The Reasoning Mining of Inner-outer Unknown Information Base on Dynamic Packet Sets

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Abstract: Packet sets (P-sets) are proposed by introducing dynamic characteristics into Cantor set $X$, and they can be returned to finite Cantor set. P-sets are combined with internal packet set $X^r$ and outer packet set $X^s$. In other words, P-sets possess dynamic characteristic, and denoted by $(X^r, X^s)$. Packet reasoning is a new reasoning method based on P-sets, this method includes internal packet reasoning and outer packet reasoning. The dining and reasoning on inner-outer unknown information is studied by using P-sets and P-reasoning, and the related theorem and criteria are given. Finally, this paper presents the application concerning the reasoning and dining of inner-outer unknown information. Copyright © 2013 IFSA.

Keywords: Packet reasoning, Inner-outer unknown information, Reasoning mining theorem, Packet sets.

1. Introduction

In 2008, the literature [1, 2] proposed packet sets (P-sets) by introducing dynamic characteristics into Cantor set $X$. P-sets are the set pair which combines internal packet set $X^r$ and outer packet set $X^s$, in other words, P-sets possess dynamic characteristic, and denoted by $(X^r, X^s)$. In 2011, the literature [3] proposed packet reasoning by using P-sets. Packet reasoning is a dynamic reasoning, and be composed of internal packet reasoning and outer packet reasoning. The literature [4-26] presented several applications of P-sets in dynamic information system.

Given the attribute set $\alpha$ on finite cantor set $X$, if we go on adding the attribute into $\alpha$, $\alpha \subseteq \alpha_1 \subseteq \alpha_2 \subseteq \cdots \subseteq \alpha_l \subseteq \alpha_r$, then $X$ becomes a series of outer p-sets: $X^r_{l-1} \subseteq X^r_l \subseteq \cdots \subseteq X^r_2 \subseteq X^r_1 \subseteq X^r$. If the finite cantor set is defined as known information, then $X^r$ is the inner unknown information of $X$, and $X^s$ is the outer unknown information of $X$. The fact of $X^r$ in $X$ is unknown to people before some attributes are added into $\alpha$, and the fact of $X^s$ out of $X$ is unknown to people before some attributes are deleted into $\alpha$. In investigating dynamic information system, people often need to know the existence of inner-outer unknown information. For example, if system malfunction occurs in the studying of system fault diagnosis, then which information cell from system output would be lost? Or which information cell in $X$ would be lost, then $X = \{x_1, x_2, \ldots, x_n\} \subset U$ becomes $X^r$ ( $X^r \subseteq X$)? People wonder if inner-outer unknown information would be found by using P-reasoning. If yes, then
what condition and criterion would be required? This paper gives some discusses about these problems by using P-sets and P-reasoning.

2. P-sets and P-reasoning

P-sets and its dynamic characteristics

Given P-sets $X = \{x_1, x_2, \ldots, x_n\} \subseteq U$, $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_l\} \subseteq V$ is the attribute set on $X$, the internal packet set derived from $X$ is denoted by $X^F$, or $X^F$ is the inner packet set, and that:

$$
X^F = X - X^- ,
$$

(1)

where $X^F = \{x_1, x_2, \ldots, x_q\}$, $p \leq q$, $p, q \in N^+$. $X^-$ is called as $F$ -element deleted set of $X$, $X^-$ is defined as:

$$
X^- = \{x \mid x \in X, f(x) = u \in X, \bar{f} \in F\}
$$

(2)

The attribute set $\alpha^F$ of $X^F$ satisfy:

$$
\alpha^F = \alpha \cup \{\alpha' \mid f(\beta) = \alpha' \in \alpha, f \in F\},
$$

(3)

where $\beta \in V, \beta \subseteq \alpha, f \in F, f(\beta) = \alpha' \in \alpha ; X^F \neq \phi$.

Given a set $X = \{x_1, x_2, \ldots, x_n\} \subseteq U$, $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_l\} \subseteq V$ is the attribute set on $X$, $X^F$ is called as the outer packet set derived from $X$, or $X^F$ is the outer packet set, and that:

$$
X^F = X \cup X^+ ,
$$

(4)

where $X^F = \{x_1, x_2, \ldots, x_q\}$, $q \leq r$, $q, r \in N^+$. $X^+$ is called as $\bar{F}$ -element supplement set of $X$, $X^+$ is defined as:

$$
X^+ = \{u \mid u \in U, u \in X, f(u) = x' \in X, f \in F\}
$$

(5)

the attribute set $\alpha^F$ of $X^F$ satisfy:

$$
\alpha^F = \alpha - \{\beta \mid \bar{f}(\alpha_i) = \beta \subseteq \alpha, \bar{f} \in \bar{F}\},
$$

(6)

where $\alpha_i \in \alpha, \bar{f} \in \bar{F}, \bar{f}(\alpha_i) = \beta \subseteq \alpha, \alpha^F \neq \phi$.

The packet sets derived from $X$ is the set pair composed of the inner packet set $X^F$ and the outer packet set $X^F$, is denoted as:

$$
(X^F, X^F),
$$

(7)

$X$ is called basic set of $(X^F, X^F)$.

Where $U$ is a finite set of the universe, $V$ is the domain of a finite attribute set, $X$ is a finite cantor set; $F = \{f_1, f_2, \ldots, f_s\}$ and $\bar{F}$ are the element transferring family; $f \in F$ and $\bar{f} \in \bar{F}$ are the element transferring operator; the characteristic of $f \in F$ is: $\exists u \in U, u \subseteq X, f \in F$ such that $f(u) = x \in X$, or $\exists \beta \in V, \beta \subseteq \alpha, f \in F$ such that $f(\beta) = \alpha' \in \alpha$ . The characteristic of $\bar{f} \in \bar{F}$ is:

$$
\exists x_i \in X, \bar{f} \in \bar{F} \text{ such that } \bar{f}(x_i) = u_i \in X , \text{ or } \exists \alpha_i \in \alpha, \bar{f} \in \bar{F} \text{ such that } \bar{f}(\alpha_i) = \beta_i \subseteq \alpha .
$$

From (3), we can get:

$$
\alpha_i^F \subseteq \alpha_2^F \subseteq \cdots \subseteq \alpha_n^F \subseteq \alpha_n^F
$$

(8)

The inner packets set satisfy (8) is:

$$
X_i^F \subseteq X_{n-1}^F \subseteq \cdots \subseteq X_2^F \subseteq X_1^F
$$

(9)

From (6), we can get:

$$
\alpha_n^F \subseteq \alpha_{n-1}^F \subseteq \cdots \subseteq \alpha_2^F \subseteq \alpha_1^F
$$

(10)

The outer packets set of (10) is:

$$
X_i^F \subseteq X_{n-1}^F \subseteq \cdots \subseteq X_2^F \subseteq X_1^F
$$

(11)

From (9) and (10), we can get:

$$
\{(X_i^F, X_j^F) \mid i \in I, j \in J\}
$$

(12)

The formula (12) is general expression of P-sets, $I$ and $J$ are the index set. The formula (12) indicates: P-sets is the set pair family composed by several set pair $(X_i^F, X_j^F)$.

From (1) - (6), (8) - (11), we can get:

**Theorem 1** (the theorem of the first relation between P-sets and finite cantor set) if $F = \bar{F} = \phi$, then packets set and finite cantor set satisfy:

$$
(X^F, X^F)_{F=\bar{F}=\phi} = X
$$

(13)

Proof: if $F = \bar{F} = \phi$, then according to (2), $X^- = \{x \mid x \in X, f(x) = u \subseteq X, \bar{f} \in F\} = \phi$, from (1), we can get $X^F = X - X^- = X$ ; according to (3), $\{(\alpha' \mid f(\beta) = \alpha' \in \alpha, f \in F) = \phi \}$, i.e. $\alpha^F = \alpha$; from (5), $X^+ = \{u \mid u \in U, u \subseteq X, f(u) = x' \in X, f \in F\} = \phi$; from (4), $X^F = X \cup X^+ = X$; from (6), then (6) becomes that $\alpha^F = \alpha$. Or, if $F = \bar{F} = \phi$, then $X^F = X = X^F$, we can get (13).

**Theorem 2** (the theorem of the second relation between P-sets and finite cantor set) if $F = \bar{F} = \phi$, then packets set and finite cantor set satisfy:
\{(X^F_i, X^F_j) | i \in I, j \in J \}_{F=F^\emptyset} = X \quad (14)

The Theorem 1, 2 indicate that: when \( F = F^\emptyset = \emptyset \), \( F \) packet set is reverted to finite cantor set \( X \) : in other words, \( F \) lose dynamic characteristics, and just become finite cantor set.

**P-reasoning and its framework**

By using \( F \), the literature [3] pointed out:

\( X^F_k \) is derived from the internal packet reasoning of \( X^F_k \), \( X^F_{k+1} \) is called as internal packet reasoning set; with regard to the attribute set \( \alpha_k^F \) of \( X^F_k \) and the attribute set \( \alpha_{k+1}^F \) of \( X^F_{k+1} \), \( X^F_k \) and \( X^F_{k+1} \) satisfy:

\[
\text{if } \alpha_k^F \Rightarrow \alpha_{k+1}^F, \text{ then } X^F_{k+1} \Rightarrow X^F_k, \quad (15)
\]

where “if \( \alpha_k^F \Rightarrow \alpha_{k+1}^F \), then \( X^F_{k+1} \Rightarrow X^F_k \)” is called as the internal packet reasoning derived from inner \( F \), just be called as inner \( F \) in short. “ \( \alpha_k^F \Rightarrow \alpha_{k+1}^F \) ” is called as the condition of inner \( F \), and “ \( X^F_{k+1} \Rightarrow X^F_k \) ” is called as the conclusion of inner \( F \).

Where \( k \in \{1, 2, \cdots, n-1\} \), “ \( \alpha_k^F \Rightarrow \alpha_{k+1}^F \) ” is equivalent to “ \( \alpha_k^F \subseteq \alpha_{k+1}^F \) ”, “ \( X^F_{k+1} \Rightarrow X^F_k \) ” is equivalent to “ \( X^F_{k+1} \subseteq X^F_k \) ”.

\( X^F_{k+1} \) is derived from the outer \( F \)-reasoning of \( X^F_k \), and just be called as the outer \( F \)-reasoning set; with regard to the attribute set \( \alpha_k^F \) of \( X^F_k \) and the attribute set \( \alpha_{k+1}^F \) of \( X^F_{k+1} \), \( X^F_k \) and \( X^F_{k+1} \) satisfy:

\[
\text{if } \alpha_k^F \Rightarrow \alpha_{k+1}^F, \text{ then } X^F_k \Rightarrow X^F_{k+1}, \quad (16)
\]

“if \( \alpha_k^F \Rightarrow \alpha_{k+1}^F \), then \( X^F_k \Rightarrow X^F_{k+1} \)” is called as the outer packet reasoning derived from outer \( F \)-sets, just be called as outer \( F \)-reasoning in short. “ \( \alpha_k^F \Rightarrow \alpha_{k+1}^F \) ” is called as the condition of outer \( F \)-reasoning, and “ \( X^F_k \Rightarrow X^F_{k+1} \) ” is called as the conclusion of outer \( F \)-reasoning.

\((X^F_k, X^F_{k+1})\) is derived from inner \( F \)-reasoning and outer \( F \)-reasoning, and be called as the \( P \)-reasoning set in short; \((\alpha_k^F, \alpha_{k+1}^F)\) and \((\alpha_k^F, \alpha_k^F)\), \((X^F_k, X^F_{k+1})\) and \((X^F_k, X^F_k)\) satisfy

\[
\text{if } (\alpha_k^F, \alpha_{k+1}^F) \Rightarrow (\alpha_k^F, \alpha_k^F), \text{ then } (X^F_k, X^F_{k+1}) \Rightarrow (X^F_k, X^F_k), \quad (17)
\]

“if \( (\alpha_k^F, \alpha_{k+1}^F) \Rightarrow (\alpha_k^F, \alpha_k^F) \), then \( (X^F_k, X^F_{k+1}) \Rightarrow (X^F_k, X^F_k) \)” is called as packet reasoning derived from \( F \)-sets, just be called as P-reasoning in short. “(\( \alpha_k^F, \alpha_{k+1}^F \)) \Rightarrow (\( \alpha_k^F, \alpha_k^F \))” is called the condition of P-reasoning, and “(\( X^F_k, X^F_{k+1} \)) \Rightarrow (\( X^F_k, X^F_k \))” is called as the conclusion of P-reasoning.

Where “(\( \alpha_k^F, \alpha_{k+1}^F \)) \Rightarrow (\( \alpha_k^F, \alpha_k^F \))” denotes that \( \alpha_k^F \Rightarrow \alpha_{k+1}^F, \alpha_k^F \Rightarrow \alpha_k^F \), and “(\( X^F_{k+1}, X^F_k \)) \Rightarrow (\( X^F_k, X^F_k \))” denotes that \( X^F_{k+1} \Rightarrow X^F_k, X^F_k \Rightarrow X^F_k \).

Fig. 1, Fig. 2 give the intuitional expression of inner P-reasoning and outer P-reasoning:

**More conceptions and characteristics of P-reasoning can be got from [3]. According to the conception from [2], we can get inner-outer unknown information and its feature characteristics.**

### 3. Inner-outer Unknown Information and its Feature Characteristics

To describing conveniently, we just denote \( X, X^F, X^F \) from 2 by \((x), (x)^F, (x)^F\) in the discussion of 3-5, in other words, \((x) = X, (x)^F = X^F, (x)^F = X^F\).

**Definition 1** \((x) = \{x_1, x_2, \cdots, x_q\} \subseteq U\) is the information, \( \forall x_k \in (x) \) is called as the information cell of \((x)\), if \((x)\) possesses the attribute set \( \alpha \), and that
\[ \alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_k\} \]  

**Definition 2** \((x)^F = \{x_1, x_2, \cdots, x_p\} \subseteq U\) is an inner information derived from \((x)\), just be called as inner information; if some attribute is added into the attribute \(\alpha\) on \((x)\), \(\alpha\) is changed to \(\alpha^F\), and that
\[
\alpha^F = \{\alpha_1, \alpha_2, \cdots, \alpha_k, \alpha_{k+1}, \cdots, \alpha_k\} \tag{19}
\]

**Definition 3** \((x)^F = \{x_1, x_2, \cdots, x_q\} \subseteq U\) is an outer information derived from \((x)\), just be called as inner information; if some attribute is deleted from the attribute \(\alpha\) on \((x)\), \(\alpha\) is changed to \(\alpha^F\), and that
\[
\alpha^F = \{\alpha_1, \alpha_2, \cdots, \alpha_{k-1}\}, \tag{20}
\]

where \(p \leq q \leq r, p, q, r \in N^+\) in Definition 1-3.

**Definition 4** the information pair composed of inner information \((x)^F\) and outer information \((x)^F\) is called as inner-outer information derived from \((x)\), and that
\[
((x)^F, (x)^F) \tag{21}
\]

From Definition 1-4, we can get:

**Theorem 3** (inner-information attribute theorem) \(\alpha_k^F\) is inner-information of \((x)\) if there exists the attribute set \(\nabla \alpha_k^F \neq \phi\) such that the attribute set \(\alpha_k^F\) of \((x)^F\) and the attribute set \(\alpha\) of \((x)\) satisfy:
\[
\alpha_k^F - \alpha = \nabla \alpha_k^F \tag{22}
\]

**Theorem 4** (outer-information attribute theorem) \(\alpha_k^F\) is outer-information of \((x)\) if there exists the attribute set \(\Delta \alpha_k^F \neq \phi\), the attribute set \(\alpha_k^F\) of \((x)^F\) and the attribute set \(\alpha\) of \((x)\) satisfy:
\[
\alpha - \Delta \alpha_k^F = \alpha_k^F \tag{23}
\]

**Theorem 5** (inner-outer information attribute theorem) \((x)^F, (x)^F\) is inner-outer information of \((x)\) if \((x)^F, (x)^F\) possesses the pair of attribute set, and that:
\[
(\alpha_k^{F_1}, \alpha_k^{F_2}), \tag{24}
\]

where \(\alpha_k^{F_1} = \alpha \cup \nabla \alpha_k^F, \alpha_k^{F_2} = \alpha - \Delta \alpha_k^F\).

The proof of Theorem 3-5 can be got from (1)-(3), (4)-(6) in 2 and Definition 1-4, and be omitted here.

### 4. P-reasoning and Dining Theorem of Inner-outer Unknown Information

**Theorem 6** (P-reasoning and dining theorem of inner-outer unknown information) \(\alpha_k^F, \alpha\) are the attribute set of \((x)_k^F, (x)\) respectively, inner P-reasoning would be satisfied:

\[
\alpha \Rightarrow \alpha_k^F \text{ then } (x)_k^F \Rightarrow (x), \tag{25}
\]

then there exists information \(\nabla (x)_k^F, (x)_k^F\) is inner unknown information after \(\nabla (x)_k^F\) is deleted from \((x)\).

Proof: \(\alpha_k^F, \alpha, (x)_k^F, (x)\) satisfying “if \(\alpha \Rightarrow \alpha_k^F\), then \((x)_k^F \Rightarrow (x)\)” is equivalent to “if \(\alpha \subseteq \alpha_k^F\), then \((x)_k^F \subseteq (x)\)”, \(\nabla (x)_k^F\) is an information, since \((x)_k^F \subseteq (x), “(x)_k^F = (x) - \nabla (x)_k^F\) holds; or \((x)_k^F\) can be got after \(\nabla (x)_k^F\) is deleted from \((x)\); from Definition 2, \((x)_k^F\) is the inner information of \((x)\); since \(\alpha \subseteq \alpha_k^F\), then we can get \(\alpha_k^F\) after some attribute is added into \(\alpha\). Before the attribute is added into \(\alpha\), \((x)_k^F\) is inner-unknown information hidden in \((x)\).

**Proposition 1** if \((x)_k^F\) is inner-unknown information, and satisfy inner P-reasoning

\[
\text{if } \alpha \Rightarrow \alpha_k^F \text{ then } (x)_k^F \Rightarrow (x), \tag{26}
\]

then many attributes are supplied into the attribute set \(\alpha\) of \((x)\).

**Theorem 7** (reasoning and dining theorem of inner unknown information) if \(\alpha_k^F, \alpha\) are the attribute set of \((x)_k^F, (x)\) respectively, satisfy inner P-reasoning

\[
\alpha_k^F \Rightarrow \alpha \text{ then } (x) \Rightarrow (x)_k^F, \tag{27}
\]

then there exists information \(\Delta (x)_k^F, (x)_k^F\) is outer-unknown information which is found after \(\Delta (x)_k^F\) is supplied into \((x)\).

The proof process is similar to Theorem 6, just be omitted.

**Proposition 2** if \((x)_k^F\) is outer-unknown information of \((x)\), and satisfy outer P-reasoning:

\[
\text{if } \alpha_k^F \Rightarrow \alpha \text{ then } (x) \Rightarrow (x)_k^F, \tag{28}
\]

then many attributes are deleted from the attribute set \(\alpha\) of \((x)\).
Theorem 8 (reasoning and dining theorem of inner-outer unknown information) if \( a_{k}^{F}, \alpha, \alpha^{F} \) are the attribute set of \((x)_{k}^{F}, (x), (x)_{k}^{F} \) respectively, and satisfy P-reasoning:

\[
if \ (\alpha, \alpha^{F}) \Rightarrow (a_{k}^{F}, \alpha) \ then \ ((x)_{k}^{F}, (x)) \Rightarrow ((x), (x)_{k}^{F}),
\]

then there exists information \( V(x)_{k}^{F}, \Delta(x)_{k}^{F}; (x)_{k}^{F} \) can be got after \( V(x)_{k}^{F} \) is deleted from \( (x), (x)_{k}^{F} \) can be got after \( \Delta(x)_{k}^{F} \) is supplied into \( (x); (x)_{k}^{F} \) and \( (x)_{k}^{F} \) constitutes inner-outer unknown information of \((x)\).

Where \((x)_{k}^{F} \subseteq (x), (x) \subseteq (x)_{k}^{F}, k \in \{1, 2, \ldots , n-1\} \). The proof can be got from Theorem 6, 7, and just be omitted.

5. The Application about the Reasoning Dining of Inner-outer Unknown Information in Information Recognition

Before discussing the application, we first give the conception of information dining coefficient and information dining round, the dining criterion of inner-unknown information and outer-unknown information.

Definition 5 \( \gamma_{k}^{F} \) is called as inner-dining coefficient of inner-unknown information \((x)_{k}^{F} \) with regard to information \((x)\), just be called as inner-dining coefficient in short, and that:

\[
\gamma_{k}^{F} = \frac{\text{card}((x)_{k}^{F})}{\text{card}((x))}
\]  \hspace{1cm} (30)

Definition 6 \( \gamma_{k}^{F} \) is called as outer-dining coefficient of outer-unknown information \((x)_{k}^{F} \) with regard to information \((x)\), just be called as outer-dining coefficient of \((x)_{k}^{F} \), and that

\[
\gamma_{k}^{F} = \frac{\text{card}((x)_{k}^{F})}{\text{card}((x))},
\]

(31)

where \text{card}=\text{cardinal number in (30) and (31).}

Apparently, the dining coefficient of known information is: \( \gamma = \frac{\text{card}((x))}{\text{card}((x))} = 1 \).

Using coordinate origin \( O \) as the centre of a circle, we draw a circle \( \mathcal{O} \) with the dining coefficient \( \gamma \) as the radius, \( \mathcal{O} \) is an information dining unit circle derived from the information \((x)\); furthermore, we can draw \( \mathcal{O}^{F} \) and \( \mathcal{O}^{F} \) by using the inner-dining coefficient \( \gamma_{k}^{F} \) and the outer-dining coefficient \( \gamma_{k}^{F} \) as the radius respectively. Fig. 3 gives the intuitionistic plot of \( \mathcal{O}, \mathcal{O}_{k}^{F} \) and \( \mathcal{O}^{F} \). \( \mathcal{O}_{k}^{F} \) and \( \mathcal{O}^{F} \) are called as inner-information dining circle and outer-information dining circle respectively.

Fig. 3. The unit circle \( \mathcal{O} \) derived from the information \((x)\) is drawn by real line; the unit circle \( \mathcal{O}_{k}^{F} \) derived from inner-unknown information \((x)_{k}^{F} \) is drawn by broken line; the unit circle \( \mathcal{O}^{F} \) derived from outer-unknown information \((x)_{k}^{F} \) is drawn by broken line; \( \gamma, \gamma_{k}^{F}, \gamma_{F}^{F} \) are the radius of \( \mathcal{O}, \mathcal{O}_{k}^{F}, \mathcal{O}^{F} \) respectively.

The reasoning dining criterion of inner-unknown information

If \((x)_{k}^{F} \) is inner-unknown information of \((x)\) by means of inner P-reasoning, then \( \mathcal{O}_{k}^{F} \) created by \((x)_{k}^{F} \) is an inner-concentric circle of \( \mathcal{O} \).

The reasoning dining criterion of outer-unknown information

If \((x)_{k}^{F} \) is outer-unknown information of \((x)\) by means of outer P-reasoning, then \( \mathcal{O}^{F} \) created by \((x)_{k}^{F} \) is an outer-concentric circle of \( \mathcal{O} \).

The dining-recognition of inner-unknown information

This example is from a kindergarten in Zhumadian china. There are ten apples in an opaque box: \( x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, x_{10} \), they have different size and weight, but all is red and melliferous. We separate them into three classes: the big is assigned to the children with 6 years old, the middle to the children with 5 years old, and the small to the children with 4 years old. We don’t which one from \( x_{1} - x_{10} \) is big, middle or small. From the discussion in 3, 4, we can get the information \((x)\) and its attribute set \( \alpha \), and that
(x) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} \\
\alpha = \{\alpha_1 = \text{red}, \alpha_2 = \text{melliferous}\} \quad (32)

If \((x)_{1}^{R}\) denotes the "big", \((x)_{2}^{R}\) denotes the "middle", and \((x)_{3}^{R}\) denotes the "small", then \((x)_{1}^{R}\), \((x)_{2}^{R}\) and \((x)_{3}^{R}\) are inner-unknown information of \((x)\), because that \((x)_{1}^{R} \subset (x)\), \((x)_{2}^{R} \subset (x)\), \((x)_{3}^{R} \subset (x)\). We now define that the diameter of the big apple is 6 cm, the diameter of the middle apple is 5 cm, and the diameter of the small apple is 4 cm, or use the attribute \(\alpha_{3} = \) the diameter with 6 cm, \(\alpha_{5} = \) the diameter with 4 cm, by using inner-P reasoning in 2, we can get:

\[
\begin{align*}
\alpha_{1}^{R} & = \alpha \cup \{\alpha_{1}^{*}\} = \{\alpha_{1}, \alpha_{2}, \alpha_{3}^{*}\} \\
\alpha_{2}^{R} & = \alpha \cup \{\alpha_{3}\} = \{\alpha_{1}, \alpha_{2}, \alpha_{3}\} \\
\alpha_{3}^{R} & = \alpha \cup \{\alpha_{5}\} = \{\alpha_{1}, \alpha_{2}, \alpha_{5}\} \\
\end{align*}
\]

where \(\alpha_{1}^{*} = \{\alpha_{1}, \alpha_{2}, \alpha_{3}^{*}\}\) and \(\alpha_{5}^{*} = \{\alpha_{1}, \alpha_{2}, \alpha_{5}\}\).

From (33), Table 1 can be got.

**Table 1. Inner-unknown information**

\[
(x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R} \text{ of } (x).
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>x_1</th>
<th>x_2</th>
<th>x_3</th>
<th>x_4</th>
<th>x_5</th>
<th>x_6</th>
<th>x_7</th>
<th>x_8</th>
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<tbody>
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<td>x_2</td>
<td>x_3</td>
<td>x_4</td>
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<td>x_6</td>
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<td>x_{10}</td>
</tr>
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<td>x_3</td>
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<tr>
<td>(x)_{3}^{R}</td>
<td>x_1</td>
<td>x_2</td>
<td>x_3</td>
<td>x_4</td>
<td>x_5</td>
<td>x_6</td>
<td>x_7</td>
<td>x_8</td>
<td>x_9</td>
<td>x_{10}</td>
</tr>
</tbody>
</table>

Table 1 shows that \((x)_{1}^{R} = \{x_1, x_7\}\), \((x)_{1}^{R}\) has the attribute set \(\alpha_{1}^{R}\); \((x)_{2}^{R} = \{x_2, x_3, x_4, x_5, x_8\}\), \((x)_{2}^{R}\) has the attribute set \(\alpha_{2}^{R}\); \((x)_{3}^{R} = \{x_6, x_7, x_{10}\}\), \((x)_{3}^{R}\) has the attribute set \(\alpha_{3}^{R}\). Apparently, we supply the attribute \(\alpha_{1}^{*}, \alpha_{2}^{*}, \alpha_{3}^{*}\) into the attribute set \(\alpha\) of \((x)\) respectively, \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) would be got; \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) are inner-unknown information of \((x)\); \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) are hidden in \((x)\); because \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) are unknown to people before no attribute is supplied into \(\alpha\); in other words, \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) is inner-unknown information of \((x)\) before no attribute is supplied into \(\alpha\).

From Definition 5, and Fig. 3, we can find: \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) create \(\mathcal{O}_{1}^{F}, \mathcal{O}_{2}^{F}, \mathcal{O}_{3}^{F}\) respectively; \(\mathcal{O}_{1}^{F}, \mathcal{O}_{2}^{F}, \mathcal{O}_{3}^{F}\) are all inner-concentric of \(\mathcal{O}\). \(\mathcal{O}\) is the information dining unit circle derived from \((x)\). From the reasoning dining criterion of inner-unknown information, we can find:

\[
\text{IDE}\{\mathcal{O}_{1}^{F}, \mathcal{O}_{2}^{F}, \mathcal{O}_{3}^{F}\} = \phi
\]

Because that some attribute is supplied into the attribute set \(\alpha\), \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) are discovered and recognized; \((x)_{1}^{R}, (x)_{2}^{R}, (x)_{3}^{R}\) have inner-dining coefficient \(\gamma_{1}^{R} = 0.2, \gamma_{2}^{R} = 0.5, \gamma_{3}^{R} = 0.3\) respectively.

**The dining-recognitio n of outer-unknown information.**

This example is from a Groups Company Zhumadian China. For the commercial secrecy, we denote the Groups by the symbol \(X\), and the filiale \(x_{k} \in X\). Table 2 and Table 3 present the merchant's profit data of this filiale, the data has been processed by a certain technique. The Groups has 8 finiales: \(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}\). \(\alpha = \{\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{6}\}\) is the attribute set of the market, \(\alpha_{5}, \alpha_{6} \in \alpha\) are the attribute of market risking.

In December, 2008 the world finance crisis arose, and the Groups are attacked and threatened from June to December 2009 (the risking attributes \(\alpha_{5}, \alpha_{6}\) are added into \(\alpha\) ) the filiales \(x_{4}, x_{7}\) are closed because of minus profit. Under this condition, the Groups Company \(X\) and it’s the attribute set \(\alpha\) are:

\[
X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_8\} \quad (35)
\]

\[
\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\} \quad (36)
\]

Table 2 presents the profit distribution under the existence of risking attribute.

**Table 2. The profit distribution under the existence of risking attribute (2009.6 -12).**

<table>
<thead>
<tr>
<th>X</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.03</td>
<td>1.16</td>
<td>0.92</td>
<td>1.07</td>
<td>1.24</td>
<td>1.10</td>
<td>1.21</td>
</tr>
</tbody>
</table>

In table, \(y_{k} \in Y\), \(y_{k} = \sum_{p=1,3,5,6,8} y_{p,k}\), \(k = 6,7,8,9,10,11,12\); \(y_{p,k}\) is the finiale, \(x_{p} \in X\) is
the profit distribution from June to December in 2009; the data of \( y_p = \{y_{p,6}, y_{p,7}, y_{p,8}, y_{p,9}, y_{p,10}, y_{p,11}, y_{p,12}\} \) is omitted, \( p = 1, 3, 5, 6, 8 \); the risking attributes \( \alpha_5, \alpha_6 \) invade in \( \alpha \) because finance risking; \( \alpha_5 \) expresses that the order is rejected, \( \alpha_6 \) expresses that the contract is terminated. The Groups \( X \) worked under low profit.

From June to December in 2010, the assault of finance risks lowed, the risking attributes \( \alpha_5, \alpha_6 \) went away, the filiales \( x_2, x_3 \) came back to the producing, and went into the Groups Company again, (35), (36) turned to:

\[
X^F = X \cup \{x_4, x_7\} \\
= \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}
\]

(37)

\[
\alpha^F = \alpha - \{\alpha_5, \alpha_6\} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}
\]

(38)

Table 3 presents the profit distribution after the risking attribute disappeared.

**Table 3.** The profit distribution after the risking attribute disappeared (2010.6-12).

<table>
<thead>
<tr>
<th>( x^F )</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^F )</td>
<td>1.96</td>
<td>1.78</td>
<td>1.92</td>
<td>1.89</td>
<td>1.69</td>
<td>1.90</td>
<td>1.73</td>
</tr>
</tbody>
</table>

In Table 3, \( y_k \in y^F, y_k = \sum_{p=1,2,...,8} y_{p,k} \), \( k = 6, 7, 8, 9, 10, 11, 12 \); \( y_{p,k} \) is the finiale, \( x_p \in X^F \) is the profit distribution from June to December in 2010; the data of \( y_p = \{y_{p,6}, y_{p,7}, y_{p,8}, y_{p,9}, y_{p,10}, y_{p,11}, y_{p,12}\} \) is omitted, \( p = 1, 3, 5, 6, 8 \).

We denote \( X, X^F \) in (35), (37) by \((x), (x)^F\) respectively, or \((x) = X, (x)^F = X^F\). We get (39) by using P-reasoning in 2:

\[
\text{if} \quad \alpha^F \Rightarrow \alpha, \text{then} \quad (x) \Rightarrow (x)^F \tag{39}
\]

(39) is outer-unknown information of \((x)\), because \((x)^F\) is unknown to people before the risking attribute \(\alpha_5, \alpha_6\) disappear; and \((x)^F\) in hidden out of \((x)\) before the risking attribute \(\alpha_5, \alpha_6\) disappear, in other words, \((x) \subseteq (x)^F\).

By Definition 6, \((x)^F\) creates \( O^F \), \( O^F \) is outer-concentric circle of \( O \), \( O \) is the information dining unit circle created by \((x)\). (40) is can be got by the reasoning dining criterion of outer-unknown-information.

\[
\text{IDE} \{\mathcal{O}^F, \mathcal{O} \} \text{IDE} = \text{identification} \tag{40}
\]

Because the attribute \(\alpha_5, \alpha_6\) are deleted from \(\alpha\) (the risking attribute \(\alpha_5, \alpha_6\) disappear from \(\alpha\)), \((x)^F\) is discovered and recognized; \((x)^F\) has outer-dining coefficient \(y^F = 1.3\).

Fig. 4 presents the profit distribution curve of Groups Company.

![Fig. 4.](image)

6. Discussion

Many problems in information science and system science has dynamic feature, and it’s difficult to research these problems with classic mathematical methods. So the literature [1, 2] proposed Packet sets (P-sets) with dynamic feature. The literature [4-26] presented several applications of P-sets. An important model of dynamic reasoning is hidden in dynamic structure of P-sets: \( X_n^F \subseteq X_{n-1}^F \subseteq \cdots \subseteq X_2^F \subseteq X_1^F \), \( X_n^F \subseteq X_{n-1}^F \subseteq \cdots \subseteq X_2^F \subseteq X_{1}^F \). The literature [3] proposed this dynamic reasoning model: P-reasoning (Packet reasoning), and some applications. This paper crosses P-reasoning and P-sets, research the reasoning dining of inner-outer unknown information, and presents some important results and applications. In fact, the information \((x)\) we fall across is not static but dynamic, unknown information \((x)^F\) is hidden in \((x)\), and unknown information \((x)^F\) is hidden out of \((x)\). People require knowing \((x)^F\) or \((x)^F\), then what methods can be used to find them? This paper presents the related discussions. P-reasoning provides a new mathematical reasoning method for studying dynamic information system.

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References


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