INVESTIGATION OF MECHANICAL RESONATORS WITH THE AID OF FIBER OPTIC INTERFEROMETER FABRY-PEROT

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Fiber optic interferometer

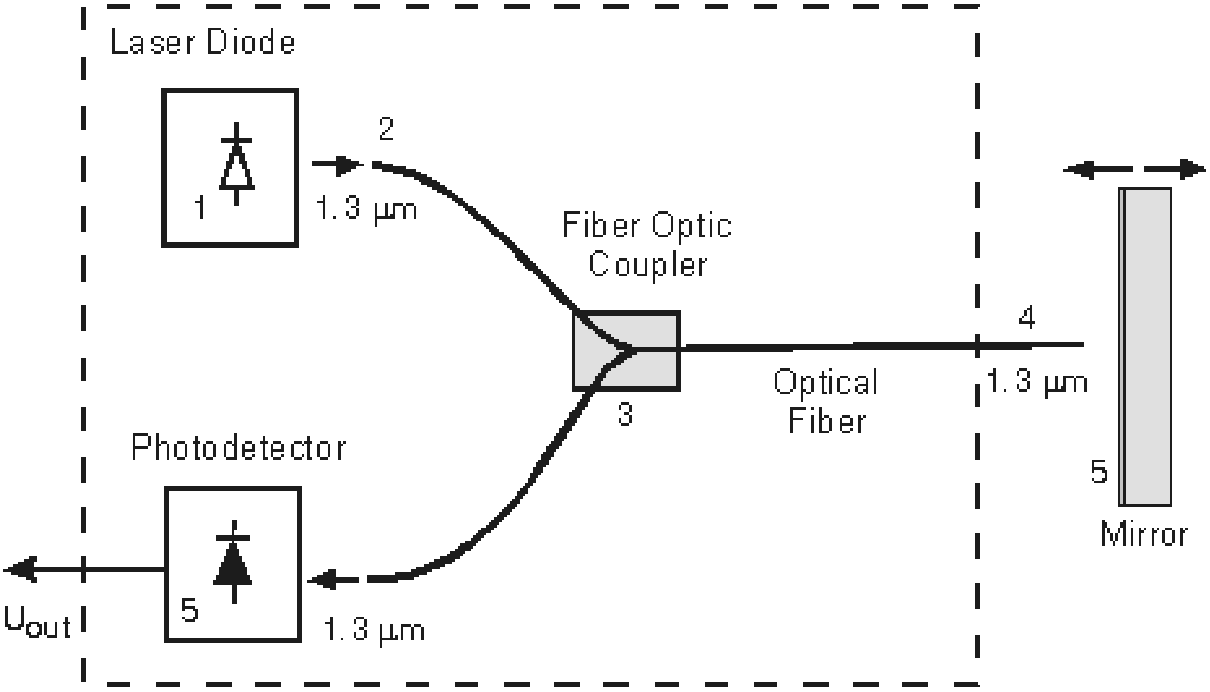
Experimental setup

- photodetector
- laser module
- coupler
- patch cord
- sensitive element
- dc-dc converter

FIBER OPTIC INTERFEROMETER FABRY-PEROT

The phenomenon of the interference of light underlies many high-precision measuring systems and displacement sensors. The use of optical fibers allows to make such devices extremely compact and economic. Two basic concepts of fiber optic interferometers are known: Mach-Zehnder and Fabry-Perot interferometers. In fiber optic interferometer Fabry-Perot the interference occurs at the partially reflecting end face surface of the fiber and an external mirror. The size of the sensitive element based on this principle can be as small as diameter of the fiber, i.e. about 0.1 mm, and the sensitivity can achieve sub-angstrom level. We can use for such an interferometer low coherence optical source (which may be even a superluminescent diode). It may be easily configured for the use in many scientific and industrial applications. Additionally, such interferometer is not sensitive to electro-magnetic interference and can be used in hostile environment.

Let us consider the principle of operation of the fiber optic Fabry-Perot interferometer.



The radiation of the laser diode 1 is coupled into the fiber 2 and propagates through the coupler 3 to fiber 4. Then, one part of radiation is reflected from the end face of the fiber 4 and other part of radiation is flashed into the air, reflected from the mirror 5 and returned back into the fiber 4. The optical beam reflected from the end face of the fiber 4 interferes with the beam reflected from the mirror. As a result the intensity of the optical radiation at photodetector 5 is periodically changed depending on the distance x_0 between the fiber and mirror as follows:

$$I = 2I_0\left(1 + \cos\left(\frac{4\pi}{\lambda}x_0 + \varphi_0\right)\right)$$

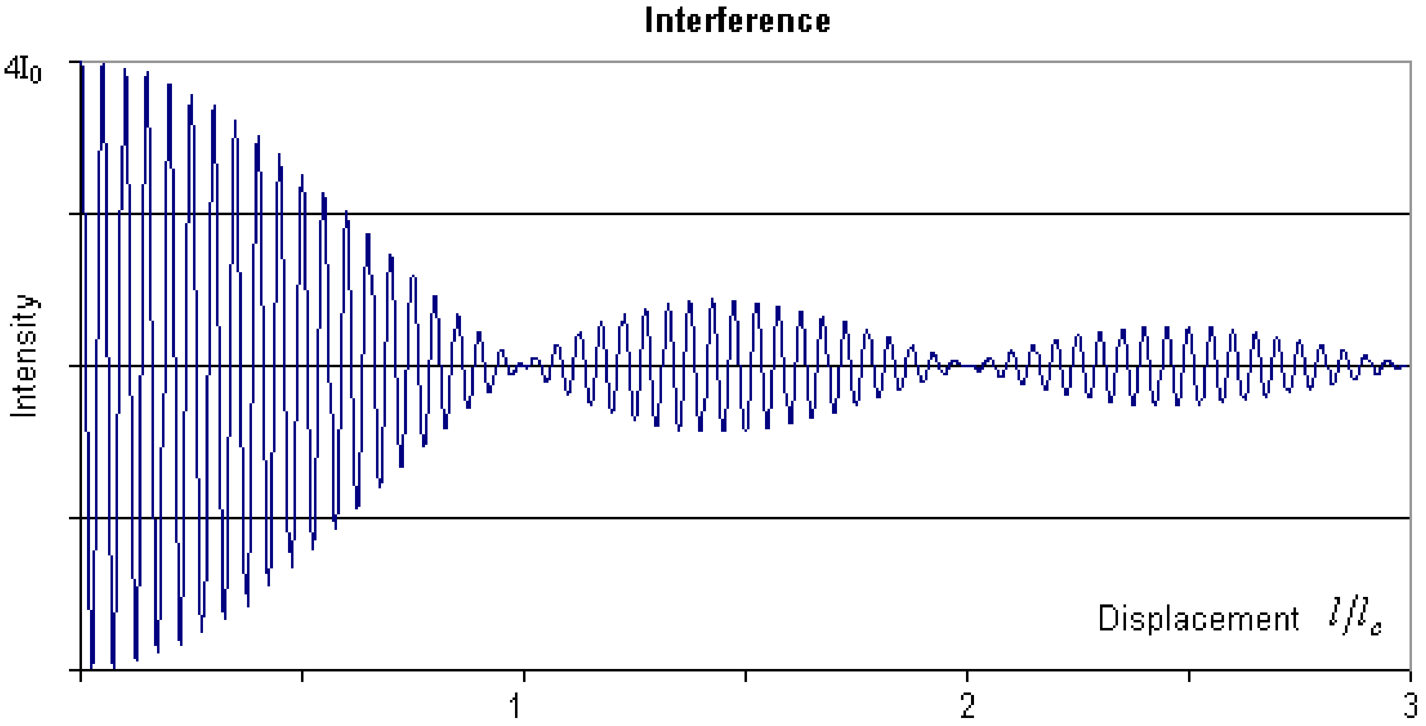
The displacement of the mirror by the half of the wavelength changes the path-length difference of the interfering rays by 2π, which corresponds to one period of variation of the radiation intensity at photodetector.

On the other hand an optical radiation can not be exactly monochromatic, and consequently it has restricted coherence length. The radiation of the laser diode consists typically of several frequency modes and the total width of the spectrum $\Delta\lambda$ is equal approximately to 3-5 nm. Coherence length l_c of such a radiation can be estimated as follows:

$$l_c = \lambda^2/\Delta\lambda$$

Substituting in this equation the typical parameters of the single-mode laser diode we can find that the coherence length equals approximately 0,5 millimiter. Using the laser diode coupled with fiber Bragg grating allows the coherence length as long as many kilometers to be acheved.

The visibility (contrast) of an interference fringes depends upon the spectrum width (and, consequently, upon the coherence length) of the light. Enlargement of the path-length difference of interfering beams decreases the visibility of interference pattern. When the path-length difference reaches the coherence length, the visibility equals 0.



The figure above shows the interference between two rays with equal intensity vs. their path-length difference l divided the coherence length l_c. This dependence is described by the equation:

$$I = 2I_0\left\{1+\frac{\sin\xi}{\xi}\cos\left(2\frac{l_c}{\lambda}\xi\right)\right\}; \qquad \xi = \pi(l/l_c)$$

where I_0 is the intensity of each of interfering beams, λ is the wavelength.

Generally, the intensity of interfering rays can be essentially different (for example, in a fiber optic interferometer where the intensity of the beam reflected from the end face of the fiber about an order of magnitude less than the intensity of the radiation reflected from the mirror and returned back into the fiber). In this case 100% visibility of interference can not be achieved even at zero path-length difference of interfering rays.

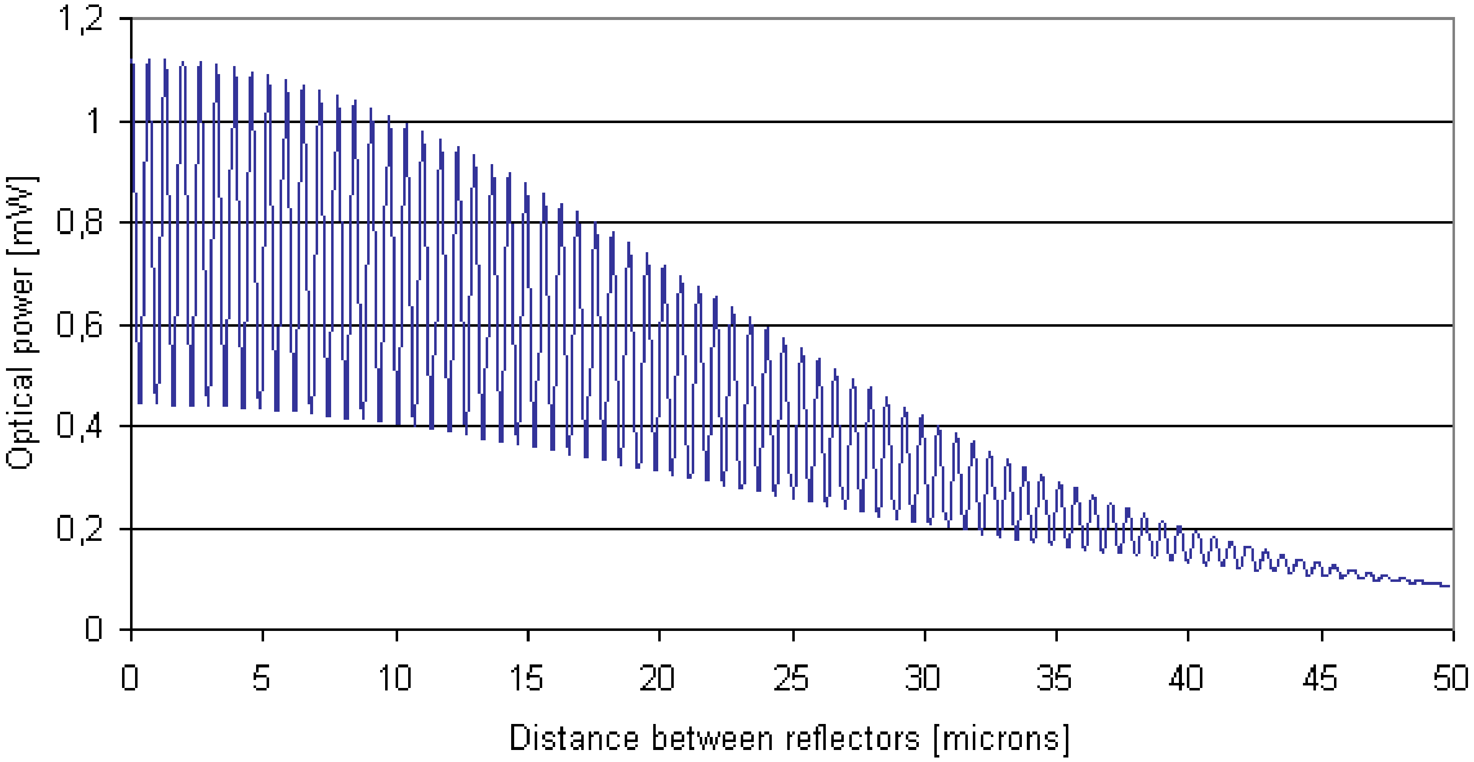
$$I = I_1 + I_2 + 2\gamma\sqrt{I_1 I_2}\cos\varphi$$

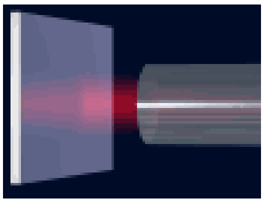
where φ is the phase difference of interfering rays, I_1 and I_2 are intensities of these two rays, γ is the degree of coherence.

In fiber optic Fabry-Perot interferometer $I_1 = R_1 I_0$ is the intensity of the light reflected from the end face surface of the fiber and $I_2 = (1-R_1)^2 R I_0$ is the intensity of the light reflected from an external mirror and returned back into the fiber, where I_0 is the intensity of the laser diode radiation coupled into the fiber, R_1 is the reflectivity of the end face of the fiber and R is the reflectivity of an external mirror. For quartz fiber R_1=0,04 is Fresnel reflectivity of the boundary surface between two substances - glass with refractive index n=1.5 and air with refractive index n=1. Thus, when the distance between interferometer mirrors equals x_0, then the light intensity detected by a photodetector is described as follows:

$$I = I_0\left\{R_1 + (1 - R_1)^2 R + 2(1 - R_1)\sqrt{RR_1}\,\frac{\sin\xi}{\xi}\cos\left(4\pi\frac{x_0}{\lambda}\right)\right\}$$

Generally, because of divergence of the light at the output of the fiber the percentage of radiation reflected from an external mirror and returned back into the fiber depends upon the distance between the fiber and mirror. The typical dependence of the optical power at photodetector upon the distance between the fiber and external mirror is given in the figure below.





Animation shows the computer simulation of fiber optic Fabry-Perot interferometer formed by a partially reflecting end face of the optical fiber and an external movable mirror. When the distance between the fiber and mirror is smaller than the coherence length, we can observe the interference and the intensity of the light in interferometer pulses with the mirror displacements. The visibility of interference increases with diminishing of the distance between the mirror and fiber. We can also see in animation the image of the fiber tip reflected in the mirror. This reflection is used sometimes in practice to align the fiber perpendicularly to mirror (in this case the fiber and its reflections lie on one line that is well visible under a microscope).

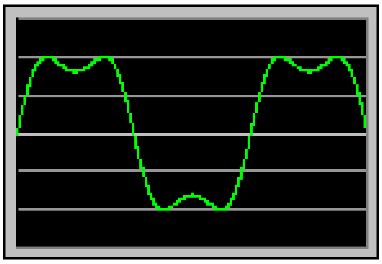
Next, we shall consider the interferometric signal appearing as a result of the reflection of the light from the vibrating surface (resonator). When the resonator oscillates, the phase difference of interfering rays is varied as follows:

$$\Delta \varphi(t) = (4\pi/\lambda)\, x_0 \sin(\omega t - \eta) = \varphi_\omega \sin(\omega t - \eta)$$

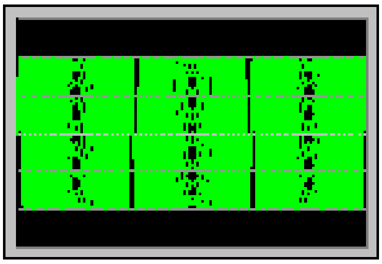
where λ is the wavelength, x_0 is the amplitude of resonator vibration. This gives rise to the following modulation of the light intensity reflected from the interferometer cavity:

$$I(t) \cong \cos\left\{\frac{4\pi}{\lambda} x_0 \sin(\omega t - \eta) + \varphi_0\right\}$$

where φ_0 is the phase difference between the interfering rays when the resonator is in equilibrium. Next two figures demonstrate the interferomentric signal when we change the mean separation between interferometer mirrors φ_0 and amplitude of the mirror vibration x_0



Change of interferometric signal during linear increase of the mean separation between mirrors

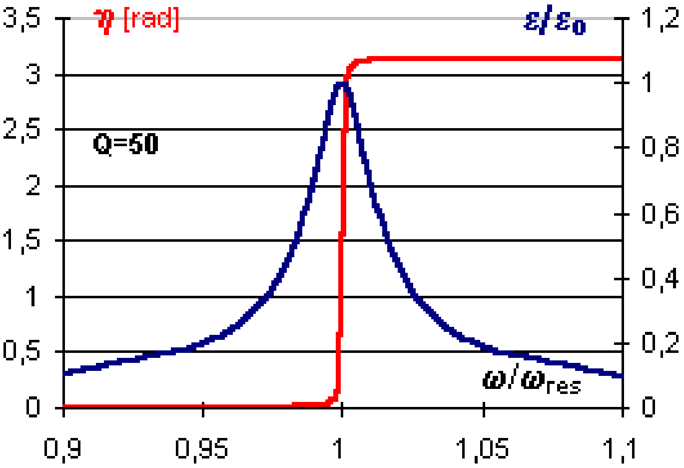


Change of interferometric signal with linear increase of the amplitude of vibration

Expanding *I*(*t*) in a Fourier series we find the alternating components of the light modulation:

$$I(t) \cong J_1(\varphi_\omega)\sin(\omega t - \eta)\sin\varphi_0 - J_2(\varphi_\omega)\cos(2\omega t - 2\eta)\cos\varphi_0 + \ldots$$

where $J_i(\varphi_\omega)$ is the Bessel functions. When $\varphi_\omega \ll 1$ and $\varphi_0 = \pi/2 + \pi k$ (*k* is an integer constant), then $J_i(\varphi_\omega)$ equals approximately $\varphi_\omega/2$ and, therefore, an alternating component of intensity *I*(*t*) will be proportional to displacement of the resonator from the equilibrium: $I_\omega \sim \sin(\omega t)$



And, finally, let us consider the case when the resonator is excited by external force (like oscillation of the cone in loudspeaker under the action of applied current, for example). In this case the resonator oscillation will depends on the frequency of the applied force as follows:

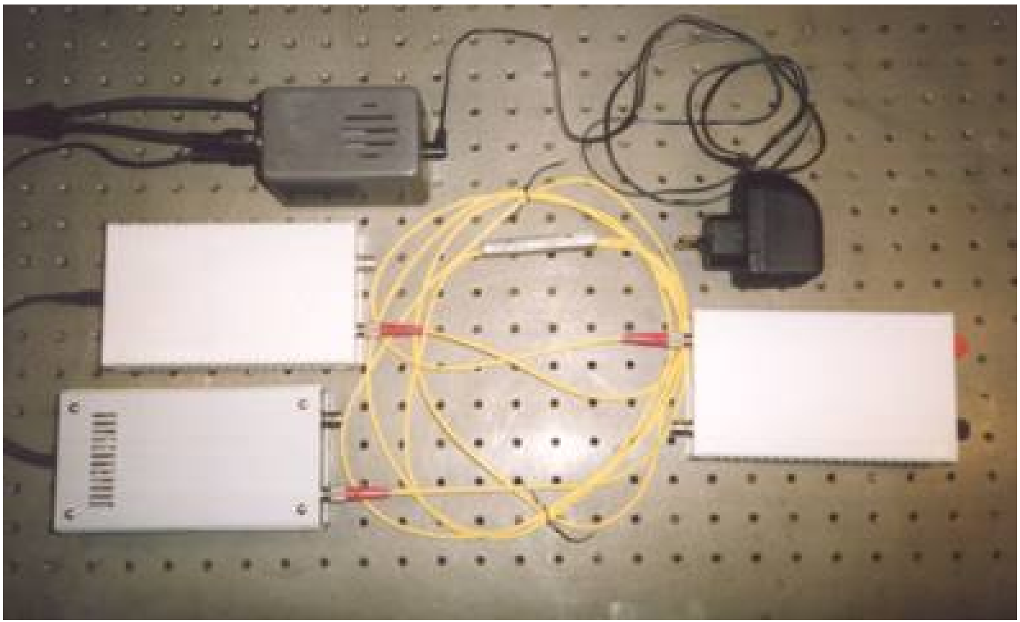
$$\varphi_\omega = \frac{4\pi}{\lambda}\cdot\frac{\varepsilon_0}{\sqrt{Q^2(1-p^2)^2+p^2}}$$

$$\operatorname{tg}\eta = \frac{1}{Q}\cdot\frac{p}{1-p^2}; \qquad p = \frac{\omega}{\omega_{\text{res}}}$$

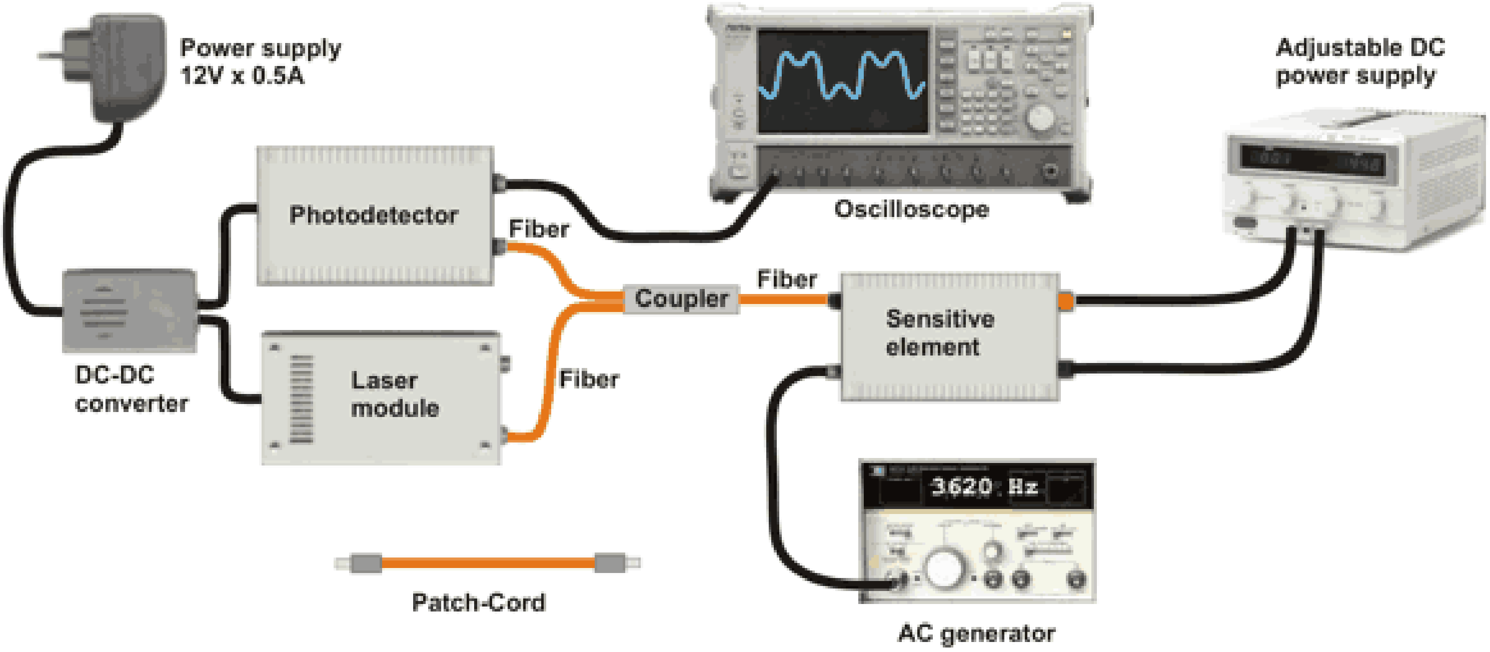
where *Q* is the quality factor of the resonator, ε_0 is the resonance amplitude of oscillation and η is the frequency-dependent phase shift between the applied force and oscillation (η varies from 0 to π, when ω varies from 0 to infinity). We can see from this equation that if $Q \gg 1$, then the amplitude of resonance oscillation is *Q* times bigger than the one at low frequency (or for quasi-static displacement of the resonator by the same force). Also, we can see that the amplitude of oscillation diminishes 1.414 times (square root of 2) as compared to resonance when the angular frequency of the applied force equals $\omega_{res} \pm \omega_{res}/2Q$. So the relative width of the resonance curve equals 1/*Q*. In general the oscillation of resonator is superposition of several oscillations with different resonance frequencies and quality factors.

EXPERIMENTAL SETUP

Introduction



In this work we consider the application of fiber-optic interferometer Fabry-Perot for investigation of the properties of mechanical resonators. We used miniature electro-magnetic sound generator HC09F as a resonator. Its semireflecting cone acts as one mirror of low-quality interferometer Fabry-Perot. The other reflector is the flat surface of the fiber tip. The sound vibrations of the cone are excited by oscillator of AC through the resistance 39 ohm. The mean separation between reflectors (working point of interferometer) is adjusted by current of controllable power supply unit applied to the red and black terminals of the sensitive element. The fiber is adjusted relatively the sound generator cone with the aid of XYZ-micropositioner. Radiation of the later module is transferred to the cavity of Fabry-Perot interferometer with the aid of single-mode coupler, which directs also the part of radiation reflected by this cavity to photodetector. Photodetector output is connected to oscilloscope. All setup is powered from external AC-DC adapter through DC-DC converter. The patch-cord is also attached for measurements of interferometer parameters.



Scheme of experimental setup.

1. Investigation of interferometer parameters.

1.1. Measurement of the optical power of the laser module.

To measure the output optical power of the laser module we have to connect the laser output with photodetector input by means of patch-cord. Measure then the voltage U_{las} at the output of photodetector. Using the data for the photodetector sensitivity we can calculate the level of the optical power in the fiber.

1.2. Fresnel reflection of the fiber end.

The boundary surface between two substances - glass with refractive index n=1.5 and air with refractive index n=1.0 serves as a reflector for the light. This was used in interferometer, where one of mirrors is the perpendicular surface of the fiber end. There is the same surface in the FC/SPS fiber adapter, which consists of ceramic capillary tube with embedded fiber. The end of the capillary tube with the fiber are highly polished to provide the minimal optical losses. To determine the Fresnel reflection of the fiber end let us connect the single-mode coupler to laser module and photodetector as shown in the figure above. Part of the light reflected by the fiber end is returned into the photodetector. Let us measure the power of this radiation. With 100% reflection of the fiber end 25% power of the laser module would be received by photodetector. Comparing photodetector signal U_{ref} with the one, measured in previous part of the work 1.1., we can find the reflection coefficient of the fiber end as follows $R=4*U_{ref}/U_{las}$.

Fresnel reflection coefficient on the boundary surface between glass and air equals about 4%. Measuring this coefficient we can find a little bit smaller value in the range 3-4%, which results from possible soiling of fiber adapter and mechanical defects of the polished fiber surface appearing during its usage. For the fiber tip used in interferometer cavity the difference can arise from the fact that fiber end surface (chip) is not exactly perpendicular to the fiber axis.

1.3. Measurement of the interferometer reflectivity and visibility of the interferometric pattern.

In this part of the work the external controllable power supply unit has to be connected to terminals of the sensitive element. Changing the applied voltage in the range 0-1 V let us measure the maximal and minimal voltage at photodetector output. The visibility of the interferometric pattern can be measured as follows: $V=(U_{max}-U_{min})/(U_{max}+U_{min})$. In our case the visibility is about 30%.

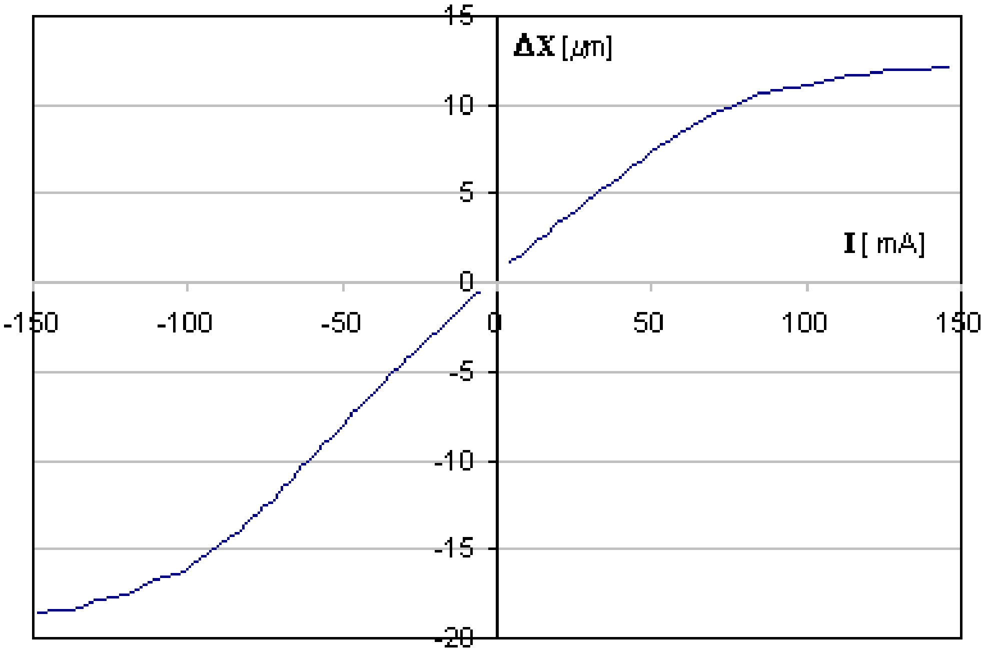
The half of the sum of maximal and minimal signals corresponds to the sum of the powers reflected at the fiber end and at the surface of mechanical resonator (cone of sound generator in our case). Comparing this value with the power of the laser module at the input of the sensitive element we can evaluate the total reflection coeffitient of the movable mirror and fiber end:

$$R = 2(U_{max}+U_{min})/U_{las} = 25\%$$

Substantially smaller reflection can indicate about disadjustment of the Fabry-Perot cavity (fiber tip relatively the vibrating reflector).

2. Current sensitivity of electro-magnetic sound generator.

When we apply current to the coil of the sound generator, its cone displaces. The value of such displacement is defined by the sensitivity of the current generator. Let us find the displacement of the sound generator cone as a function of current through its coil. For this purposes the controllable DC power supply has to be connected to terminals of the sensitive element. Changing the voltage applied to the sensitive element we shall put down the values at which the output of photodetector achieves maximums and minimums. Recalculating the applied voltage to the current through the sound generator coil and numbers of the interferometric fringes to the displacement of cone, we can draw the graph as shown below.



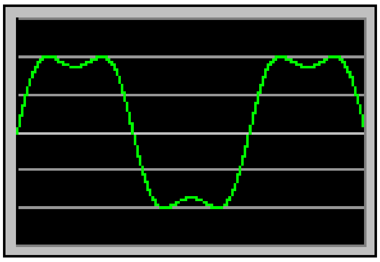
Displacement of the cone vs. the current through the coil of sound generator. (more detail graph is here).

From this figure we can find the inclination of the curve near to zero that corresponds to initial sensitivity of the sound generator to current. In our case this sensitivity equals 0,16 μm/mA. Curvature of this dependence at currents less than 80 mA is defined by non-linearity of the sound generator itself, while at bigger currents the zener overload protection scheme plays the main role (in this case the coil of sound generator is intensively heated). In the permissible 80 mA current limits the curve is approximated well by polynomial of second order: $\Delta x = -0{,}0003I^2 + 0{,}152I + 0{,}382$ and the non-linearity of sound generator equals about 10%.

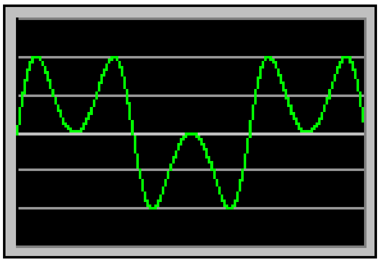
3. Investigation of the frequency parameters of sound generator.

The final part of the work is devoted to investigation of the frequency properties of the sound generator. For this purposes we needs:

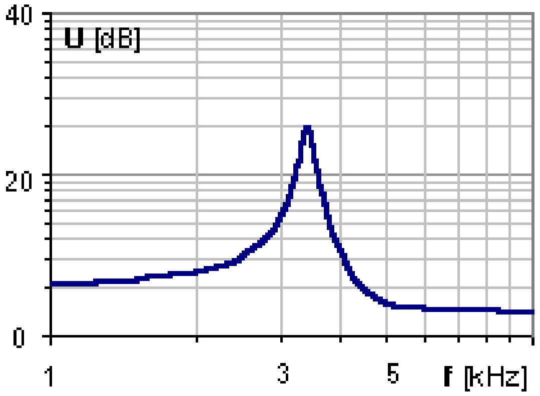
- oscilloscope connected to output of photodetector
- AC voltmeter connected across oscilloscope
- AC generator with 50ohm output connected to input of the sensitive element
- DC power supply which allows the voltage to be controlled in the limits 0-1 V.



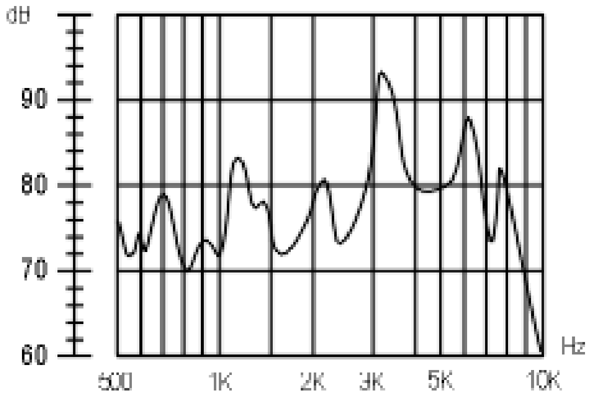
In the beginning we have to apply to sensitive element the alternating voltage ~ 20 mV (effective value). Changing the frequency of oscillator we can find the resonance of the sound generator (the amplitude of the signal is considerably increased in this case). With the aid of DC power supply we have to adjust then the working point of interferometer having obtained symmetrical picture, as shown in figure. For linearity of the interferometer output signal we have to diminish then the amplitude of oscillator twice comparing with the value which corresponds to the resonance phase modulation in π (see figure). Changing the frequency of oscillator in the range 0-10 kHz and putting the values of the alternating photodetector signal down we can draw the dependence of the displacement of the cone on the applied current (resistance of the coil is 79 ohm). The points near to resonance (3-4 kHz) have to be measures especially carefully (about 20 points). Then we can find the quality factor of resonator as ratio of the resonance frequency to the width of the resonance curve on the level 0,707 of the maximal value. In our case quality factor Q equals about 16. Compensing the photodetector offset voltage 1.4 mV we can make sure that ratio of the resonance vibrations to quasistatic displacement (or displasement at frequensies less than 100 Hz) of the cone equals to the quality factor of the resonance.



It is convenient to determine the resonance frequency of resonator when the depth of the phase modulation is equals to 2π. In this case the the amplitude of the resonator oscillations equals a quarter of the wavelength and we can observe the signal as shown in the figure and no necessity in adjusting of the interferometer working point.



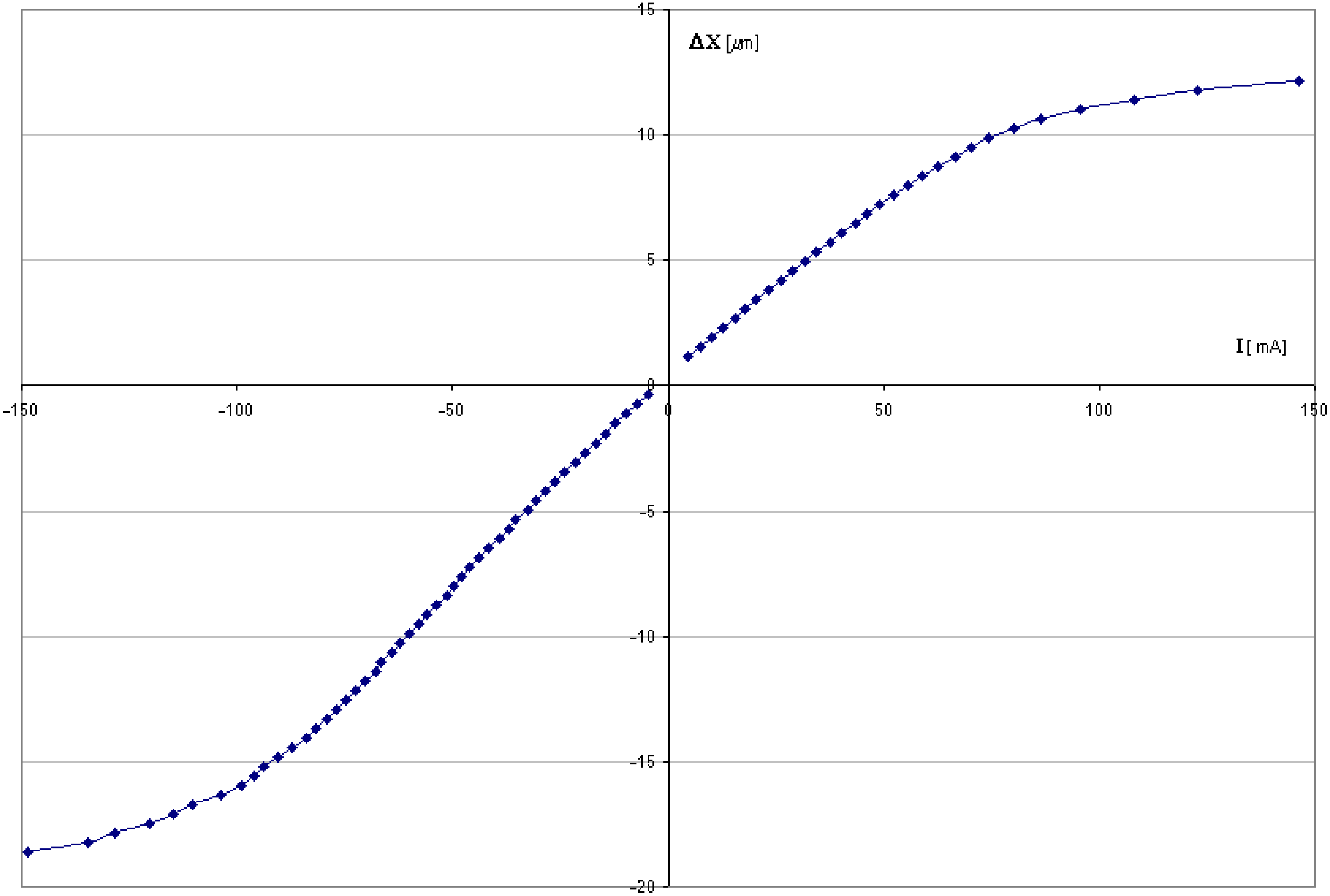
Dependence of the amplitude of the cone oscillation on the frequency of the applied voltage (resonance curve) obtained with the aid of fiber optic interferometer Fabry-Perot. Resonance frequency is 3.43 kHz. Quality factor of resonance equals 16. More detail figure is here.

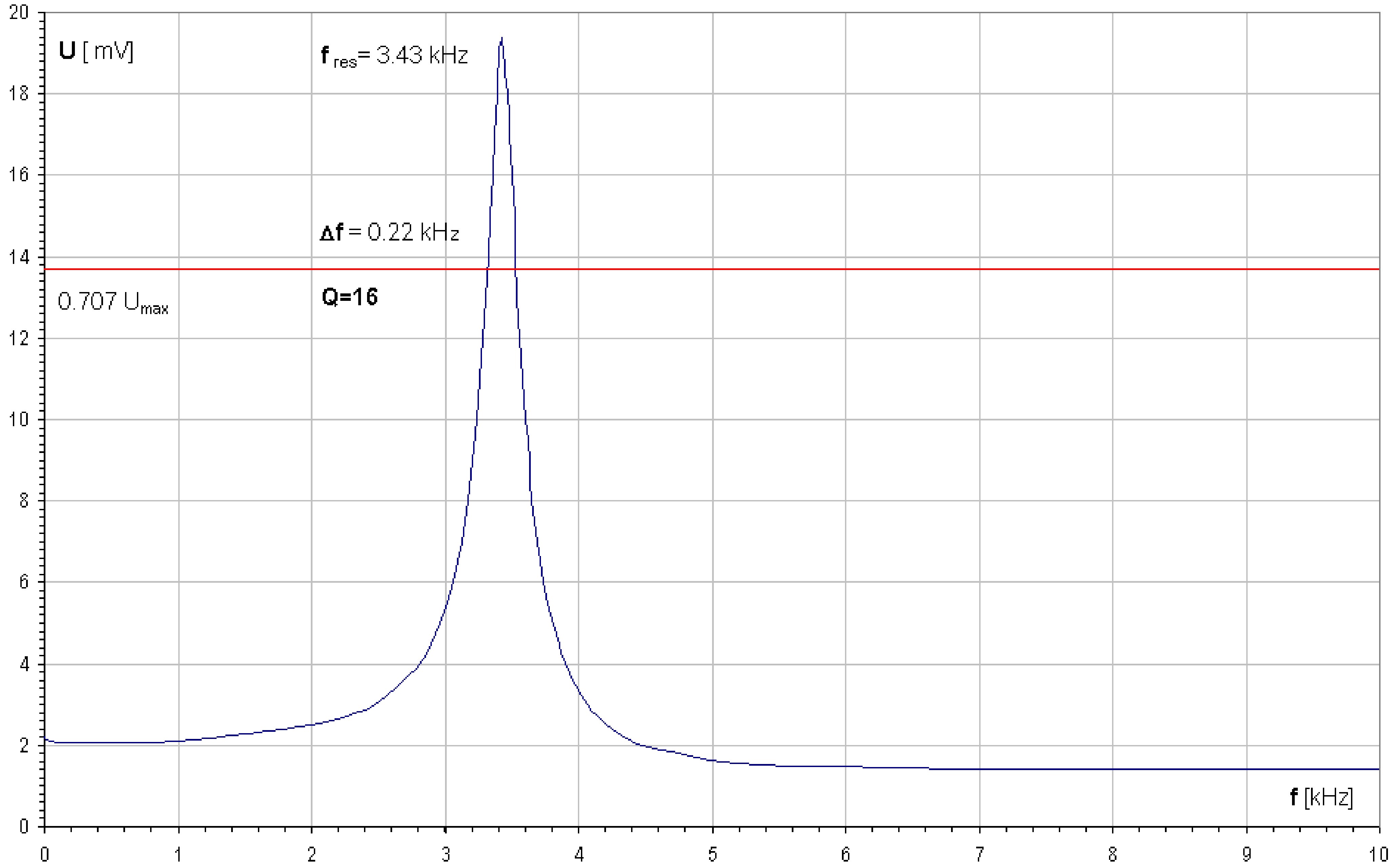


Frequency response of electro-magnetic sound generator HC09F measured with microphone at distance 10 cm (according to technical data of producer). The voltage 1.5 V was applied to the sound generator with 50% duty square wave. Resonance frequency is 3.2 kHz.

4. Conclusion.

In this work we described the application of the fiber optic interferometer Fabry-Perot for the measurement of the parameters of the mechanical resonators. The methodic of such measurements was described. Electro-magnetic sound generator played a role of the resonator in our experiments. Oscillation of the cone was excited by alternating current applied to the coil of the sound generator. The same methodic can be applied for investigation of different micromechanical resonant structures made of quartz, piezoceramic, or silicon. When the resonator can not be excited neither voltage or current, nor magnetic or electrostatic field, the miniature sound generator or vibrating platform can be used to excite the oscillation in the resonator of interest. Sensitivity of the interferrometric interrogation system proved to be enough for the work with the amplitude of oscillation as small as parts of nanometer.


ΔX [μm]
I [mA]
15
10
5
0
-5
-10
-15
-20
-150
-100
-50
0
50
100
150


U [mV]
f_{res}= 3.43 kHz
Δf = 0.22 kHz
Q=16
0.707 U_{max}
f [kHz]
20
18
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10