{S_3}{\varepsilon_3}(-\hat{b}_n z_n - l_2^z z_2 + k_n^v v_n + \chi_3),
\]
(17)

So,
\[
\left| \dot{S}_3 + \frac{S_3}{\varepsilon_3} \right| \leq B_1(z_1, z_2, \cdots z_n, S_2, S_3, \hat{b}_n, y_d, \dot{y}_d, \ddot{y}_d).
\]
where \( B_1(z_1, z_2, \cdots z_n, S_2, S_3, \hat{b}_n, y_d, \dot{y}_d, \ddot{y}_d) \) is the bounded function.
\[
S_3 \dot{S}_3 \leq \frac{S_3^2}{\varepsilon_3} + B_1 S_3 \leq \frac{S_3^2}{\varepsilon_3} + S_3^2 + \frac{1}{4} B_1^2,
\]
(18)

Step i, \( i = 3, 4 \cdots \rho \).

Define \( z_i = v_{m,i} - \chi_i \), then
\[
\dot{z}_i = \dot{v}_{m,i} - \dot{\chi}_i = v_{m,i+1} - k_i v_{m,i} - \dot{\chi}_i, \quad i = 1, 2 \cdots n.
\]
(19)

Design virtual control input \( \alpha_i \)
\[
\alpha_i = -z_{i+1} - l_i z_i + k_i v_{m,i} + \dot{\chi}_i, \quad i = 1, 2 \cdots n.
\]
(20)

where \( l_i \) is the plus parameter.

Introduce a new state variable \( \chi_{i+1} \), define first-order filter as following
\[
e_{i+1} \dot{\chi}_{i+1} + \chi_{i+1}(0) = \alpha_i(0)
\]
(21)

By \( v_{m,i+1} = z_{i+1} + \chi_{i+1}, \) get
\[
\dot{z}_i = z_i (v_{m,i+1} - k_i v_{m,i} - \dot{\chi}_i)
\]
\[
= z_i z_{i+1} + z_i S_{i+1} + z_i \alpha_i + z_i (-k_i v_{m,i} - \dot{\chi}_i)
\]
\[
\leq \frac{1}{2} z_i^2 + \frac{1}{2} z_{i+1}^2 + \frac{1}{4} S_{i+1}^2 - l_i z_i z_{i+1} - z_{i+1} z_i
\]
(22)

\[
\leq \frac{1}{2} z_i^2 - (l_i - 2) z_i^2 + \frac{1}{2} z_{i+1}^2 + \frac{1}{4} S_{i+1}^2,
\]

Step n, Define \( \dot{z}_n = v_{m,n} - \dot{\chi}_n \), then
\[
\dot{z}_n = \dot{v}_{m,n} - \dot{\chi}_n = \sigma_0(\nu) u + v_{n,e+1} - k_n v_{m,n} - \dot{\chi}_n,
\]
(23)

Select the control law as
\[
u = -\frac{1}{\sigma_0(\nu)}(-l_n z_n - v_{n,e+1} + k_n v_{m,n} + \dot{\chi}_n),
\]
(24)

where \( l_n \) is the plus parameter, adaptive law
\[
\dot{\theta} = \mu_1 \sigma_0(\nu),
\]
(25)

where \( \mu_1, \mu_2 \) are the plus constants.

### 2.3. Robustness Analysis

Define Lyapunov function as follows
\[
V = \frac{1}{2} \sum_{i=1}^{\rho} \gamma_i + \frac{1}{2d} \sum_{i=1}^{\rho} \delta_i \| \dot{\theta} \| + \frac{1}{2} \widehat{\theta} \| \dot{\theta} \| + \frac{1}{2} \rho^2 + \frac{1}{2} \rho^2 S_i^2, \quad (25)
\]

where \( \gamma_i = \left[ z_i, z_2, \cdots z_n \right]^T \). Suppose bounded function \( B_i < M_i, M_i > 0, \) \( i = 2 \cdots n \). From equations (24) and (25), it can be shown that the Lyapunov function satisfies
\[
\dot{V} = \sum_{i=1}^{\rho} \frac{1}{2d} \sum_{i=1}^{\rho} \delta_i \| \dot{\theta} \| + \frac{1}{2} \widehat{\theta} \| \dot{\theta} \| - \frac{1}{\mu_1} \gamma_i \dot{\theta}
\]
(26)

From suppose we can get that \( \nu = \frac{1}{4} \sum_{i=2}^{\rho} M_i^2 \) is bounded, so if we choose parameters accurately, we can achieve \( \dot{V}(t) \leq -c V(t) + \nu, c > 0 \), then
\[
\frac{d}{dt} (V(t) e^\nu) \leq c V(t) e^\nu,
\]
(27)

So
\[
0 \leq V(t) \leq \frac{V(0)}{c} e^{-c t} \leq \frac{V(0)}{c} - V(t) = v_0,
\]
\[
\| \dot{\theta} \| \leq \frac{2 \mu_1 \nu}{c}, \quad z_i \leq \frac{2 \nu}{c}, \quad \| \delta_i \| \leq \frac{2d \nu}{c \mu_{min} \left( P \right)},
\]
\[
\kappa^2 \leq \frac{2 \mu_2 \nu}{c}, \quad S_i^2 \leq \frac{2 \nu}{c}, \quad i = 1, 2,
\]

where \( \dot{\theta}, z_i, \delta, K, S_i \) are bounded. Therefore, system semi global boundedness of the state is guaranteed. The tracking error is ultimately confined to a ball of radius.
### 3. Hypersonic Aircraft Adaptive Fault Tolerant Controller Design

Consider the following longitudinal dynamic equations of a generic hypersonic aircraft cruising at a Mach number of 15 and at altitudes of 110,000 ft [9, 10].

\[
\begin{align*}
\dot{x}_1 &= x_2 + f_1(x_1) \\
\dot{x}_2 &= f_2(x_1, x_2) + g_1(x_1)u_1 + g_2u_2 \\
y &= x_1
\end{align*}
\]  

where

\[
f_1(x_1) = c_1(0.6203\alpha + 0.02318\sin \alpha) + \frac{\mu - v^2r}{v^2r},
\]

\[
c_1 = 0.5\rho v^2S/mv,
\]

\[
f_2(x_1, x_2) = -0.035\alpha + 0.036617\alpha + 5.3261 \times 10^{-6},
\]

\[
C_{u0}(\alpha) = -0.035\alpha + 0.036617\alpha + 5.3261 \times 10^{-6},
\]

\[
C_{u}(q) = \frac{c}{2\nu}(q(-6.796\alpha^2 + 0.3015\alpha - 0.2289)),
\]

where \( v \) is the velocity, \( \gamma \) is the flight path angle, \( h \) is the altitude, \( \alpha \) is the attack angle, \( q \) is the pitch rate. \( T, D, L \) and \( M_{yy} \) represent thrust, drag, lift-force and pitching moment respectively. \( M, I_{yy}, \mu \) and \( R_e \) represent the mass of aircraft, moment of inertia about pitch axis, gravity constant and the radial distance from center of the earth. \( S \) is the reference area, \( \delta_e \) is the elevator deflection and \( \beta_0 \) is the throttle setting. \( c \) and \( c_e \) are the constants.

Suppose \( \cos \gamma = 1 \), \( x_1 = \alpha \), \( x_2 = q \), \( u_1 = \beta_0 \), \( u_2 = \delta_e \), where suppose \( q \) is unmeasured, then

\[
\begin{align*}
\dot{x}_1 &= x_2 + f_1(x_1) \\
\dot{x}_2 &= f_2(x_1, x_2) + g_1(x_1)u_1 + g_2u_2 \\
y &= x_1
\end{align*}
\]

where

\[
f_1(x_1) = c_1(0.6203\alpha + 0.02318\sin \alpha) + \frac{\mu - v^2r}{v^2r},
\]

\[
c_1 = 0.5\rho v^2S/mv,
\]

\[
f_2(x_1, x_2) = -0.035\alpha + 0.036617\alpha + 5.3261 \times 10^{-6},
\]

\[
\begin{align*}
f_{12} &= c_1[-0.035\alpha + 0.036617\alpha\alpha] + c_1 \cdot 5.3261 \times 10^{-6} \\
&\quad + (c/2\nu)(q(-6.796\alpha^2 + 0.3015\alpha - 0.2289))
\end{align*}
\]

\[
m = 9375, I_{yy} = 7, S = 3603, c = 80,
\]

\[
g_1(x_1)u_1 = 0.5\rho vSCT\sin \alpha/m, c_e = 0.0292,
\]

\[
\rho = 0.24325.
\]

Suppose actuator failure take place on the throttle setting \( \beta_0(u_t = \overline{u}) \), \( \delta_e \) is on the rails \( (u = u_t) \).

Then

\[
\begin{align*}
\dot{x}_1 &= x_2 + f_1(x_1) \\
\dot{x}_2 &= f_2(x_1, x_2) + \sigma(x_1)u_1 + b_2 \cdot g_2u \\
y &= x_1
\end{align*}
\]

where \( \sigma(x_1) = \frac{0.5\rho vS}{m} \cdot \sin \alpha \).

### 4. Simulation Results

Select \( K \)- filter as

\[
\dot{\eta} = A\eta + K(y - \varphi(y)),
\]

\[
\dot{\xi} = A\xi + H(y),
\]

\[
\dot{\lambda} = A_0\lambda + e_c\sigma(y)u,
\]

where

\[
\varphi(y) = [f_1(x_1), f_2(x_1, x_2)]^T, a = [0, a]^T, b = b_2,
\]

\[
H(y) = \begin{bmatrix} 0 & 0 \\ 0 & \sigma(x_1) \end{bmatrix}, v_0 = \lambda, k_1 = 3, k_2 = 2, l_1 = 7, c_2 = 0.2,
\]

\( \sigma_0(y) = g_2 \) is constant. \( x_{td} = 2 \), simulation condition is \( v = 15 \text{ M}_{as}, \text{ altitude } h = 30 \text{ km.} \) Results as follows.
Where $x_1$ is the attack angle $\alpha$, $x_2$ is the flight path angle $\gamma$. Fig. 3 and Fig. 4 is the error outputs of $\alpha$ and $\beta$. In Fig. 1 the output of $\alpha$ has small jump, which cause by the adaptive parameter of $\alpha_1$, and its change scope is less, so we can take it as well-balanced. So simulation verification shows the simpleness and effective of this method.

5. Conclusions

This paper presented a nonlinear adaptive controller for a hypersonic aircraft with states vector unmeasured. K-filter is adopted to reconstruct state vectors. The nonlinear adaptive controller is designed using dynamic surface backstepping control techniques, Lyapunov stability theory is used to prove the stability of the system and derive the tuning rules for updating all parameters. Finally, a nonlinear numerical simulation of a hypersonic aircraft model is performed to demonstrate the good performance of the proposed method.

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