*Sensors & Transducers, Vol. 167, Issue 3, March 2014, pp. 151-160*

www.sensorsportal.com

**Sensors & Transducers**



# Resource Allocation Algorithm Based on Bandwidth Demands for OFDMA Systems with Imperfect Channel Quality Information

**[1,2] Zhixin ZHAO, [2] Min JIA, [2] Qing GUO**

[1] College of Electronic and Information Engineering, Heilongjiang University of Science and Technology, No.2468 Puyuan Road, Songbei District, Harbin, 150022, China

[2] Communication Research Center， Harbin Institute of Technology, No. 2 Yikuang Street, Nangang District, Harbin, 150001, China

[1] Tel.: +8618845612192, fax: +8645157601683

[1] E-mail: zhaozhixin0830@hotmail.com

*Received: 5 March 2014 Accepted: 29 March 2014 Published: 31 March 2014*

**Abstract:** For the problem to maximize spectral efficiency of OFDMA-TDD systems while satisfying bandwidth demands and the average bit error rate (BER) constraints of users, a resource allocation algorithm based on bandwidth demands of users is proposed. The user weights are introduced and the Max weighted-SNR scheduling strategy based on bandwidth demands of users is proposed. Based on this strategy，the closed expression of average BER with channel estimation error and Channel Quality Information (CQI) delay is achieved, and above problem is decomposed into the subchannel allocation problem and the problem of rate adaptation threshold vector optimization. The proposed Max weighted-SNR scheduling strategy is used to allocate subchannel to satisfy bandwidth demands of users. By Lagrange multiplier approach, this (*J-1*) dimensional optimization problem of rate adaptation threshold vector is transformed into one-dimensional optimization problem, resulting in optimal solution. This algorithm not only can satisfy bandwidth demands of users，but also adapt to the change of Imperfect Channel Quality Information (ICQI) accuracy and provides the corresponding optimal rate adaptation threshold vector so as to maximize spectral efficiency while satisfying the average BER constraints of users. *Copyright © 2014 IFSA Publishing, S. L.*

**Keywords:** ICQI, Subchannel allocation, Average BER, Rate adaptation thresholds, CQI delay.

## 1. Introduction

In the actual downlink of OFDMA-TDD systems, there must be the channel estimation error due to reasons like the restrictions of pilot overhead, and there is a time delay from the instant of uplink subchannel estimation in base station (BS) to the instant of downlink data transmission using resource allocation scheme based on the estimated SNR on subchannel. In this paper, the SNR on subchannel is used as channel quality information (CQI) to perform adaptive resource allocation and this time delay is called CQI delay. The change of channel state during CQI delay will lead to inaccuracy of CQI. So, CQI obtained by BS is always inaccurate. The inaccurate CQI obtained by BS, i. e., the estimated SNR on subchannel, is called ICQI. The application of adaptive resource allocation with assuming CQI accurate will cause the degradation of performance of systems [1, 2]. The degree of the accuracy of ICQI with respect to the actual channel state at the instant of data transmission, i. e., ICQI accuracy, has a great influence on the performance of systems. Therefore, studying how to design resource allocation algorithm adaptive to ICQI accuracy to ensure user's QoS while maximizing the spectral efficiency of systems is very necessary.

Currently, research on resource allocation for OFDMA systems with ICQI generally only consider the impact of the channel estimation error on the performance of systems [1, 3-7], or only consider the impact of CQI delay on it [8-12]. The authors in [13] analyzed the impact of channel estimation error, CQI delay, the number of CQI quantization bits and CQI feedback errors on throughput of OFDMA-FDD system, but a Max-SNR scheduler was employed so that the bandwidth demands for different users couldn't be fulfilled. So, a resource allocation algorithm based on bandwidth demands of users is proposed. The user weight is introduced and Max weighted-SNR scheduling strategy based on bandwidth demands of users is presented. Based on this strategy, the closed expression of average BER with channel estimation error and CQI delay is achieved, and the problem of resource allocation based on bandwidth demands of users is decomposed into the subchannel allocation problem and the problem of rare adaptation threshold vector optimization and thus is solved. This algorithm can not only meet bandwidth demands of users, but also adapt to the change of ICQI accuracy so as to provide the corresponding optimal rare adaptation threshold vector and maximize spectral efficiency of systems while satisfying average BER constraints for users. As the closed expression of average BER for users is achieved and ICQI accuracy parameters (the variance of channel estimation error and CQI delay) also can be obtained by measurement in BS, the average BER of users can be calculated out analytically. Therefore, this algorithm can be easily applied to the actual TDD system.

## 2. Adaptive Scheduling Model and ICQI

### 2.1. Adaptive Scheduling Model

A one cell SISO OFDMA downlink scenario in an TDD system with one BS and $U$ users with user index $u=1,\cdots,U$ is considered. Furthermore, the downlink bandwidth is subdivided into $K$ orthogonal subchannels, with each subchannel consisting of a group of subcarriers. The channel transfer function $H_{u,k}(n)$ of user $u$ on subchannel $k$ with $k=1,\cdots,K$ at subframe $n$ is modeled by a complex Gaussian distributed random variable with variance one [13] and the mean of $H_{u,k}(n)$ is set to zeros. In frequency domain, $H_{u,k}(n)$ is assumed to be flat on one subchannel and it is assumed that $H_{u,k}(n)$ is uncorrelated to $H_{u,k-1}(n)$ [1, 13]. In time domain, $H_{u,k}(n)$ is assumed to be constant during a subframe. The total transmission power $P_{\mathrm{T}}$ of BS is equally allocated to $K$ subchannels, i.e. the transmission power on each subchannel is $P_{\mathrm{SC}}=P_{\mathrm{T}}/K$. The SNR of user $u$ on subchannel $k$ at subframe $n$ is given by:

$$\gamma_{u,k}(n)=P_{\mathrm{SC}}\cdot\left|H_{u,k}(n)\right|^2\Big/\left(L_{u,\mathrm{P}}\left(B_{\mathrm{SC}}\cdot N_0\right)\right),$$

where $L_{u,\mathrm{P}}$ is path/ shadowing loss of user $u$ assumed to be unchanged with $n$, $N_0$ is the power spectral density of additive white Gaussian noise in the system, and $B_{\mathrm{SC}}$ is the bandwidth of each subchannel. The mean of $\gamma_{u,k}(n)$ is given by:

$$E[\gamma_{u,k}(n)]=P_{\mathrm{SC}}\big/(L_{u,\mathrm{P}}(B_{\mathrm{SC}}\cdot N_0))=\overline{\gamma}_u,$$

with $E[\left|H_{u,k}(n)\right|^2]=1$ and $E[\gamma_{u,k}(n)]$ being independent of $n$ and $k$. Let $\overline{\gamma}_u$ denote the mean of SNR on subchannel of user $u$ and then

$$\gamma_{u,k}(n)=\overline{\gamma}_u\cdot\left|H_{u,k}(n)\right|^2.$$

Firstly, using the reciprocity of uplink and downlink channels in TDD systems, BS obtains the estimated SNR of user on each downlink subchannel through estimating the SNR on each uplink subchannel, i. e. BS obtains ICQI of user. Then, the resource scheduler in BS assigns different subchannles to different users and then adapts the transmit rate of each subchannel allocated to the scheduled user according to the bandwidth demand vector $\boldsymbol{\lambda}=[\lambda_1,\cdots,\lambda_u,\cdots,\lambda_U]$ and ICQI. Finally, the data symbols of the scheduled user are modulated by inverse fast Fourier transform, cyclic prefix is inserted and then the data symbols are transmitted to the user. $\lambda_u$ （$0\le\lambda_u\le1$） denotes the bandwidth demands factor of user $u$, representing the ratio of bandwidth demands of user $u$ to downlink bandwidth. Resource allocation and CQI are updated on a one subframe cycle.

### 2.2. ICQI

When $\overline{\gamma}_u$ is a constant value, $\gamma_{u,k}(n)$ is determined only by $H_{u,k}(n)$. For convenience of expression, we use $H_{u,k}(n)$ instead of $\gamma_{u,k}(n)$ to analyze ICQI and subframe index is omitted hereinafter. Subchannel index is omitted in this section.

#### 2.2.1. The Channel Estimation Error

Let $H_u^{(E)}$ denote the channel transfer function of user $u$ on subchannel at the instant of channel estimation. The estimate of $H_u^{(E)}$ is denoted by $\hat{H}_u$ and the estimation error of user $u$ by

$$E_u = H_u^{(E)} - \hat{H}_u$$

The random variables $\hat{H}_u$ and $E_u$ are uncorrelated [13], where $E_u$ and $\hat{H}_u$ are complex Gaussian distributed with mean 0 and with variance $\sigma^2_{\mathrm{E},u}$ and $1-\sigma^2_{\mathrm{E},u}$ respectively. Using orthogonal pilot signal to estimate subchannel, the variance of $E_u$ is

$$\sigma^2_{\mathrm{E},u} = 1/(1+N_{\mathrm{r},u}\gamma_{\mathrm{S},u}),$$

where $N_{\mathrm{r,u}}$ is the number of training symbols and $\gamma_{\mathrm{S},u}$ is SNR of pilot signals for user $u$ [13]. Let $\gamma_{\mathrm{S},u} = \bar{\gamma}_u$ [13], and then:

$$\sigma^2_{\mathrm{E},u} = 1/(1+N_{\mathrm{r},u}\bar{\gamma}_u) \cdot \quad (1)$$

#### 2.2.2. Outdated CQI

Due to CQI delay, the available CQI at BS is outdated. Outdated CQI is modelled by correlation, i.e. $H_u^{(E)}$ and the actual channel transfer function of user $u$ on subchannel at the instant of transmission denoted by $H_u$ are modeled as two complex Gaussian distributed random variables with a correlation coefficient $\rho_u$. Assuming Jakes scattering model, the correlation coefficient [13]

$$\rho_u = J_0\left(2\pi f_{\mathrm{D},u}\tau\right),$$

with $J_0(x)$ denoting the 0-th order Bessel function, $f_{\mathrm{D},u}$ denoting the maximum Doppler frequency of user $u$, $\tau$ denoting CQI delay and $f_{\mathrm{D},u}\tau$ denoting normalized Doppler frequency of user $u$.

## 3. Resource Allocation Based on Bandwidth Demands of Users

### 3.1. Problem of Resource Allocation Based on Bandwidth Demands of Users

The spectral efficiency of systems shall be maximized over the user weights vector $\boldsymbol{w}$ and the rate adaptation threshold vector $\boldsymbol{s}^{(u)}$ $(u=1,2,\cdots,U)$ subject to the bandwidth demands and the average BER constraint of users

$$\max_{\boldsymbol{w},\boldsymbol{s}^{(u)}} \sum_{u=1}^{U} P_u(\boldsymbol{w}) \cdot \bar{R}_u\left(\boldsymbol{w},\boldsymbol{s}^{(u)}\right), \quad (2)$$

subject to:

C1: $\overline{BER}_u\left(\boldsymbol{w},\boldsymbol{s}^{(u)}\right) \le \overline{BER}_{u,0}, \forall u \in \{1,2,\cdots,U\}$

C2: $P_u(\boldsymbol{w}) = \lambda_u, \forall u \in \{1,2,\cdots,U\}$.

The user weights vector is given by $\boldsymbol{w} = [w_1, w_2, \cdots, w_U]$ with $w_u$ denoting the weights of user $u$, the rate adaptation threshold vector of user $u$ is given by $\boldsymbol{s}^{(u)} = [s_0^{(u)}, \cdots s_j^{(u)}, \cdots, s_J^{(u)}]$ with $s_0^{(u)} = 0$, $s_J^{(u)} = \infty$ and $J$ denoting the number of available modulation schemes. Since $U$ different users are competing for the subchannels, the probability of user $u$ to get access to a subchannel, i. e. the channel access probability $P_u(\boldsymbol{w})$ of user $u$, depends on $\boldsymbol{w}$. $\overline{BER}_u\left(\boldsymbol{w},\boldsymbol{s}^{(u)}\right)$ and $\overline{BER}_{u,0}$ denote the average BER and average BER constraint of user $u$ respectively. $\bar{R}_u(\boldsymbol{w},\boldsymbol{s}^{(u)})$ denotes spectral efficiency of user $u$. Constraint C1 ensures that the average BER of user $u$ is not more than its average BER constraint $\overline{BER}_{u,0}$. Constraint C2 ensures that the channel access probability is equal to the bandwidth demands factor of user $u$ to satisfy the bandwidth demands of user $u$.

### 3.2. Subchannel Allocation Based on Bandwidth Demands of Users

#### 3.2.1. Max Weighted-SNR Scheduling Strategy (MSSS)

In many subframes, the actual average bandwidth allocated to user $u$ is given by $B_u = B_{\mathrm{SC}} \cdot K \cdot P_u(\boldsymbol{w})$. To make $B_u$ equal to the bandwidth demands of user $u$ given by $B_{\mathrm{SC}} \cdot K \cdot \lambda_u$, i.e. to make constraint C2 satisfied, besides the estimated SNR of users on subchannel, the user weights should also be used to carried out subchannel allocation so as to achieve the purpose of proportional allocation of bandwidth to different users according to the user weights vector $\boldsymbol{w}$. Thus, Max weighted-SNR Scheduling Strategy (MSSS) based on bandwidth demands of users is proposed, i.e. subchannel $k$ is allocated to the user with the maximal weighted and estimated SNR on this subchannel:

$$u^*(k) = \arg\max_u \left(w_u \cdot \hat{\gamma}_{u,k}\right) \cdot \quad (3)$$

### 3.2.2. The Channel Access Probability Under MSSS

As the estimated SNRs for one user on different subchannels are identically distributed, subchannel index is omitted below. The probability density function (PDF) of the estimated SNR of user $u$ on subchannel is exponential distribution with the mean $\bar{\gamma}_{\mathrm{E},u} = E[\hat{\gamma}_u] = \bar{\gamma}_u\left(1-\sigma^2_{\mathrm{E},u}\right)$ taking the relation of $\hat{H}_u$ and $E_u$ into account.

Since the estimated SNRs of different users on the one subchannel are independent distributed, the joint PDF of estimated SNR of all $U$ users on the one subchannel is given by:

$$p_{\hat{\gamma}_1,\hat{\gamma}_2,\cdots,\hat{\gamma}_U}(\hat{\gamma}_1,\hat{\gamma}_2,\cdots,\hat{\gamma}_U) = p_{\hat{\gamma}_1}(\hat{\gamma}_1)\cdot p_{\hat{\gamma}_2}(\hat{\gamma}_2)\cdots p_{\hat{\gamma}_U}(\hat{\gamma}_U)$$

with $p_{\hat{\gamma}_u}(\hat{\gamma}_u)$ denoting the PDF of estimated SNR of user $u$ on subchannel. With the MSSS of (3), if a subchannel is allocated to user $u$, all the $U-1$ other users must have smaller weighted and estimated SNR on the subchannel, so the estimated SNR $\hat{\gamma}_u$ of user $u$ on the subchannel fulfills following in equation:

$$\hat{\gamma}_u > (w_i\cdot\hat{\gamma}_i)/w_u\,,\ \forall i \neq u\,, \tag{4}$$

The channel access probability of user $u$

$$\begin{aligned} P_u(\boldsymbol{w}) &= \int_{\hat{\gamma}_u=0}^{\infty}\int_{\hat{\gamma}_1=0}^{\frac{w_u\hat{\gamma}_u}{w_1}}\cdots\int_{\hat{\gamma}_{u-1}=0}^{\frac{w_u\hat{\gamma}_u}{w_{u-1}}}\int_{\hat{\gamma}_{u+1}=0}^{\frac{w_u\hat{\gamma}_u}{w_{u+1}}}\cdots\int_{\hat{\gamma}_U=0}^{\frac{w_u\hat{\gamma}_u}{w_U}} p_{\hat{\gamma}_1}(\hat{\gamma}_1) \\ &\cdot p_{\hat{\gamma}_2}(\hat{\gamma}_2)\cdots p_{\hat{\gamma}_U}(\hat{\gamma}_U)d_{\hat{\gamma}_U}\cdots d_{\hat{\gamma}_{u+1}}d_{\hat{\gamma}_{u-1}}d_{\hat{\gamma}_1}d_{\hat{\gamma}_u} \\ &= \int_0^{\infty}\frac{1}{\bar{\gamma}_{\mathrm{E},u}}\exp\left(-\frac{\hat{\gamma}_u}{\bar{\gamma}_{\mathrm{E},u}}\right)\cdot\prod_{\substack{i=1\\i\neq u}}^{U}\left(1-\exp\left(\frac{w_u\hat{\gamma}_u}{w_i\bar{\gamma}_{\mathrm{E},i}}\right)\right)d_{\hat{\gamma}_u} \end{aligned} \tag{5}$$

Performing some transformations to the second term of the integral, the channel access probability of user $u$ is given by:

$$\begin{aligned} P_u(\boldsymbol{w}) &= \int_0^{\infty}\sum_{j=1}^{U}(-1)^{j-1}\sum_{|\boldsymbol{\theta}|=j-1}\frac{1}{\bar{\gamma}_{\mathrm{E},u}}\exp\left[-\left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}}+\sum_{\substack{i=1\\i\neq u}}^{U}\theta_i\frac{w_u}{w_i\bar{\gamma}_{\mathrm{E},i}}\right)\hat{\gamma}_u\right]d_{\hat{\gamma}_u} \\ &= \sum_{j=1}^{U}(-1)^{j-1}\sum_{|\boldsymbol{\theta}|=j-1}\left(1+\sum_{\substack{i=1\\i\neq u}}^{U}\theta_i\frac{w_u\bar{\gamma}_{\mathrm{E},u}}{w_i\bar{\gamma}_{\mathrm{E},i}}\right)^{-1}, \end{aligned} \tag{6}$$

with the vector $\boldsymbol{\theta} = [\theta_1,\theta_2,\cdots,\theta_U]$ with $\theta_i \in \{0,1\}\,\forall i\neq u$. The equation $|\boldsymbol{\theta}| = j-1$ denotes $C_{U-1}^{j-1}$ possible vectors of $\boldsymbol{\theta}$ when the number of 1 in vector $\boldsymbol{\theta}$ is equal to $j-1$. $\sum_{|\boldsymbol{\theta}|=j-1} f(\boldsymbol{\theta})$ denotes the sum of $C_{U-1}^{j-1}$ function values $f(\boldsymbol{\theta})$ corresponding to a specific vector $\boldsymbol{\theta}$.

## 3.3. Spectral Efficiency and Average BER Taking ICQI into Account

### 3.3.1. The Distribution of the Estimated SNR of a Subchannel Allocated to User

According to MSSS, the estimated SNR of a subchannel allocated to user $u$ fulfills (4) by which the upper limit of the integral can be determined. The PDF of the estimated SNR $\hat{\gamma}$ of a subchannel allocated to user $u$ is then the marginal PDF calculated by determining the integral over the joint PDF $p_{\hat{\gamma}_1,\hat{\gamma}_2,\cdots,\hat{\gamma}_U}(\hat{\gamma}_1,\hat{\gamma}_2,\cdots,\hat{\gamma}_U)$ leading to:

$$\begin{aligned} p_{u,\hat{\gamma}}(\hat{\gamma}) &= a_u\int_{\hat{\gamma}_1=0}^{\frac{w_u\hat{\gamma}}{w_1}}\cdots\int_{\hat{\gamma}_{u-1}=0}^{\frac{w_u\hat{\gamma}}{w_{u-1}}}\int_{\hat{\gamma}_{u+1}=0}^{\frac{w_u\hat{\gamma}}{w_{u+1}}}\cdots\int_{\hat{\gamma}_U=0}^{\frac{w_u\hat{\gamma}}{w_U}} p_{\hat{\gamma}_1}(\hat{\gamma}_1) \\ &\cdots p_{\hat{\gamma}_{u-1}}(\hat{\gamma}_{u-1})p_{\hat{\gamma}}(\hat{\gamma})p_{\hat{\gamma}_{u+1}}(\hat{\gamma}_{u+1})\cdots p_{\hat{\gamma}_U}(\hat{\gamma}_U) \\ &\quad d_{\hat{\gamma}_U}\cdots d_{\hat{\gamma}_{u+1}}d_{\hat{\gamma}_{u-1}}\cdots d_{\hat{\gamma}_1} \\ &= a_u\cdot\sum_{j=1}^{U}(-1)^{j-1}\sum_{|\boldsymbol{\theta}|=j-1}\frac{1}{\bar{\gamma}_{\mathrm{E},u}} \\ &\cdot\exp\left[-\left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}}+\sum_{\substack{i=1\\i\neq u}}^{U}\theta_i\frac{w_u}{w_i\bar{\gamma}_{\mathrm{E},i}}\right)\hat{\gamma}\right]. \end{aligned} \tag{7}$$

where the factor $a_u$ ensures that:

$$\int_0^{\infty} p_{u,\hat{\gamma}}(\hat{\gamma})d_{\hat{\gamma}} = 1\,, \tag{8}$$

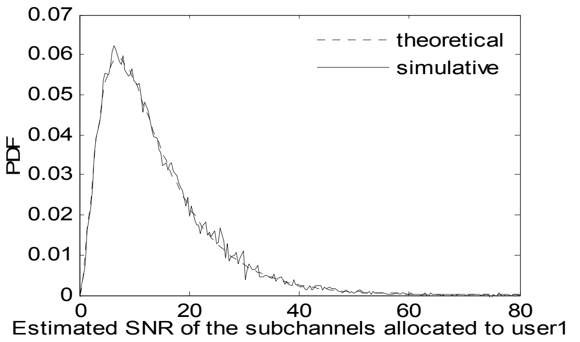
Substituting $\hat{\gamma}$ for $\hat{\gamma}_u$ in (5), it can be seen that the integrals in (5) and (8) are identical except for the factor $a_u$, leading to:

$$a_u = 1/P_u(\boldsymbol{w})\,, \tag{9}$$

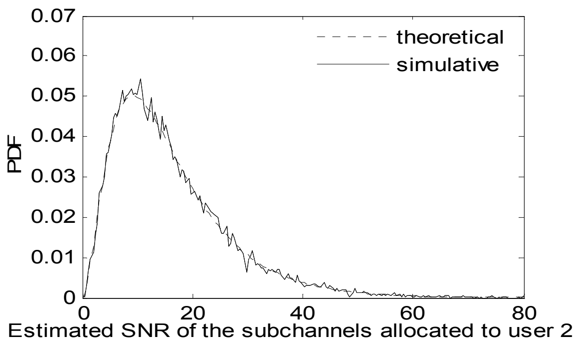
Inserting (9) into (7) leads to:

$$\begin{aligned} p_{u,\hat{\gamma}}(\hat{\gamma}) &= \frac{1}{P_u(\boldsymbol{w})}\cdot\sum_{j=1}^{U}(-1)^{j-1}\sum_{|\boldsymbol{\theta}|=j-1}\frac{1}{\bar{\gamma}_{\mathrm{E},u}} \\ &\cdot\exp\left[-\left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}}+\sum_{\substack{i=1\\i\neq u}}^{U}\theta_i\frac{w_u}{w_i\bar{\gamma}_{\mathrm{E},i}}\right)\hat{\gamma}\right] \end{aligned}, \tag{10}$$

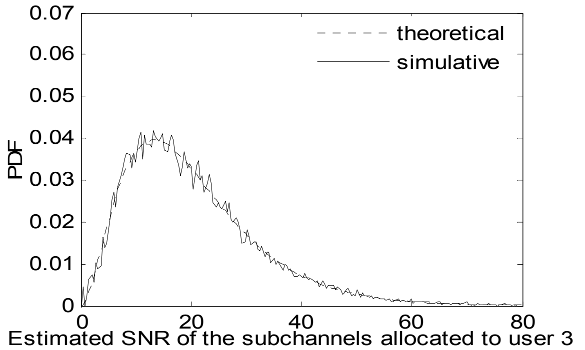
Let us assume a system with the user weights vector of $\boldsymbol{w} = [0.75, 0.5, 0.25]$ and $U$=3 users where all users have the same mean of SNR on subchannel $\bar{\gamma}_u = 10\,\text{dB}$ and the same number of training symbols $N_{\mathrm{r},u} = 1$. In Fig. 1 (a), the PDF of the estimated SNR of the subchannels allocated to user 1 is depicted. The dashed curve represents the theoretical PDF according to (10) and the solid lines represent the PDF evaluated from 10000 simulation runs. Fig. 1(b) and Fig. 1(c) the PDFs for user 2 and user 3 is depicted.




(a)




(b)




(c)

**Fig. 1.** The distribution of the estimated SNR of the subchannels allocated to user. The PDF of estimated SNR of the subchannels allocated to: (a) user 1; (b) user 2; (c) user 3.

One can see that the theoretical PDFs are consistent with the simulative ones. Furthermore, it can be seen that the probability of small estimated SNR is larger for user 1 than for user 2 and user 3. The reason for that is the higher weights of user 1 compared to users 2 and user 3, i.e., in order to successfully compete against the other users, the estimated SNR of user 1 does not have to be as high due to the highest weights of user 1 while for user 3, the estimated SNR have to be rather high in order to be considered for allocation.

### 3.3.2. Spectral Efficiency

According to the estimated SNR $\hat{\gamma}$ of the subchannels allocated to user $u$ and rate adaptation threshold vector $\boldsymbol{s}^{(u)} = [s_0^{(u)}, s_1^{(u)}, \cdots, s_J^{(u)}]$, the resource scheduler in BS adapts the transmit rate of each subchannel allocated to user $u$, i.e., the $j$-th modulation scheme is selected and used for transmission on the subchannel whenever $s_{j-1}^{(u)} \le \hat{\gamma} < s_j^{(u)}$. Let $c_j$ denote the number of transmitted bits on a subcarrier during a OFDM symbol when $j$-th modulation scheme is selected and let $c_j = j-1$. The spectral efficiency of user $u$ is defined by [3, 9, 14, 15].

$$\begin{aligned}\bar{R}_u\left(\boldsymbol{w}, \boldsymbol{s}^{(u)}\right) &= \sum_{j=1}^{J} \int_{s_{j-1}^{(u)}}^{s_j^{(u)}} c_j \cdot p_{u,\hat{\gamma}}(\hat{\gamma})\, d_{\hat{\gamma}}, \\ &= \sum_{j=1}^{J} c_j \cdot \left[G_{u,\hat{\gamma}}\left(s_j^{(u)}\right) - G_{u,\hat{\gamma}}\left(s_{j-1}^{(u)}\right)\right],\end{aligned} \tag{11}$$

with

$$G_{u,\hat{\gamma}}(x) = \frac{1}{P_u(\boldsymbol{w})} \cdot \sum_{j=1}^{U} (-1)^{j-1} \sum_{|\boldsymbol{\theta}|=j-1} \frac{1}{1 + \sum\limits_{\substack{i=1\\ i\neq u}}^{U} \theta_i \frac{w_u \bar{\gamma}_{\mathrm{E},u}}{w_i \bar{\gamma}_{\mathrm{E},i}}} \cdot \left\{1 - \exp\left[-\left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}} + \sum_{\substack{i=1\\ i\neq u}}^{U} \theta_i \frac{w_u}{w_i \bar{\gamma}_{\mathrm{E},i}}\right) x\right]\right\}.$$

### 3.3.3. Average BER

According to [3, 9, 14, 15], the average BER of user $u$ is defined by

$$\overline{BER_u}\left(\boldsymbol{w}, \boldsymbol{s}^{(u)}\right) = \frac{\sum_{j=1}^{J} \int_{s_{j-1}^{(u)}}^{s_j^{(u)}} c_j \cdot \widehat{BER_u}(\hat{\gamma}, j)\, p_{u,\hat{\gamma}}(\hat{\gamma})\, d_{\hat{\gamma}}}{\bar{R}_u\left(\boldsymbol{w}, \boldsymbol{s}^{(u)}\right)}, \tag{12}$$

where, according to [9, 15], $\widehat{BER_u}(\hat{\gamma}, j)$ denotes the average BER as a function of the applied $j$-th modulation scheme and $\hat{\gamma}$ given by:

$$\begin{aligned}\widehat{BER}_u(\hat{\gamma}, j) &= E_{\gamma|\hat{\gamma}}[BER(\gamma, j)] \\ &= \int_0^\infty BER(\gamma, j) p_{u,\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) d_\gamma\end{aligned}, \quad (13)$$

where $BER(\gamma, j)$ denotes the BER of the applied $j$-th modulation scheme and $p_{u,\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ denotes the conditional PDF of the actual SNR $\gamma$ at the instant of data transmission and the estimated SNR $\hat{\gamma}$ on a subchannel for user *u*. In the following, the expression of $p_{u,\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ will be given.

According to [13,16], the relation of $H_u$ and $\hat{H}_u$ can be modeled by

$$H_u = \rho_u \cdot (\hat{H}_u + E_u) + \sqrt{1-\rho_u^2} \cdot X, \quad (14)$$

with $X$ denoting a complex Gaussian distributed random variable with variance one and mean zeros and is independent of $\hat{H}_u$ and $E_u$. The conditional PDF of $H_u$ and $\hat{H}_u$ is given by:

$$p_{H_u|\hat{H}_u}\left(H_u \middle| \hat{H}_u\right) = CN(\rho_u \hat{H}_u, \sigma_{\mathrm{r},u}^2), \quad (15)$$

with $CN(\rho_u \hat{H}_u, \sigma_{\mathrm{r},u}^2)$ denoting the complex Gaussian distribution with mean $\rho_u \hat{H}_u$ and variance $\sigma_{\mathrm{r},u}^2 = 1-\rho_u^2(1-\sigma_{\mathrm{E},u}^2)$. $p_{H_u|\hat{H}_u}\left(H_u \middle| \hat{H}_u\right)$ reflects the degree of the accuracy of the estimated channel transfer function $\hat{H}_u$ relative to $H_u$ [1, 7], i.e. ICQI accuracy, determined by $\sigma_{\mathrm{E},u}^2$ and $f_{\mathrm{D},u}\tau$ ,which are called the parameter of ICQI accuracy.

According to (15) and the book [17], the conditional PDF $p_{u,\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ is a Non-central chi-square distribution with 2 degree of freedom given by

$$p_{u,\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{1}{\bar{\gamma}_u \sigma_{\mathrm{r},u}^2} \exp\left(-\frac{\hat{\gamma}\rho_u^2 + \gamma}{\bar{\gamma}_u \sigma_{\mathrm{r},u}^2}\right) I_0\left(\frac{2\rho_u \sqrt{\hat{\gamma}\gamma}}{\bar{\gamma}_u \sigma_{\mathrm{r},u}^2}\right), \quad (16)$$

with $I_0(x)$ denoting the 0-order modified Bessel function of first kind.

The BER $BER(\gamma, j)$ of M-QAM modulation can be approximated by:

$$BER(\gamma, j) = 0.2 \cdot \exp(-\beta_j \gamma), \quad (17)$$

with $\beta_j = 1.6/(2^{c_j}-1)$ using M-QAM modulation and $\beta_j = 7/(2^{1.9c_j}+1)$ using M-PSK modulation [14]. For special cases, $BER(\gamma,1)$ equals average BER constraint with $j$=1 and $\beta_2 = 1$ with $j$=2.

Inserting (16) and (17) in (13) leads to:

$$\widehat{BER}_u(\hat{\gamma}, j) = \frac{0.2}{\beta_j \bar{\gamma}_u \sigma_{\mathrm{r},u}^2 + 1} \cdot \exp\left(-\frac{\beta_j \rho_u^2 \cdot \hat{\gamma}}{\beta_j \bar{\gamma}_u \sigma_{\mathrm{r},u}^2 + 1}\right), \quad (18)$$

Inserting (10) and (18) in (12) leads to

$$\overline{BER}_u\left(\boldsymbol{w}, \boldsymbol{s}^{(u)}\right) = \frac{\sum_{j=1}^{J} \frac{0.2 \cdot c_j}{\beta_j \bar{\gamma}_u \sigma_{\mathrm{r},u}^2 + 1}\left[F_u\left(s_j^{(u)}\right) - F_u\left(s_{j-1}^{(u)}\right)\right]}{\bar{R}_u\left(\boldsymbol{w}, \boldsymbol{s}^{(u)}\right)}, \quad (19)$$

With

$$F_u(x) = \frac{1}{P_u(\boldsymbol{w})} \sum_{v=1}^{U} (-1)^{v-1} \sum_{|\boldsymbol{\theta}|=v-1} \frac{1}{1 + \sum_{\substack{i=1 \\ i \neq u}}^{U} \theta_i \frac{\bar{\gamma}_{\mathrm{E},u} w_u}{w_i \gamma_{\mathrm{E},i}} + \frac{\bar{\gamma}_{\mathrm{E},u} \beta_j \rho_u^2}{\beta_j \bar{\gamma}_u \sigma_{\mathrm{r},u}^2 + 1}} \cdot \left\{1 - \exp\left[-\left(\frac{1}{\gamma_{\mathrm{E},u}} + \sum_{\substack{i=1 \\ i \neq u}}^{U} \theta_i \frac{w_u}{w_i \gamma_{\mathrm{E},i}} + \frac{\beta_j \rho_u^2}{\beta_j \bar{\gamma}_u \sigma_{\mathrm{r},u}^2 + 1}\right) x\right]\right\}.$$

### 3.4. Resource Allocation Algorithm Based on Bandwidth Demands of Users

#### 3.4.1. Decomposing the Problem of Resource Allocation based on Bandwidth Demands of Users into Two Problems

According to (6), $P_u(\boldsymbol{w})$ is dependent on $\boldsymbol{w}$ and independent of $\boldsymbol{s}^{(u)} (u = 1, 2, \cdots, U)$, so we only need to find out the user weights vector $\boldsymbol{w}^*$ making constraint C2 satisfied. For getting the unique $\boldsymbol{w}^*$ to fulfill constraint C2, the weights of the user with the highest bandwidth demands factor is set to 1.

With the given $\boldsymbol{w}^*$, $\bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right)$ and $\overline{BER}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right)$ are only dependent on $\boldsymbol{S}^{(u)}$. Thus, for each user *u*, the spectral efficiency of user *u* can be maximized over the its own rate adaptation threshold vector $\boldsymbol{S}^{(u)}$ subject to the average BER constraint of user *u*. when the spectral efficiency of each user reaches the maximum simultaneously, the spectral efficiency of systems is also reaches the maximum. So, the problem of resource allocation based on bandwidth demands of users can be done by solving the following two problems:

*Subchannel allocation problem*: to fulfill the bandwidth demands of all users, with MSSS, finding out the user weights vector $\boldsymbol{w}^*$ leads to

$$P_u(\boldsymbol{w}^*) = \lambda_u, \forall u \in \{1, 2, \cdots, U\}, \quad (20)$$

*The problem of rate adaptation threshold vector optimization*: According to $\boldsymbol{w}^*$ obtained from (20), for each user $u$, maximize the spectral efficiency of user $u$ over its own rate adaptation threshold vector $\boldsymbol{S}^{(u)}$ subject to the average BER constraint of user $u$

$$\max_{\boldsymbol{s}^{(u)}} \bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right), \tag{21}$$

subject to : $\overline{BER}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right) \le \overline{BER}_{u,0}$.

### 3.4.2. Subchannel Allocation Problem

Without loss of generality, let $w_U$=1. Subchannel allocation problem (20) can be done by solving the following constrained nonlinear optimization problem:

$$\boldsymbol{w}^* = \arg\min_{\boldsymbol{w}} \sum_{u=1}^{U-1} \left|P_u(\boldsymbol{w}) - \lambda_u\right|$$

subject to : $0 < w_u \le 1,\ \forall u \in \{1, 2, \cdots, U-1\}$,
which can be solved easily using numerical methods.

### 3.4.3. The problem of Rate Adaptation Threshold Vector Optimization

To solve (21), a Lagrange multiplier approach similar to the method in [9], is performed where the objective function is given by:

$$\begin{aligned}\Phi_u(\boldsymbol{s}^{(u)}) &= \bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right) + \mu\Big[\bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right) \\ &\quad \cdot \overline{BER}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right) - \bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right)\overline{BER}_{u,0}\Big], \\ &= \left(1 - \mu\overline{BER}_{u,0}\right)\bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right) \\ &\quad + \mu\bar{R}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right)\overline{BER}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)}\right),\end{aligned} \tag{22}$$

with $\mu$ denoting the Lagrange multiplier. The optimal rate adaptation threshold vector $\boldsymbol{S}^{(u)*}$ should satisfy :

$$\left.\frac{\partial \Phi_u(\boldsymbol{s}^{(u)})}{\partial s_j^{(u)}}\right|_{\boldsymbol{s}^{(u)} = \boldsymbol{s}^{(u)*}} = 0,\ j = 1, 2, \cdots, J-1, \tag{23}$$

and

$$\overline{BER}_u\left(\boldsymbol{w}^*, \boldsymbol{s}^{(u)*}\right) - \overline{BER}_{u,0} = 0, \tag{24}$$

Inserting (22) in (23) results in the following equation given by:

$$\widehat{BER}_u(s_1^{(u)}, 2) = \frac{\left[c_{j+1}\widehat{BER}_u(s_j^{(u)}, j+1) - c_j\widehat{BER}_u(s_j^{(u)}, j)\right]}{c_{j+1} - c_j}, \quad j = 2, 3, \cdots, J-1. \tag{25}$$

From (25), it can be seen that each element $s_j^{(u)} (j = 2, 3, \cdots, J-1)$ of $\boldsymbol{S}^{(u)*}$ can be calculated using an initial value $s_1^{(u)}$. Therefore, the optimal threshold vector $\boldsymbol{s}^{(u)*}$ is completely determined by $s_1^{(u)}$, i.e. $\boldsymbol{s}^{(u)*} = f(s_1^{(u)})$, where $s_1^{(u)}$ is selected to satisfy the constraint in (21). Thus the ($J$ − 1)-dimensional problem of rate adaptation threshold vector optimization (21) is converted into a one-dimensional optimization problem given by:

$$\max_{s_1^{(u)}} \bar{R}_u\left(\boldsymbol{w}^*, f(s_1^{(u)})\right),$$

subject to : $\overline{BER}_u\left(\boldsymbol{w}^*, f(s_1^{(u)})\right) \le \overline{BER}_{u,0}$,

which can be solved easily using numerical methods.

## 4. Numerical Results

In this section, we will present numerical results for a OFDMA-TDD system with $U = 4$ users and assumes that all users have the same average BER constraints $10^{-3}$ and the same number of training symbols $N_{\mathrm{r},u} = 1$. The following modulation schemes are considered: $M_0$, BPSK, QPSK, 8 PSK, 16 QAM, 32 QAM, 64 QAM and 128 QAM with $M_0$ denoting no transmission. It is also assumed that $\sigma_{\mathrm{E},u}^2$ and $f_{\mathrm{D},u}\tau$ is known to BS. When the average BER constraint of user $u$ can't be satisfied, the proposed algorithm lets $\bar{R}_u(\boldsymbol{w}, \boldsymbol{s}^{(u)}) = 0$ and $\overline{BER}_u(\boldsymbol{w}, \boldsymbol{s}^{(u)}) = \overline{BER}_{0,u}$.

### 4.1. Impact of the User Weights on the Channel Access Probability

Assuming all users have the same mean of SNR $\bar{\gamma}_u = 10\,\mathrm{dB}$ on subchannel and making the weights $w_1$ of user 1 change from 0.1 to 0.8 with $w_2$=1, $w_3$=0.2 and $w_4$=0.3, the channel access probability is depicted versus the weights $w_1$ of user 1 in Fig. 2.

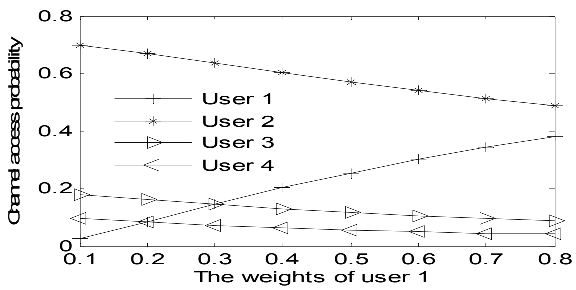



**Fig. 2.** The channel access probability vs. the weights $w_1$ of user 1.

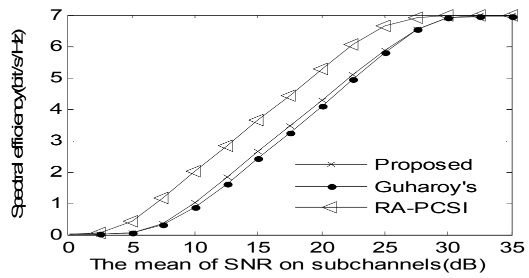
It can be seen that as $w_1$ increases, the channel access probability of user 1 also increases and the channel access probability of 3 other users decreases simultaneously. This behavior shows that MSSS can proportionally allocate required bandwidth to users by adjusting the user weights vector $\boldsymbol{w}$.

### 4.2. Comparison with Other Algorithms

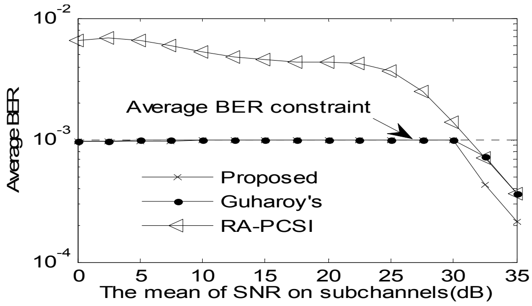
The authors in [12] proposed a rate adaptation threshold vector optimization algorithm based on single parameter $\theta$ with assuming perfect channel estimation is available at BS. To ensure fair comparison, using (11) and (19) to achieve the optimal parameter $\theta$ through simulation, for different normalized Doppler frequency $f_{\mathrm{D},u}\tau$ and different mean of SNR on subchannel. In this paper, the resource allocation algorithm based on MSSS and the rate adaptation threshold vector optimization algorithm [12] is called Guharoy's algorithm. To compare with Guharoy's algorithm, let λ = [0.25, 0.25, 0.25, 0.25]. Moreover, Resource Allocation algorithm with Perfect CQI (RA-PCQI) is also involved, which is based on MSSS and the fixed rate adaptation threshold vector [0, 5.30, 11.30, 23.18, 49.67, 102.65, 208.62, 420.55, ∞].

Assuming all users have the same mean of SNR on subchannel where $\bar{\gamma}_u = \bar{\gamma}_v, \forall u, v \in \{1, 2, \cdots, U\}$, Fig. 3 illustrates the performance of the proposed algorithm, Guharoy's algorithm and RA-PCQI versus the mean of SNR on subchannel for normalized Doppler frequency $f_{\mathrm{D},u}\tau = 0.1$ with $\forall u \in \{1, 2, \cdots, U\}$. For the proposed algorithm, with the mean of SNR on subchannel increasing, i.e. with ICQI accuracy increasing, the average BER of systems maintains at the average BER constraint $10^{-3}$ until the spectral efficiency of systems reaches the maximum, and then the average BER decreases. This behavior results from the fact that the optimal rate adaptation threshold vector is obtained on the boundary of the average BER constraint C1 in (2) and the average BER decreases when the highest modulation order (128QAM) is selected on subchannels. Simulation results show that the proposed algorithm can adapt to the change of the parameter $\sigma^2_{\mathrm{E},u}$ of ICQI accuracy (determined by $\bar{\gamma}_u$ ,see (1)) and provide the corresponding optimal rate adaptation threshold vector resulting in the average BER of user maintaining at the average BER constraint $10^{-3}$ so as to maximize spectral efficiency of users.

Assuming all users have the same normalized Doppler frequency where $f_{\mathrm{D},u}\tau = f_{\mathrm{D},v}\tau, \ \ \forall u, v \in \{1, 2, \cdots, U\}$, Fig. 4 illustrates the performance of the proposed algorithm, Guharoy's algorithm and RA-PCQI with respect to normalized Doppler frequency for the mean of SNR on subchannel $\bar{\gamma}_u = 15\mathrm{dB}$ with $\forall u \in \{1, 2, \cdots, U\}$.

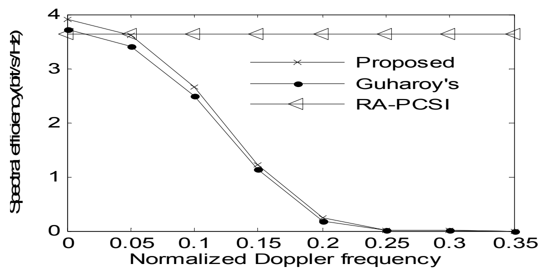



(a)

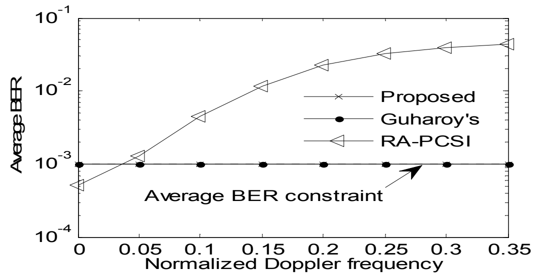



(b)

**Fig. 3.** Performance of 3 algorithms vs. the mean of SNR on subchannel. (a) Spectral efficiency of systems for different algorithms vs. the mean $\bar{\gamma}_u$ of SNR on subchannel; (b) Average BER of systems for different algorithms vs. the mean $\bar{\gamma}_u$ of SNR on subchannel.




(a)




(b)

**Fig. 4.** Performance of 3 algorithms vs. normalized Doppler frequency $f_{\mathrm{D},u}\tau$. (a) Spectral efficiency of systems for different algorithms vs. normalized Doppler frequency $f_{\mathrm{D},u}\tau$; (b) Average BER of systems for different algorithms vs. normalized Doppler frequency $f_{\mathrm{D},u}\tau$.

For the proposed algorithm, in order to ensure the average BER constraint is satisfied, with the normalized Doppler frequency increasing, i.e. with ICQI accuracy decreasing, the threshold value becomes high and the probability of selection of the low-order modulation scheme increases, so the spectral efficiency of the system decreases with the average BER maintaining at average BER constraint $10^{-3}$. Simulation results show that the proposed algorithm can adapt to the change of the parameter $f_{\mathrm{D},u}\tau$ of ICQI accuracy and provide the corresponding optimal rate adaptation threshold vector resulting in the average BER of user maintaining at average BER constraint $10^{-3}$ so as to maximize spectral efficiency of users.

Fig. 3 and Fig. 4 show that the average BER constraint can be fulfilled for both the proposed and Guharoy's algorithm, but the spectral efficiency of the proposed algorithm is higher than that of Guharoy's algorithm. As RA-PCQI assumes CQI accurate, the average BER constraint can't be satisfied. Thus, the proposed algorithm is better than Guharoy's algorithm and RA-PCQI.

## 5. Conclusion

The resource allocation algorithm based on bandwidth demands of users is proposed. This algorithm can not only meet bandwidth demands of users, but also adapt to the change of ICQI accuracy so as to provide the corresponding optimal rate adaptation threshold vector and maximize spectral efficiency while meeting the average BER constraint of users. As the closed expression of average BER for user is achieved and ICQI accuracy parameters ( $f_{\mathrm{D},u}\tau$ and $\sigma_{\mathrm{E},u}^{2}$ ) can be obtained by measurement at BS, the average BER of users can be calculated out analytically. Therefore, this algorithm can be easily applied to the actual TDD system.

## Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant No. 61201143 and Innovation Foundations of CAST (ITS) (Grant No.F-W-YY-2013-016).

## References

[1]. S. Stefanntos, C. Papathanasiou, N. Dimitriou, Downlink Mobile OFDMA Resource Allocation with Minimum User Rate Requests, in *Proceedings of Global Telecommunications Conference*, Honolulu, USA, November 30- December 4, 2009, pp. 1-5.

[2]. Y. X. Wang, K. Ssndrasegaran, X. N. Zhu, C. C. Lin, Packet Scheduling in LTE with Imperfect CQI, *International Journal of Advanced Research in Computer Science and Software Engineering,* Vol. 3, Issue 6, 2013, pp. 6-13.

[3]. X. Yu, W. Tan, S. H. Leung, Y. Rui, X. Yin, X. Liu, Discrete-Rate Adaptive Modulation with Optimum Switching Thresholds for Space-Time Coded Multiple-Input Multiple-Output System with Imperfect Channel State Information, *IET Communications*, Vol. 7, Issue 6, 2013, pp. 521-530.

[4]. I. C. Wong, B. L. Evans, OFDMA Downlink Resource Allocation for Ergodic Capacity Maximization with Imperfect Channel Knowledge, in *Proceedings of Global Telecommunications Conference,* Washington, DC, USA, 26-30 November, 2007. pp. 3729-3733.

[5]. Wong, I. C. and B. L. Evans, Optimal OFDMA Subcarrier, Rate, and Power Allocation for Ergodic Rates Maximization with Imperfect Channel Knowledge, in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP),* Honolulu, UAS, 15-20 April, 2007, pp.89-92.

[6]. F. Brah, L. Vandendorpe, J. Louveaux, Constrained Resource Allocation in OFDMA Downlink Systems with partial CSIT, in *Proceedings of IEEE International Conference on Communications,* Beijing, China, 19-23 May, 2008, pp. 4144-4148.

[7]. S. Stefanatos, N. Dimitriou, Downlink OFDMA Resource Allocation Under Partial Channel State Information, in *Proceedings of IEEE International Conference on Communications*, Dresden, Germany, 14-18 June, 2009, pp. 1-5.

[8]. M. Torabi, J. F. Frigon, B. Sanso, Performance Analysis of Adaptive Modulation in Multiuser Selection Diversity Systems with OSTBC Over Time-Varying Channels, *IEEE Signal Processing Letters,* Vol. 19,Issue 4, 2012, pp. 211-214.

[9]. Q. Ma, C. Tepedelenlioglu, Practical Multiuser Diversity with Outdated Channel Feedback, *IEEE Transactions on Vehicular Technology,* Vol. 54, Issue 4, 2005,pp. 1334-1345.

[10]. Y. Xin, X. B. Yu, Y. Liu, W. Tan, X. M. Chen, Performance Analysis of Multiuser MIMO System with Adaptive Modulation and Imperfect CSI, in *Proceedings of IET International Conference on Information and Communications Technologies,* Beijing, China, 27-29 April, 2013, pp. 571-576.

[11]. S. Guharoy, N. B. Mehta, Joint Evaluation of Reduced Feedback Scheme, Scheduling, and Rate Adaptation in OFDMA Systems with Feedback Delays, in *Proceedings of Global Communications Conference IEEE*, Anaheim, USA, 3-7 December, 2012, pp. 4566-4571.

[12]. S. Guharoy, N. B. Mehta, Joint Evaluation of Channel Feedback Schemes, Rate Adaptation, and Scheduling in OFDMA Downlinks with Feedback Delays, *IEEE Transactions on Vehicular Technology,* Vol. 62, Issue 4, 2013, pp. 1719-1731.

[13]. A. Kuhne, A. Klein, Throughput Analysis of Multi-User OFDMA-Systems Using Imperfect CQI Feedback and Diversity Techniques, *IEEE Journal on Selected Areas in Communications,* Vol. 26, Issue 8, 2008, pp. 1440-1450.

[14]. S. T. Chung, A. J. Goldsmith, Degrees of Freedom in Adaptive Modulation: a Unified View, *IEEE Transactions on Communications,* Vol. 49, Issue 9, 2001, pp. 1561-1571.

[15]. S. Falahati, A. Svensson, T. Ekman, M. Sternad, Adaptive Modulation Systems for Predicted Wireless

Channels, *IEEE Transactions on Communications,* Vol. 52,Issue 2, 2004, pp. 307-316.
[16]. Z. X. Zhao, Q. Guo, Throughput Analysis of Adaptive Uplink SC-FDMA System with Imperfect Channel State Information, *Journal of Sichuan University (Engineering Science Edition)*, Vol. 44, Issue 5, 2012,pp. 149-154.
[17]. J. Proakis, Digital Communications, *Mc Graw Hill*, 1998.