

Optimal Design Based on Rough Set and Implementation of Worm Gear in Valve Actuator

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Abstract: Rough set optimization method was applied to design the worm gear in valve actuator. The superfluous constraints are reduced based on the methodology of rough set at the model of optimization that was established. The importance of each constraint condition and the coordination of the objective function can be got. Through example it can be seen that the optimal scheme made worm gear accord with the practical situation. So the volume of valve actuator was reduced and the efficiency was increased. Copyright © 2013 IFSA.

Keywords: Valve electric actuator, Worm gear, Rough set, Attribute reduct, Multi-objective optimization.

1. Introduction

The valve is an important part of the pipeline control system. It is widely used in metallurgical, chemical, oil and other industries. Valve electric actuator is the essential implementation components of a valve centralized control, automatic control and remote control. A conventional valve electric actuator is constituted by reduction gears, mechanical stroke control mechanism, torque control mechanism, valve position indication institutions, valve position signal feedback mechanism and electrical control part. Because of its complex structure, low control accuracy, it can not meet the requirements of automatic control, not to meet the evolving needs of industrial automation control and computer intelligent control [2, 14]. Now, new valve electric actuator has used mechatronics technology [3]. Its asynchronous motor directly drives worm drive opening and closing of the valve. The flexible valve shutdown, accurate positioning is achieved by its built-in inverter and fuzzy control system. The mechanical transmission is simplified. In

this actuator, as an important part of drive and power, worm drive's design reasonable or not directly related to the quality and efficiency. In the previous design, the uncertainty, vagueness of various factors affecting the worm drive were not taken into account, such as the level of manufacturing, material, component design parameters uncertainties and the importance of judgment on the fuzziness design. These make ordinary design method is often difficult to conform to the actual situation [5].

Rough Set optimal design method combines rough set theory and mechanical design methods, analyzes engineering uncertainties, and optimizes the processing. Optimization design of worm gear in valve actuator is actually a multi-objective programming problem. Each condition is different to the binding force of the each objective function. In this paper, based on the methodology of rough set, superfluous constraints are reduced. The importance of each constraint condition and the coordination of the objective function can be got. This result provides a more dependable basis for the optimization design.

2. Build Mathematical Model to Optimize Design

2.1. Initial Conditions

We use the data of the literature [2] to the initial settings. Where, the worm speed $n_1 = 1380$ r/min, and the transmission ratio $i = 38$, the input power $P = 5.5$ kW. Valve electric device transmission principle is shown in Fig. 1.

2.2. Determine the Design Variables

According to the design requirements of electric devices, the main parameters of the worm drive are: total number of worm Z ; modulus m ; worm diameter factor q .

The design variables: $X = [x_1, x_2, x_3]^T = [Z, m, q]^T$.

2.3. Establish the Objective Functions

1) Volume objective function.

To meet the requirements, electric device should be saving material, that is, the volume of the crown to a minimum, which also allows the smallest of the electric device. Shown in Fig. 2, the volume is:

$$F_1(X) = V = \pi/4 \{d_w^2 - [d_2 - 2(1.2m + e)^2]\}B, \quad (1)$$

where

d_w^2 is the worm gear tooth crown outside diameter (mm);

$$d_w = d_2 + 3m = mZ + 3m;$$

d_2 is the worm gear pitch diameter (mm) $d_2 = mZ_2$;

Z_2 is the worm gear teeth;

e is the tooth crown smallest thickness (mm) $e = 2m$;

B is the worm wheel width (mm) $B = 0.65d_a$;

d_a is the worm tooth crest circle diameter (mm)

$$d_a = mq + 2m;$$

2) Efficiency objective function.

The transmission efficiency of electric apparatus is highest if the worm rotation efficiency is highest. That is, amount of wear and heat are the minimum. It should enable the tooth surface's relative sliding velocity minimum. There are:

$$F_2(X) = \frac{mn}{19100} \sqrt{q^2 + Z^2} \quad (2)$$

This is a multi-objective optimization. With weighted coefficient method, a total objective functions:

$$F(X) = \omega_1 F_1(X) + \omega_2 F_2(X), \quad (3)$$

where ω_1, ω_2 are the weighting factors, and $\omega_1 + \omega_2 = 1$.

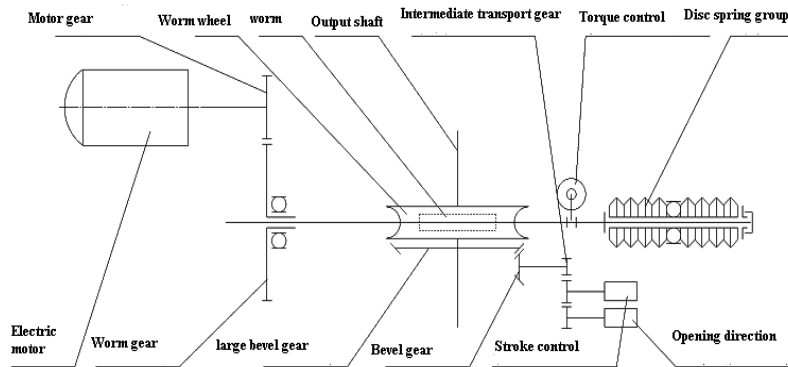


Fig. 1. Valve electric device transmission principle.

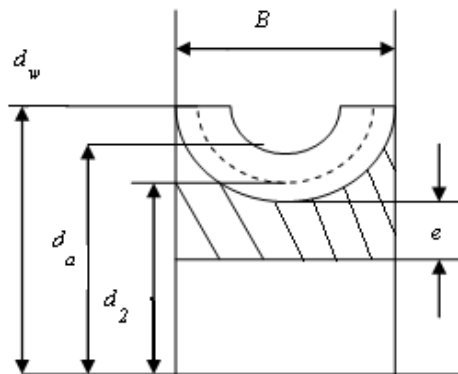


Fig. 2. Worm crown.

3. Determine the Constraint Conditions

The constraints are mainly two aspects: performance constraints and boundary constraints. There is a transition process from fully available to completely unavailable. Constraints and boundary constraints that affect the performance of the worm drive can be seen as a subset of the design space. The obtained constraint conditions are as follows:

a) Worm gear tooth contact strength constraints

$$\sigma_H = Z_E \sqrt{\frac{9KT_2}{m^3 q Z^2}} \leq [\sigma]_H, \quad (4)$$

where σ_H is the worm gear calculation of contact stress (N/mm²);

Z_E is the material factor;

K is the load factor;

T_2 is the worm gear suffered torque (N·mm);

$[\overline{\sigma}]_H$ is the fuzzy allowable contact strength (N/mm²)

b) Worm gear bending strength constraints.

$$\sigma_F = \frac{1.629KT_2}{m^3qZ_2} Y_F Y_\beta \leq [\overline{\sigma}]_F, \quad (5)$$

where

σ_F is the worm wheel dedendum calculated bending stress;

$[\overline{\sigma}]_F$ is the fuzzy allowable bending stress (N/mm²);

Y_F is the worm gear tooth form factor;

Y_β is the helix angle factor;

c) Worm stiffness constraints.

The maximum deflection should not exceed m/50 when the worm work, that is

$$y = \frac{\sqrt{F_{Y1}^2 + F_{T1}^2}}{48EJ} L^3 \leq [\overline{y}], \quad (6)$$

where

$[\overline{y}]$ is the worm fuzzy permissible deflection, mm;

F_{Y1} , F_{T1} are the radial force, the circumferential force of the worm, respectively, N

L is the worm span (mm);

E is the worm material elastic modulus (N/mm²);

J is the inertia torque of the worm at the dangerous section (mm⁴), $J = \frac{\pi d_{f1}^4}{64}$

d_{f1} is the worm tooth root circle diameter (mm).

d) Upper and lower constraints of design variables

According to the specifications and experience, the design variables will be within an approximate range of values, namely:

$$\underline{Z} \leq Z \leq \overline{Z} \quad (7)$$

$$\underline{m} \leq m \leq \overline{m} \quad (8)$$

$$\underline{q} \leq q \leq \overline{q} \quad (9)$$

From the above analysis, the design variables and the objective functions are determined; the only constraint is vague and uncertain. So, the worm drive optimization model of electric device is an ordinary uncertain constraints asymmetric optimization model.

4. Constraints and Application Based on Attribute Reduction

Rough set theory is a new data analysis theory to deal with imprecise, uncertain data, proposed by Polish mathematician Z. Pawlak at 1982 [7]. It has been widely used in the field of artificial intelligence, pattern recognition and intelligent information [9]. It can effectively analyze and deal with imprecise, inconsistent, incomplete information, and discover hidden knowledge, revealing potential rules [13]. The rough set method simulates the human abstract logical thinking. It is based on indiscernibility and knowledge reduction, and derived logic rules from data as a knowledge system model. The knowledge space can be reduced. Inference rules are obtained from the sample data. Next, we first described the concept of attribute reduction and core, and then provide the calculation method of attribute reduction.

4.1. Attribute Reduction and Core

In an information system $S=(U,A,V,f)$, a subset $P \subseteq A$ can determine a binary indiscernibility relationship $IND(P)$: $IND(P) = \{(x,y) \in U \times U | \forall a \in P, f(x,a) = f(y,a)\}$. Obviously, $IND(P)$ is an equivalence relation on the domain U , and

- 1) $IND(P) = \bigcap_{a \in P} PIND(\{a\})$;
- 2) If $P, Q \subseteq A, P \subseteq Q$, then $IND(Q) \subseteq IND(P)$.

Definition 1. Let $S=(U,A,V,f)$ be an information system. Attribute a is called unnecessary in A (redundant) if $IND(A - \{a\}) = IND(A)$. Otherwise, a is called a necessary in the A ; If every attribute $a \in A$ in A are necessary, then A is independent, otherwise known as the A dependence.

Definition 2. Let $S=(U,A,V,f)$ be an information system, $P \subseteq A$. P is a reduct of A if P Satisfies:

- 1) $IND(P) = IND(A)$;
- 2) P is independent.

Rough approximation is used to deal with imprecise and uncertain information. It has a strong ability of identification data. To this end, we constitute effective solution set U (finite) of a new model that removes some of the constraints which are not important, denoted $U = \{x_1, x_2, \dots, x_m\}$, called the domain. Efficient solution classification knowledge in U is expressed in the form of relational tables. Rows of relational tables are corresponding to the object (the effective solution of the simplified model). Columns are corresponding to the object's properties (including condition attribute and decision attribute). The information of an object is specified by each attribute's value of the object to express. Each constraint is seen as condition attributes, while the effective solution of the U is or not is the effective solution of the original problem as a decision attribute. Then, we can get a decision table. The relationship constituted by condition attributes and decision attributes is denoted by R . From the literature [14]: An attribute is corresponding

to an equivalence relation. Table 1 can be seen as a family of equivalence relations, namely knowledge base.

Table 1. Optimal solutions of the objective function to get rid of the constraints.

	C ₁	C ₂	...	C _p	F ₁	...	F ₁
x ₁	v ₁₁	v ₁₂	...	v _{1p}	t ₁₁	...	t ₁₁
x ₂	v ₂₁	v ₂₂	...	v _{2p}	t ₂₁	...	t ₂₁
...
x _m	v _{m1}	v _{m2}		v _{mp}	t _{m1}		t _{m1}

In the previous table, the attribute value v_{ij} is 1 if x_i satisfies the constraint condition of the original problem C_j . If it is not the case, v_{ij} is 0. Decision attribute value t_{ik} is taken as 1 if x_i is the optimal solution of the objective function f_k . Otherwise, t_{ik} is 0. This can get a decision information table.

So for $\forall X \subseteq U$, define lower approximation and upper approximations of X are:

$$\underline{R}X = \cup \{Y \sqsubseteq U/R \mid Y \subseteq X\},$$

$$\overline{R}X = \cup \{Y \in U/R \mid Y \cap X \neq \emptyset\}.$$

So any effective solution set X of simplified model can be expressed as: $(\underline{R}X, \overline{R}X)$. Lower approximation of X is certainly part of the original problem solution set; upper approximation of X is the set of effective solutions that may belong to the original problem. $\underline{R}X = \overline{R}X$, when removing redundant constraints conditions, that is, effective solution of the simplified model is the original model of effective solutions.

If "-" indicates unacceptable solution, "0" is the solution may not be able to accept, "+" is perfectly acceptable solution, we can describe "against", "neutral" and "fully agreed" attitude of the objective function to a feasible solution. According to this view, there is a decision information table of relevant objective function with feasible solution, as shown in Table 2.

Table 2. A decision information table of objective function and feasible solution.

Candidate feasible solution	F ₁	F ₂	F ₃	F ₄	F ₅
x ₁	+	+	-	-	-
x ₂	0	+	+	0	+
x ₃	+	-	-	-	+
x ₄	0	-	-	-	-
x ₅	+	-	+	0	-

There are the following relationships between the objective functions.

Suppose that F_i, F_k is objective function. For any feasible solution x_i :

- 1) Coordination $R_{x_i}^1(F_i, F_k) = 1$, if F_i and F_k holds the same attitude to x_i ,
- 2) Independence $R_{x_i}^0(F_i, F_k) = 0$, if at least a neutral attitude in F_i and F_k ,
- 3) Competition $R_{x_i}^{-1}(F_i, F_k) = -1$, If F_i and F_k have different attitude to x_i .

We know that $R_{x_i}^1(F_i, F_k)$ is an equivalence relation for solution x_i by [7]. According to Table 2, Table 3-6 indicates the relationship between the objective functions and each candidate solutions, respectively.

Table 3 shows that: the objective function F_2, F_3, F_4 for candidate solution x_1, x_3, x_4 , is a coordinated relationship, but they compete with F_1, F_5 . Table 4 shows that: the objective function F_2, F_3, F_5 with candidate solution x_2 is a coordination relationship. F_1, F_4 hold neutral stance to candidate solution x_2 . Similarly, other cases can be analyzed.

Decision makers can determine a priority of non-inferior solution based on the satisfaction of the objective function to the feasible solutions.

$$x_3 \prec x_1 \parallel x_5 \prec x_4 \prec x_2 \tag{10}$$

In (10), x_2 is the best non-inferior solutions. It can be seen by the description: The objective function F_2, F_3, F_5 show "fully agreed" support x_2 . F_1, F_4 remain as neutral. By contrast, x_3 is the worst non-inferior solutions. Therefore, taking satisfaction 0.80, we can get a really efficient solution x_2 .

Table 3. The objective functions and candidate solutions x_1, x_3 .

x _{1,3}	F ₁	F ₂	F ₃	F ₄	F ₅
F ₁	1				
F ₂	1	1			
F ₃	-1	1	1		
F ₄	-1	1	1	1	
F ₅	-1	1	1	1	1

Table 4. The objective functions and candidate solutions x_2 .

x ₂	F ₁	F ₂	F ₃	F ₄	F ₅
F ₁	1				
F ₂	0	1			
F ₃	0	1	1		
F ₄	0	0	0	1	
F ₅	0	1	1	0	1

Table 5. The objective functions and candidate solutions x_4 .

x ₄	F ₁	F ₂	F ₃	F ₄	F ₅
F ₁	1				
F ₂	0	1			
F ₃	0	1	1		
F ₄	0	1	1	1	
F ₅	0	-1	-1	-1	1

Table 6. The objective functions and candidate solutions x_5 .

x_5	F_1	F_2	F_3	F_4	F_5
F_1	1				
F_2	-1	1			
F_3	-1	1	1		
F_4	0	0	0	1	
F_5	-1	1	1	0	1

4.2. Constraints Analysis Based Attribute Reduction

Suppose that we denote the constraint condition that we are concerned with the condition attributes as c_1, c_2, \dots, c_4 . The set of condition attributes is denoted as $C = \{c_1, c_2, \dots, c_4\}$. Its attribute values are numeric data. The optimal solution of objective function and removing the constraint condition c_1, c_2, \dots, c_4 , respectively, can be obtained using Lindo linear programming software. Their related information is shown as Table 7.

Here, we compare optimal values of F_1, F_2, F_3, F_4 under the conditions $C - \{c_1\}, C - \{c_2\}, C - \{c_3\}, C - \{c_4\}$ with the optimal value of the original constraints groups (6) at the last line, respectively. It is not difficult to find: the optimal values in $C - \{c_4\}$ and $C = \{c_1, c_2, \dots, c_4\}$ are identical. This indicates that the conditions c_4 is relatively redundant constraints, getting rid of them without changing the original problem. Thus, the original optimization problem (*) and removing redundant constraints optimization problem (**) have the same solution.

Table 7. Optimal solutions of the objective function to get rid of the constraints.

	F_1	F_2
$C - \{c_1\}$	40107.25	0.8244
$C - \{c_2\}$	30097.11	0.8466
$C - \{c_3\}$	37886.67	0.8317
$C - \{c_4\}$	285667.83	0.8696
C	285667.83	0.8696

It is easy to see from Table 7: All objective functions are constrained by constraint c_1 . To give up it may produce an unfeasible solution contrary to the original constraints. c_2, c_3 bound F_2 . c_4 constrains F_2 . In summary, the importance of the four necessary constraint conditions: c_1 is the most important; c_2, c_3 , followed; c_4 is unimportant.

According to the above analysis, it is noted that F_1 and F_2 is coordination. It can take F_1 weighting coefficient as 0.8, while F_2 is a weight coefficient of 0.2. The original problem of conversion:

$$Max f(x) = 0.8 F_1 + 0.2 F_2$$

$$\begin{cases} \sigma_H = Z_E \sqrt{\frac{9KT_2}{m^3 q Z_2^2}} \leq [\sigma]_H \\ \sigma_F = \frac{1.629KT_2}{m^3 q Z_2} Y_F Y_\beta \leq [\sigma]_F \\ y = \frac{\sqrt{F_{V1}^2 + F_{T1}^2}}{48EJ} L^3 \leq [y] \end{cases}$$

We can get a non-inferior solution $X = (3.0612, 4.9547, 10.0328)^T$. This solution can enable the two objective functions F_1, F_2 to achieve optimum values. The fuzzy optimization result [2] is $X = [3.0612, 4.9547, 10.0328]^T$.

In this case, the number of worm head and the worm diameter coefficient is an integer, taking $Z=3, q=10$. Modulus should be taken as the standard value, and the upper and lower approximation of the modulus calculated values are 5, 4.5. This takes $m=5$, i.e. $X = [3, 5, 10]^T$. Crown volume: $V_0 = 285667.83 \text{ mm}^3$, efficiency $\eta_0 = 0.8696$.

The results with the rough set method are $X = [3, 4, 9]^T, V_0 = 278966.23 \text{ mm}^3, \eta_0 = 0.8966$.

After the comparison of the two results, we can see that crown volume is reduced by 2.34 % and its efficiency is improved by 3.1 %.

5. Conclusions

After instance, analysis and comparison, we obtain the following conclusions: (a) Rough set optimal design method can not only make the minimum volume of the crown, and thus make electric devices the smallest. Also electric device can be the most efficient. Its performance can be improved. (b) This method for multi-objective programming problem has a certain practicality. It is more flexible, simple to use in large-scale multi-stage dynamic programming.

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References

- [1]. A. M. Alfredo, A. C. Carlos, and M. M. Efrén. Evolutionary Algorithms Applied to Multi-Objective Aerodynamic Shape Optimization, *Computational Optimization, Methods and Algorithms*, 2011, pp. 211-240.
- [2]. X. M. Han, Z. S. Zhao, and Z. X. Tie, Fuzzy optimization design of worm gear in new type valve actuator, *Modern Machinery*, No. 6, 2004, pp. 20-21, 33.
- [3]. S. Koji, Y. Shu, J. Shinkyu, O. Shigeru, and Y. Yasuyuki, Multi-Objective Design Optimization for a Steam Turbine Stator Blade Using LES and GA,

- Journal of Computational Science and Technology*, Vol. 5, No. 3, 2011, pp. 134-147.
- [4]. V. S. Luis, G. H. Alfredo, M. Julián, A. C. Carlos, C. Rafael, DEMORS: A hybrid multi-objective optimization algorithm using differential evolution and rough set theory for constrained problems, *Computers & Operations Research*, Vol. 37, No. 3, 2010, pp. 470-480.
- [5]. S. B. Liu, A. S. Shui, J. P. Chen, R. Zhou, and H. Long, Research and Simulation on Intelligent Velocity Controller of Valve Electric Actuator, *Journal of Logistical Engineering University*, Vol. 27, No. 2, 2011, pp. 75-80.
- [6]. S. Obayashi. (2011). Extraction of design rules from multi-objective design exploration (MODE) using rough set theory, *Fluid Dynamics Research*, Vol. 43, No. 3, p. 70-80.
- [7]. Z. Pawalak, An inquiry into anatomy of conflicts, *Journal of Information Sciences*, No. 14, 1998, pp. 65-78.
- [8]Z. Pawlak, Rough sets, *International Journal of Computer and Information Sciences*, Vol. 11, No. 5, 1982, pp. 341-356.
- [9]Z. Pawlak, A. Skowron, Rudiments of rough sets, *Information Sciences*, Vol. 177, No. 1, 2007, pp. 3-27.
- [10]. L. Sen, T. S. Felix, and S. H., Chung, A study of distribution center location based on the rough sets and interactive multi-objective fuzzy decision theory, *Robotics and Computer-Integrated Manufacturing*, Vol. 27, No. 2, 2011, pp. 426-433.
- [11]. O. Shigeru, J. Shinkyu, and S. Koji, Multi-Objective Design Exploration and its Applications, *International Journal of Aeronautical and Space Science*, Vol. 11, No. 4, 2010, pp. 247-265.
- [12]. J. P. Xu, L. H. Zhao, A multi-objective decision-making model with fuzzy rough coefficients and its application to the inventory problem, *Information Sciences*, Vol. 180, No. 5, 2010, pp. 679-696.
- [13]. W. X. Zhang, W. Z. Wu, J. Y. Liang, and D. Y. Li, Rough Sets Theory and Method, *Science Press*, Beijing, (In Chinese), 2005.
- [14]. R. R. Zhu, S. W. Zhang, G. H. Shu, The design of an intelligent controller for use with electric valve-actuators, *Industrial Instrumentation & Automation*, No. 4, 2005, pp. 26-28.
- [15]. B. Cai, Y. Liu, A. Abulimiti, R. Ji, Y. Zhang, X. Dong, Y. Zhou, Optimal Design Based on Dynamic Characteristics and Experimental Implementation of Submersible Electromagnetic Actuators, *Strojniški vestnik - Journal of Mechanical Engineering*, Vol. 59, No. 7-8, 2013, pp. 473-482.