Effect of Design Geometry on the Performance Characteristics of Linear Variable Differential Transformers

1 Mohammad KILANI, 2 Sinan TAIFOUR and 3 Lutfi AL-SHARIF

1 Department of Mechatronics Engineering, The University of Jordan, Amman 11942, Jordan
Tel.: +962 6 5355000, ext. 23025

2 Technische Universität München, Arcisstrasse 21, 80333 Munich, Germany
Tel.: +49 157 785 62559

3 Department of Mechatronics Engineering, The University of Jordan, Amman 11942, Jordan
Tel.: +962 6 5355000, ext. 22993
E-mail: mkilani77@yahoo.com, sinan@taifour.com, lal-sharif@theiet.org

Received: 29 November 2012 / Accepted: 19 March 2013 / Published: 29 March 2013

Abstract: The effect of design geometry on the output voltage, linearity and sensitivity of linear variable differential transformers (LVDTs) is presented. The effect of varying six geometric design parameters, including primary coil length, secondary coil length, inner and outer coil radii, and the length and radius of the core, on the transfer characteristics of LVDT is investigated using Finite element simulations. Output voltage vs. core displacement figures are used to determine the effect of the parameters investigated on the stroke and sensitivity.

Keywords: Linear variable differential transformer, Transducer, Sensor, Displacement, Sensitivity, Stroke, Finite element modeling.

1. Introduction

A linear variable differential transformer (LVDT) is an inductive transducer whose input is the linear displacement of a core and whose output is a pair of ac voltages proportional to core displacement from a null point [1]. A typical LVDT consists of a single primary winding positioned between two identical secondary windings wound on a tubular ferromagnetic former. Fig. 1 shows a schematic representation of an LVDT with the primary coil energized by an external harmonic reference source, and the two secondary coils connected in series opposition so that the induced output voltages oppose each other. As the core moves inside the windings, the induced voltage associated with one secondary coil increases while the other decreases, with the difference between the two varying linearly with displacement. The range over which the output voltage varies linearly distance is known as the linear stroke of the LVDT, or simply the stroke, s.

Compared to potentiometric displacement transducers, LVDTs provide the advantages of frictionless operation, infinite mechanical life, excellent resolution and good repeatability [1]. Their main disadvantage is that response is dynamic and is influenced by the exciting frequency. In practice a signal conditioning IC such as AD598 (by Analog Devices, Inc.) is used to convert the sinusoidal differential to a dc voltage proportional to
the armature displacement. LVDTs are often used as secondary transducers whereby a primary transducer converts the measured quantity (pressure, acceleration, or force) into displacement [2, 3].

LVDT design parameters include geometric dimensions and material properties. This work investigates the effect of geometric parameters on the output voltage, sensitivity and stroke. The classical method of analysis is based on approximate formulas which neglect end effects, and cross-coupling between secondaries [4]. More novel methods for analysis employ finite element methods [5, 6], and artificial neural networks [7, 8]. The dynamic response of the LVDT is discussed in [9]. The LVDT has also been integrated into linear actuators [10]. This work utilizes finite element simulations to investigate the effect of dimensional parameters on the output voltage, stroke and sensitivity of an LVDT.

2. LVDT Construction and Design Parameters

Fig. 2 shows a cross-sectional view of a representative LVDT illustrating its components and geometric dimensions. It consists of a cylindrical transformer, with a single primary coil of length $l_p$ and two secondary coils of length $l_s$ each. All three coils are wound onto a hollow cylindrical bobbin, with a resulting inner coil radius $r_i$ and an outer coil radius $r_o$. A moveable core with length $l_c$ and radius $r_c$ is fitted concentrically within the bobbin, and is free to move axially within the body. One end of the core is normally fixed to an extension rod which emerges through the end of the transducer. The separation distance between the coils, $c_s$, is determined by the thickness of the former on which the coils are wound, and the center-to-center distance between the wires in the coils, $w_s$, is determined by the thickness of the wire insulation. The diameter of the coil wires determines the overall resistance of the coil, and is not considered a geometric design parameter. The remaining six geometric dimensions, i.e., $l_p$, $l_s$, $l_c$, $r_i$, $r_o$, and $r_c$, are free to be specified by the designer. The effect of these parameters is investigated in this work.

In use, the transducer body is clamped in position while the extension rod is attached to the component being measured. When the core is at the center position (null position) between the coils, the output voltages from the two secondary windings cancel one another. A displacement, $x$, of the core from the center position offsets the balance between the windings and results in a non-zero differential voltage, which is used to measure the displacement. Electrical connections to the internal windings of the transducer normally emerge through the side of the casing in a multi-core cable.

![Fig. 1. Operating principle of an LVDT [1].](image1)

![Fig. 2. A cross section through a representative LVDT showing relevant dimensions.](image2)

3. Methodology

The effect of varying the six geometric design parameters on the output voltage, stroke and sensitivity is investigated in this work using finite element simulations. A finite element model for the LVDT was constructed to simulate the effect of varying each of the six geometric parameters on transfer characteristics. Table 1 shows the default values and range of each of the investigated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default value (mm)</th>
<th>Range (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary coil length, $l_p$</td>
<td>80</td>
<td>26 – 133</td>
</tr>
<tr>
<td>Secondary coil length, $l_s$</td>
<td>75</td>
<td>36 – 125</td>
</tr>
<tr>
<td>Coils’ inner radius, $r_i$</td>
<td>10</td>
<td>4.8 – 6.7</td>
</tr>
<tr>
<td>Coils’ outer radius, $r_o$</td>
<td>35</td>
<td>16.8 – 58.3</td>
</tr>
<tr>
<td>Core length, $l_c$</td>
<td>130</td>
<td>62 – 216</td>
</tr>
<tr>
<td>Core radius, $r_c$</td>
<td>4.5</td>
<td>2.1 – 7.5</td>
</tr>
</tbody>
</table>

The Finite Element Method Magnetics (FEMM) software [11] was used to construct and simulate the FEM models. Since the problem is axially symmetric around the axis of the core, a 2D model for half of the LVDT’s cross section, as shown in Fig. 3, was sufficient to fully represent the 3D problem. The material of the core is assumed to be pure iron, with a
relative permeability $\mu_r = 14 \times 10^3$. A sinusoidal waveform with frequency $f = 50$ Hz was applied to the primary coil to achieve a peak current of $I = 50$ mA. The geometric parameters $c_s$ and $w_s$ were assumed to be 5 mm and 0.3 mm, respectively.

To investigate the effect of a certain geometric parameter, the parameter of interest was varied in the model through its range, while all other parameters kept constant at their default value. The dc output voltage $V_o$ vs. core displacement $x$ for each parameter set was obtained by finding the total flux linkage between the primary coil and each of the two secondaries, $M_{ps1}$ and $M_{ps2}$, for the range of $x$ using the FEMM simulations. The amplitudes of the two secondary ac voltages $v_{s1}$ and $v_{s2}$ for each $x$ were then calculated from Faraday’s law as:

$$v_{s1,s2} = \frac{d\phi_{s1,s2}}{dt} = M_{ps1,ps2} \frac{dI}{dt} = 2\pi f IM_{ps1,ps2} \quad (1)$$

where $\phi_{s1,s2}$ and $M_{ps1,ps2}$ are, the total flux linkage and the mutual inductance between the primary coil and each of the secondaries. It is assumed that the two ac signals from the two secondary coils are processed by a full wave rectifier, which smoothes the signals before subtracting them and hence the dc output voltage $V_o$ at a core displacement $x$ is calculated as the difference between the peak ac voltages of the two secondary coils:

$$V_o(x) = v_{s1} - v_{s2} \quad (2)$$

LVDT output voltage, $V_o$ vs. core position $x$ for each parameter set was produced by carrying out a series of finite element simulations. A MATLAB [12] code was used to control the calls to FEMM for value of $x$.

4. Results and Discussion

A large number of FEM simulations were carried out to investigate the effect of each geometric parameters of interest on the transfer characteristics of LVDTs. Fig. 4, Fig. 5 and Fig. 6 show, respectively, the effect of these parameters on the output voltage $V_o$, stroke, $s$, and sensitivity, $\sigma$, for the same values of primary coil current, frequency, and LVDT magnetic material properties.

Fig. 4 shows a plot of $V_o$ vs. core displacement $x$, from the center position (null position of the LVDT). It can be seen that $V_o(x)$ follows a generally sinusoidal profile with the axis of symmetry of the sinusoidal waveform at the origin. Both the amplitude and the period of the sinusoid changes with LVDT geometry, but the relative amount of variation differs depending on the variable under investigation. Both $s$ and $\sigma$ are therefore influenced by the geometry as discussed next.

The stroke, $s$, of an LVDT is defined as the range of $x$ over which $V_o(x)$ changes linearly with $x$ [1]. Since $V_o(x)$ follows a generally sinusoidal profile, it will inevitably deviate from a straight line, and a truly linear range does not exist. The definition of $s$, therefore, needs to allow for a nominal deviation from a linear $V_o(x)$. We define $s$ as the range of $x$ around the origin over which the percentage non-linearity, $\varepsilon(x)$ does not exceed 0.5%. The percentage non-linearity at a certain core displacement $x$, $\varepsilon(x)$ is defined as:

![Fig. 3. Determination of flux linkage between the coils using FEM (a) FEM model, and (b) sample results.](image-url)
Fig. 4. $V_o$ vs. $x$ for different values of (a) $l_p$, (b) $l_s$, (c) $r_i$, (d) $r_o$, (e) $l_c$, and (f) $r_c$.

$$
\varepsilon(x) = \left[ \frac{V_o(x) - ax}{V_o(s/2) - V_o(-s/2)} \right] \times 100\% \quad (3)
$$

where $a$ is the slope of $V_o(x)$ at the null point ($x = 0$)

$$
a = \left. \frac{dV_o(x)}{dx} \right|_{x=0} \quad (4)
$$

The definition of $\varepsilon(x)$ in Eq. (3) assumes prior knowledge of $s$, which is not available. However, since $V_o(x)$ follows a sinusoidal profile, the deviation from linearity increases monotonically with $x$, and a certain $x$ lies within $s$ if the following condition is satisfied:

$$
\left| \frac{V_o(x) - ax}{2ax} \right| \times 100\% \leq 0.5\% \quad (5)
$$

Fig. 5 shows the effect of the investigated geometric design parameters on $s$. The plots in Fig. 5 (a) show the effect of the core and coil lengths $l_p$, $l_s$, $l_c$, on, and show that $s$ increases with each of these parameters. The plot of the $s(l_p)$ curve flattens at high values of $l_p$ while the $s(l_s)$ flattens at low value of $l_s$.
Fig. 5 (b) shows the effects of LVDT’s coil and core radii $s$. The varying range for the variables in the horizontal axis in the different plots is due to the different ranges considered for the variables investigated. Fig. 5 (b) shows that $s$ increases with each of $r_1$ and $r_o$, and $r_c$, with a change in $r_1$ producing the most tangible effect on $s$ followed by $r_o$ and finally $r_c$.

The sensitivity $\sigma$ of the LVDT is a measure of the change in $V_o (x)$ for a given change in $x$, for a given input primary core excitation voltage, $V_i$. The units of $\sigma$ are mm$^{-1}$, and are in many cases cited as mV/mm/V. The value of $\sigma$ for an LVDT is the slope of $V_o (x)$ at the null point of the transfer characteristics when the amplitude of the primary coil excitation $V_i = 1$ Volts. It is thus related to $\alpha$ by:

$$\sigma = \frac{\alpha}{V_i} \quad (6)$$

The effect of the geometric design parameters on $\sigma$ is shown in Fig. (6). Fig. 6 (a) shows that $\sigma$ increases with $l_s$ and $l_c$, and decreases with $l_p$. The figure also shows that a change in $l_p$ produces a significantly larger change on $\sigma$ than does a similar change on $l_s$ or $l_c$. Additionally, Fig. 6 (b) shows that $\sigma$ increases with $r_c$ and $r_o$, and decreases with $r_i$. The relative influence of these parameters is approximately equal.

5. Conclusions

This work presented an investigation of the effect of varying the six geometric design parameters $l_p$, $l_s$, $l_c$, $r_i$, $r_o$ and $r_c$ on $V_o$, $s$ and $\sigma$ of LVDTs using FEM simulations. Variables were varied one at a time while keeping other parameters fixed. Simulation results show that $V_o (x)$ follows a sinusoidal profile with $x$, that $s$ increases with each of $l_p$, $l_s$, $l_c$, $r_i$, $r_o$, and $r_c$, that $\sigma$ increases with each of $l_s$, $l_c$, $r_o$, and $r_c$, and
decreases rapidly with $l_p$ with $r_i$. Results also show that the $s(l_p)$ curve exhibits a flattening behavior at high values of $l_p$, while the $s(l_c)$ exhibits such a behavior at low value of $l_c$. Simulation results also show that increasing $l_p$ produces a significantly larger drop on $\sigma$ than the gain obtained by increasing $l_c$ or $l_s$, which means that simultaneously increasing both $r_i$ and $r_c$ causes an increase in $\sigma$, as does an increase in $r_o$.

The values of $l_p$ and $l_s$ where the curves tend to flatten are significant design parameters. For example, increasing $l_p$ beyond its flattening value produces little effect on $s$ while adversely influencing $\sigma$ besides increasing space and cost.

This work examined the effect of individually varying six of the eight geometric LVDT parameters, while keeping $w_s$ and $c_s$ constant. Future research efforts may include examining the effects of varying $w_s$ and $c_s$, and also the effect varying the parameters at the same time to see the overall effect on the transfer characteristic. Additionally, the effect of varying non-geometric parameters on the transfer characteristic may be investigated. Of particular importance is the effect of the $f$ on the design.

Acknowledgements

This work was supported by the Deanship of Scientific Research in the University of Jordan, and by the Arab Science and Technology Foundation’s ALJ grant through the project No. MC 06192.

References