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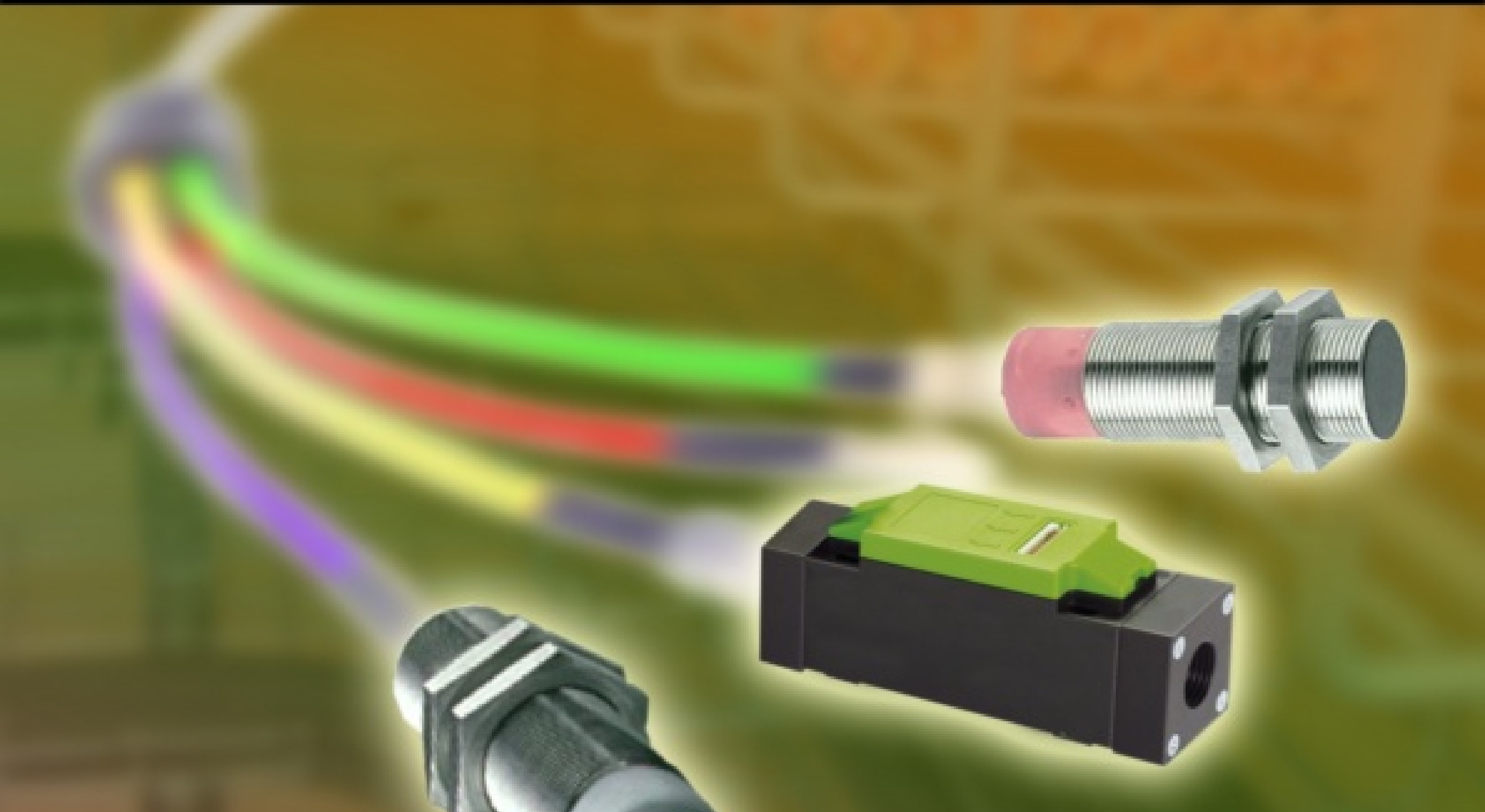

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An Analysis of Sawtooth Noise in the Timing SynPaQ III GPS Sensor

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Abstract: This paper addresses a probabilistic analysis of sawtooth noise in the one pulse per second (1PPS) output of the timing SynPaQ III GPS Sensor. We show that sawtooth noise is uniformly distributed within the bounds caused by period of the Local Time Clock of the sensor and that the probability density function (pdf) of this noise is formed with 1ns sampling interval used in the sensor to calculate the negative sawtooth. We also show that the pdf has at zero a spike of 1ns width caused by roll-off. It is demonstrated that an unbiased finite impulse response filter is an excellent suppresser of such a noise in the estimates of the time interval errors of local clocks. *Copyright © 2007 IFSA.*

Keywords: GPS timing, Time interval error, Time sensor, Sawtooth noise, Probability density function.

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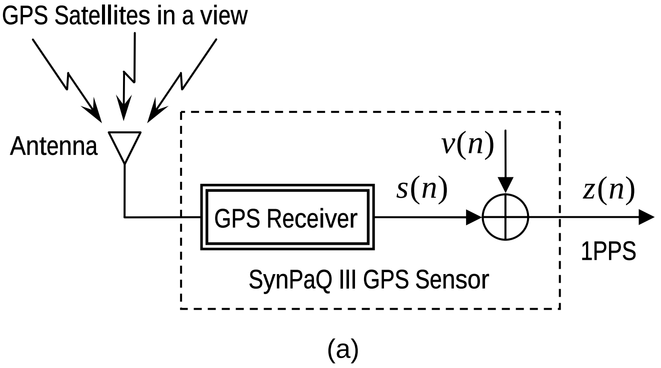
1. Introduction

In modern timekeeping [1]—[5], time interval errors (TIEs) of local clocks are measured using GPS time sensors such as the Motorola family [6] SynPaQ III GPS Sensor (Synergy Systems, LLC, San Diego, CA). The sensor receives the navigation message from available in a view the GPS satellites, exploits the Local Time Clock (LTC), and generates the one pulse per second (1PPS) signal that, owing to the principle of the 1PPS formation, is perturbed by sawtooth noise $v(n)$, where n is an integer associated with discrete time t_n and 1s time step (sample time) $\tau = t_n - t_{n-1} = 1\text{s}$. Regarding the 1PPS output, the sensor model diagram can be shown as in Figure 1a. Here an ideal 1PPS signal is represented with $s(n)$ and an additive sum of $s(n)$ and $v(n)$ forms the noisy 1PPS output $z(n)$. The noise $v(n)$ ranges within the bounds $\pm\Delta$ as sketched in Figure 1b, and the bound Δ is calculated as a reciprocal of the LTC frequency f_{LTC} by $\Delta[\mathrm{ns}]=\frac{10^3}{2f_{\mathrm{LTC}}[\mathrm{MHz}]}$. In SynPaQ III GPS Sensors, f_{LTC} is chosen to be $f_{\mathrm{LTC}}=10\,\mathrm{MHz}$ and the bound is thus $\Delta=50\,\mathrm{ns}$.



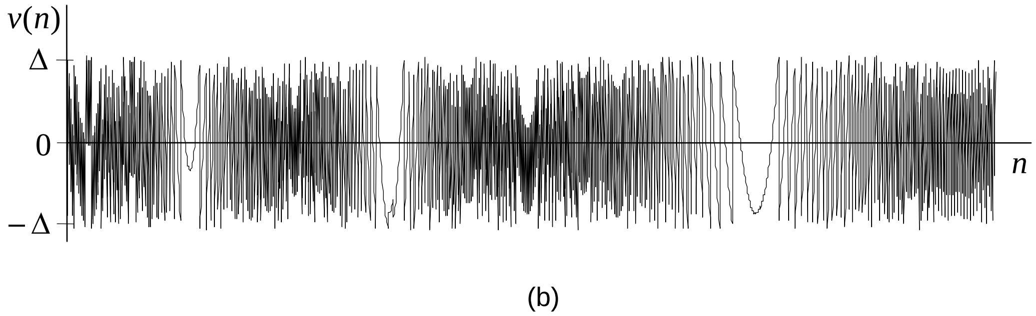


Fig. 1. SynPaQ III GPS Sensor: (a) model diagram and (b) sawtooth noise.

In this paper, we shown that the sawtooth structure of $v(n)$ represents the modulo 2Δ Brownian TIE associated with the nonstationary phase of the LTC. We provide a probabilistic analysis of $v(n)$ and give a receipt of how to suppress sawtooth noise in the estimates of the clock time errors with high efficiency.

2. Probability Density of the Sawtooth Noise

In a low precision LTC oscillator used in the SynPaQ III GPS Sensor, the white frequency Gaussian noise $\tilde{f}(t)$ with zero-mean, $\langle\tilde{f}(t)\rangle=0$, and variance $\sigma_f^2=S_f\delta(\tau)$, where S_f is the uniform power spectral density, dominates at the Fourier frequency $\mathrm{f}=1\mathrm{Hz}$ associated with the 1PPS signal. The oscillator continuous-time noisy phase $\tilde{\varphi}(t)$ can thus be modeled as the Wiener (or Brownian) process by the equation $\frac{d\tilde{\varphi}(t)}{dt}=2\pi\tilde{f}(t)$, $\tilde{\varphi}(0)=\varphi_0$, which solution is

$$\tilde{\varphi}(t)=\varphi_0+2\pi\int_0^t\tilde{f}(\theta)d\theta\,. \tag{1}$$

By multiplying (1) with $1/2\pi f_{\mathrm{LTC}}$, we go to the equation for the instantaneous time error noise,

$$\tilde{v}(t) = v_0 + \int_0^t \frac{\tilde{f}(\theta)}{f_{\mathrm{LTC}}} d\theta = v_0 + \int_0^t \tilde{y}(\theta) d\theta \,, \tag{2}$$

where $\tilde{v} = \tilde{\varphi} / 2\pi f_{\mathrm{LTC}}$ and $v_0 = \tilde{v}(0)$. Here $\tilde{y}(t) = \tilde{f}(t) / f_{\mathrm{LTC}}$ is an instantaneous noisy fractional frequency offset with zero-mean, $\langle \tilde{y}(t) \rangle = 0$, and variance $\sigma_y^2 = \mathrm{h}_0 \mathrm{f}^0 \delta(\tau)$, where h_0 is a constant postulated by the IEEE Standard [7]. Because $\tilde{v}(t)$ is the Gaussian process, we define its mean value $\langle \tilde{v}(t) \rangle = v_0$ and variance

$$\sigma_v^2 = E\{[\tilde{v}(t) - v_0]^2\} = \int_0^t \int_0^t \langle \tilde{y}(t_1) \tilde{y}(t_2) \rangle dt_1 dt_2 = \mathrm{h}_0 \mathrm{f}^0 \int_0^t \int_0^t \delta(t_2 - t_1) dt_1 dt_2 = \mathrm{h}_0 \mathrm{f}^0 t \,, \tag{3}$$

and write the nonstationary conditional normal probability density function (pdf)

$$p(\tilde{v}; t \,|\, v_0) = \frac{1}{\sqrt{2\pi \mathrm{h}_0 \mathrm{f}^0 t}} \exp\left[-\frac{(\tilde{v} - v_0)^2}{2 \mathrm{h}_0 \mathrm{f}^0 t} \right] \tag{4}$$

regarding the instantaneous time error noise existing within the infinite bounds, $-\infty < \tilde{v} - v_0 < \infty$.

The principle of the 1PPS signal formation utilized to the SynPaQ III GPS Sensor presumes changing the code of the counter if $\tilde{v}(t)$ ranges beyond the bounds $\pm \Delta = \pm \dfrac{1}{2 f_{\mathrm{LTC}}}$. This means that the time error noise $v(t) \bmod 2\Delta$ becomes sawtooth and its pdf is formed by

$$p(v; t \,|\, v_0) = \frac{1}{\sqrt{2\pi \mathrm{h}_0 \mathrm{f}^0 t}} \sum_{k=-\infty}^{\infty} \exp\left[-\frac{(\tilde{v} - v_0 - 2\Delta k)^2}{2 \mathrm{h}_0 \mathrm{f}^0 t} \right], \tag{5}$$

where $-\Delta \le v \le \Delta$. An actual pdf of the sawtooth noise is formed by overlapping all of the possible values of $v(t)$ available over time. With time t tending to infinity, the infinite sum in (5) can be substituted with integration over infinite bounds that yields

$$\begin{aligned} p(v \,|\, v_0) &= \lim_{t \to \infty} p(v; t \,|\, v_0) = \frac{1}{\sqrt{2\pi \mathrm{h}_0 \mathrm{f}^0 t}} \int_{-\infty}^{\infty} \exp\left[-\frac{(\tilde{v} - v_0 + 2\Delta \varsigma)^2}{2 \mathrm{h}_0 \mathrm{f}^0 t} \right] d\varsigma \\ &= \frac{1}{2\Delta} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \mathrm{h}_0 \mathrm{f}^0 t / 4\Delta^2}} \exp\left\{ -\frac{[\varsigma - (v_0 - \tilde{v}) / 2\Delta]^2}{2 \mathrm{h}_0 \mathrm{f}^0 t / 4\Delta^2} \right\} d\varsigma \,. \end{aligned} \tag{6}$$

Because the integrand in the last expression of (6) represents the normal pdf, the integral becomes unity, and we arrive at

$$p(v \,|\, v_0) = p(v) = \begin{cases} \dfrac{1}{2\Delta}, & -\Delta \le v \le \Delta \\ 0, & \text{otherwise} \end{cases}, \tag{7}$$

meaning that the pdf of the sawtooth noise is uniform and unconditional. It is not, however, unexpected, since the sawtooth noise originates from the random Brownian phase of the LTC that, in its mod 2π version, is also uniformly distributed [8]. The *m*-order moment and variance corresponding to (7) are, respectively,

$$\left\langle v^m \right\rangle = \int_{-\Delta}^{\Delta} v^m p(v)dv = \frac{\Delta^m}{2(m+1)}[1-(-1)^{m+1}], \tag{8}$$

$$\sigma_v^2 = \left\langle v^2 \right\rangle = \frac{\Delta^2}{3}. \tag{9}$$

Figure 2 sketches the probability density of the negative sawtooth measured in the SynPaQ III GPS Sensor for the database of $M = 2\times10^5$ points. It is seen that the function has almost ideal rectangularity fitting (7) and is a step function with 1ns sampling interval used in the sensor to calculate the negative sawtooth. We notice that of 1ns width a spike exists in the function being caused by roll-off at zero.

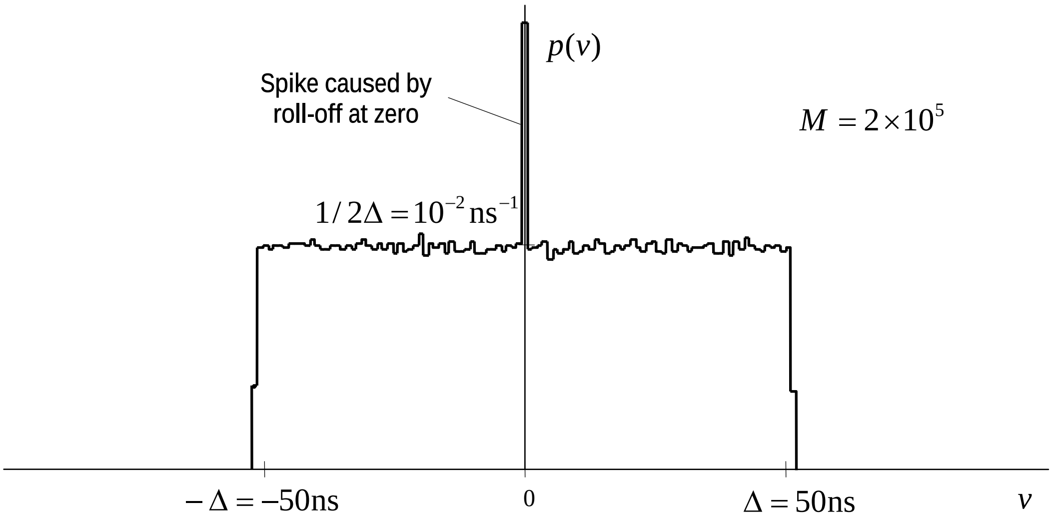


Fig. 2. Probability density function of the sawtooth noise $v(n)$ measured in the SynPaQ III GPS Sensor for $M = 2\times10^5$.

3. Filtering the Sawtooth Noise

Because sawtooth noise is not Gaussian, although its pdf (6) is uniform and symmetric (Figure 2), special care must be done for noise suppression in estimates of the TIE. We notice that Kalman filtering may produce estimates noisy and biased if noise is non Gaussian. Referring to this fact, an unbiased finite impulse response (FIR) filter was designed in [9] especially for local clocks with linear TIE models on a horizon of the filter memory. The filter was then developed and generalized in [10] for an arbitrary TIE model.

The linear TIE model fits well the most widely used crystal and rubidium clocks, although for some crystal clocks, the quadratic TIE model is more relevant. We therefore give below an example of sawtooth noise suppression with a linear unbiased FIR filter. Assuming that measurements $\xi(n)$ of the linear TIE model $x(n)$ are obtained with negligible errors in the presence of an additive sawtooth noise $v(n)$, we write

$$\xi(n) = x(n) + v(n)\,. \tag{10}$$

To provide the unbiased FIR estimate $\hat{x}(n)$ of $x(n)$ via (10), the discrete-time convolution is used,

$$\hat{x}(n) = \sum_{i=0}^{N} h_1(i)\xi(n-i)\,, \tag{11}$$

In which $h_1(i)$ is derived in [9] and [10] the unique impulse response

$$h_1(i) = \frac{2(2N-1)-6i}{N(N+1)}\,, \tag{12}$$

allowing for unbiased estimates of linear models. An important feature of (12) is that it produces, by (11), practically optimal estimates if *N* is large, $N >> 1$. We notice that large *N* is typical for precision and accurate local clocks.

Figure 3 brings the results of GPS-based sawtooth measurements of the TIE $x(n)$ of a local crystal clock (GPS sawtooth measurement) and estimates $\hat{x}(n)$ obtained with the unbiased FIR (12). Measurements were provided with the SynPaQ III GPS Sensor and Standford Frequency Counter SR620 for the crystal clock imbedded to SR620. Simultaneously, to watch for an actual trend, the same clock was measured for the rubidium clock (Rb-measurement) attached to the Standford Frequency Counter SR625.

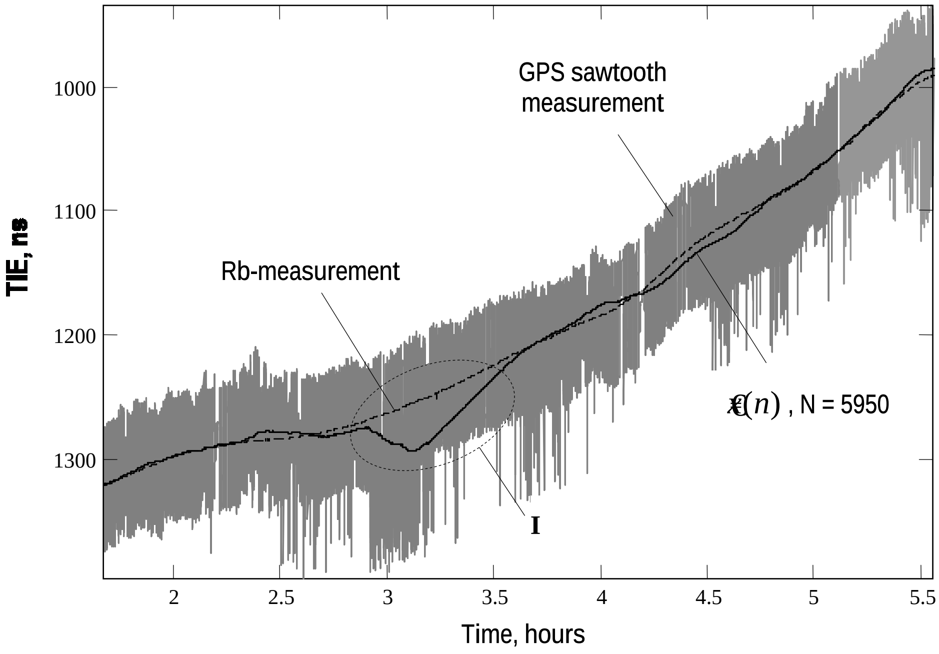


Fig. 3. GPS-based sawtooth measurements of the TIE of a crystal clock provided using the SynPaQ III GPS Sensor and estimates of the TIE obtained with a linear unbiased FIR filter.

As can be seen, the unbiased FIR filter removes sawtooth noise in the estimate $\hat{x}(n)$ of the TIE $x(n)$ with high efficiency, except for the regions, were the GPS time demonstrates temporary uncertainty. One of these regions (I) is circled in Figure 3. We notice that the other benefit of the FIR (12) follows from the fact that, in the presence of sawtooth noise, it produces estimates having the Allan variance

[7] much lower than in the Kalman filter [10]. An excellent noise performance of (12) was also recently demonstrated in the studies provided in [11] for application in GALELEO.

5. Conclusions

In this paper, we provided a probabilistic analysis of the sawtooth noise induced by the SynPaQ III GPS Sensor to GPS-based measurements of the TIE of local clocks owing to the principle utilized for 1PPS output formation. We showed that this noise is uniformly distributed and its pdf is a step function formed with 1ns sampling interval used in the sensor to calculate the negative sawtooth. We also showed that a spike of 1ns width exists in this pdf being caused by roll-off at zero. Kalman filtering is not appropriate for noise that is not white Gaussian, otherwise the Kalman filter may produce noisy and biased estimates. We therefore used recently designed the unbiased FIR filter that becomes practically optimal with a large number N in the average. We notice that large N is typical for precise clocks. Based on the TIE measurements of the local crystal clock, we demonstrated that the linear unbiased FIR filter removes sawtooth noise in the estimates of the TIE with high efficiency.

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Guide for Contributors

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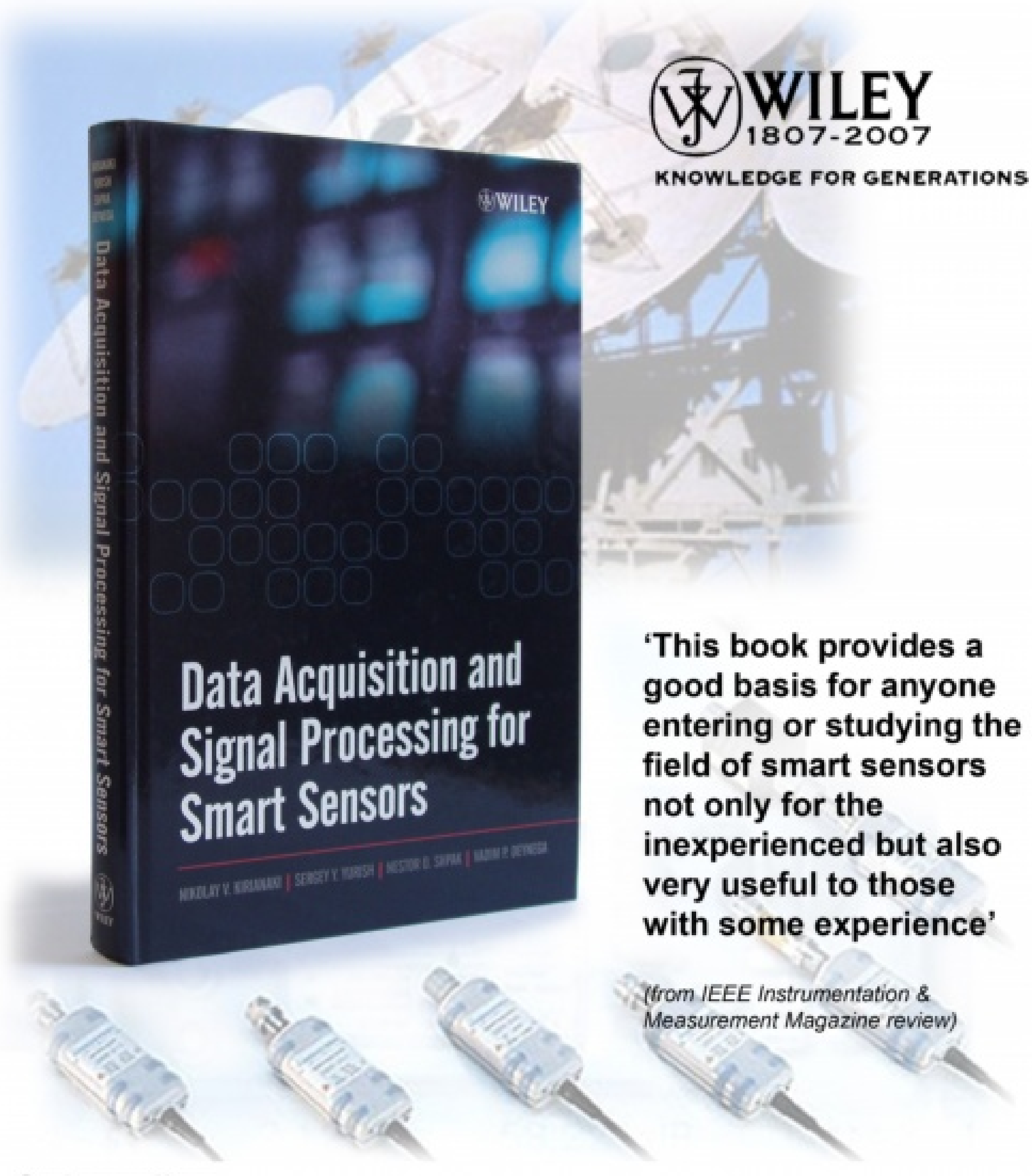
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