

## Collision Energy Dissipation Calculation and Experiment for Impact Damper with Particles

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**Abstract:** The simulation model of impact damper with particles was established based on the theory of collision energy dissipation to study the displacements and velocities of the adjacent particles before and after the collision between the particles and the damper wall. The particle parameters were studied experimentally, such as the number of particles, the density of particles, the size of particles, the recovery coefficient of granular material and the initial velocity of damper, which have a great influence on the damping characteristics. The experimental results show good agreement with the calculation prediction, which verifies the proposed simulation model. The results have shown that the impact damper with particles can effectively reduce the kinetic energy of structure, and offer considerable damping effect. This simulation model will provide the engineering applications with the theoretical guidance and design criteria. *Copyright © 2013 IFSA.*

**Keywords:** Impact damper, Particles, Collision, Energy dissipation, Experimental study.

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### 1. Introduction

Particle damping technology is a passive vibration control technology. The kinetic energy of structural system is dissipated through the collision and friction between the particles and the damper, so as to suppress the vibration and the noise [1-2]. This vibration damping technology has a lot of advantages, for example, simple structure, low costs, small modification of original structure, low additional mass, remarkably improving the damping ratio of system, applicable for harsh environment, without damping performance degradation over time etc [3-4]. Therefore, it has wide prospect of engineering applications in the fields of aerospace, automobile and precision machinery etc [5-7].

This concept of non-obstructive particle damping (NOPD) was put forward firstly in 1991 and had been successfully applied to the turbine blades [8]. However, the mechanism of vibration energy

dissipation in the particle damper is very complex and involves in the mechanical behavior of particles media, which has hindered the application of the particle damping. Recently, many studies have been carried out based on the particle damping and the particulate matter dynamics. The research methods are mainly divided into the theoretical simulation and the experiment [9-14]. The theoretical methods mainly include the discrete element method (DEM), the collision theory, the regression design and the optimization design etc., and many new theories and methods are being introduced into the research of particle damping [15-19]. DEM is used to simulate the dynamics of particle system contacting with each other by establishing the mechanical model of single particle [20-23]. However, during the numerical computation, because the nonlinear contact characteristics of particles limit the time-step of contact parameters, DEM is unsuitable for any large simulation of the nonlinear contact model with

particle number more than  $10^7$  [20]. However, in the engineering applications, the particle number in the damper is usually more than  $10^8$ , which limit the application of DEM in the engineering field.

According to the present research of mechanism and the defects of the existing model [24-26], we establish the collision model based on the movement of among the particles, between the particle unit and the damper boundary, and among the adjacent particles after colliding with the damper wall. The energy method is used to calculate the energy dissipation factor, which is compared with the experimental results. This paper investigates the particle parameters between the particle group and the damper, such as the number of particles, the density of particles, the size of particles, the recovery coefficient of granular material and the initial velocity of collision, which have a great influence on the damping characteristics. The simulation model of impact damper with particles which characterizes the nonlinear damping will be employed in the engineering design.

## 2. Energy Dissipation Model of Collision for Particles

During the energy dissipation of impact damper with particles, when the amplitude of the excitation force is more than 1/5 of system gravity [15], the relative movement among the particles is violent so that the particles are separated. At this time, the vibration energy of system is dissipated mainly by the collision among the particles and between the particles and the damper wall. This paper will establish the energy dissipation model of impact damper with particles based on the collision theory [12, 16, 26].

### 2.1. Collision Model Among the Particles

In order to be convenient for analysis and calculation, it is assumed that the particle is the sphere unit. The analysis of interaction among the particles requires judging the relationship of particle positions at different moments. The local coordinates are established in Fig. 1.

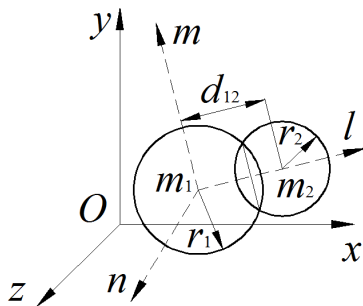


Fig. 1. Collision coordinates of two particles.

It is assumed that the masses of particle 1 and particle 2 are respectively  $m_1$  and  $m_2$ , the radius of particle 1 is  $r_1$ , its position coordinates are  $(x_1, y_1, z_1)$ , its velocity is  $(x_1', y_1', z_1')$ , and the radius of particle 2 is  $r_2$ , its position coordinates are  $(x_2, y_2, z_2)$ , its velocity is  $(x_2', y_2', z_2')$ . The contact conditions of particle 1 and particle 2 can be expressed as

$$d_{12} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} < r_1 + r_2 \quad (1)$$

Before the collisions happen, the velocities of particle 1 and particle 2 in local coordinates are respectively  $(v_{1l}, v_{1m}, v_{1n})$  and  $(v_{2l}, v_{2m}, v_{2n})$ . It is assumed that  $l$  axis coincides with the connecting line for centers of two particles in local coordinates, and  $\cos \alpha$ ,  $\cos \beta$ ,  $\cos \gamma$  are respectively direction cosine of  $l$  axis.

The corresponding relations of the movement velocity of particle 1 in local coordinates and original coordinates before the collision are

$$v_{1l} = x_1' \cos \alpha + y_1' \cos \beta + z_1' \cos \gamma \quad (2)$$

$$v_{1m} = \frac{-\cos \beta}{\sqrt{\cos^2 \alpha + \cos^2 \beta}} x_1' - \frac{\cos \alpha}{\sqrt{\cos^2 \alpha + \cos^2 \beta}} y_1' \quad (3)$$

$$v_{1n} = \frac{-\cos \alpha \cdot \cos \gamma}{\sqrt{\cos^2 \alpha + \cos^2 \beta}} x_1' - \frac{\cos \beta \cdot \cos \gamma}{\sqrt{\cos^2 \alpha + \cos^2 \beta}} y_1' + \sqrt{\cos^2 \alpha + \cos^2 \beta} \cdot z_1' \quad (4)$$

After the collisions, the velocity components of particle 1 and particle 2 in local coordinates are respectively  $(V_{1l}, V_{1m}, V_{1n})$  and  $(V_{2l}, V_{2m}, V_{2n})$ . Hence, the equations according to the momentum theorem are written as

$$\begin{aligned} m_1 \cdot V_{1l} + m_2 \cdot V_{2l} &= m_1 \cdot v_{1l} + m_2 \cdot v_{2l} \\ m_1 \cdot V_{1m} + m_2 \cdot V_{2m} &= m_1 \cdot v_{1m} + m_2 \cdot v_{2m} \\ m_1 \cdot V_{1n} + m_2 \cdot V_{2n} &= m_1 \cdot v_{1n} + m_2 \cdot v_{2n} \end{aligned} \quad (5)$$

The normal Newton recovery coefficient at the contact point of two spheres is given by

$$e = \frac{V_{2l} - V_{1l}}{v_{1l} - v_{2l}} \quad (6)$$

The friction coefficient  $\lambda_l$  in local coordinates is the ratio of impulses in two directions, which is

written as  $\lambda_1 = \frac{I_m}{I_l}$   $\lambda_2 = \frac{I_n}{I_l}$ . The momentum equations become

$$\begin{cases} m_1 \cdot V_{1m} + \lambda_1 \cdot m_2 \cdot V_{2l} = m_1 \cdot v_{1m} + \lambda_1 \cdot m_2 \cdot v_{2l} \\ m_1 \cdot V_{1n} + \lambda_2 \cdot m_2 \cdot V_{2l} = m_1 \cdot v_{1n} + \lambda_2 \cdot m_2 \cdot v_{2l} \end{cases} \quad (7)$$

The friction coefficient  $\lambda_1$  and  $\lambda_2$  is defined by

$$\text{sgn } \lambda_1 = \text{sgn}\left(\frac{v_{2m} - v_{1m}}{v_{2l} - v_{1l}}\right), \text{sgn } \lambda_2 = \text{sgn}\left(\frac{v_{2n} - v_{1n}}{v_{2l} - v_{1l}}\right),$$

where symbolic function  $\text{sgn}[\ ]$  is expressed as

$$\text{sgn}[x] = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

It is assumed that the vector

$$V = \{V_{1l}, V_{1m}, V_{1n}, V_{2l}, V_{2m}, V_{2n}\}^T,$$

$$v = \{v_{1l}, v_{1m}, v_{1n}, v_{2l}, v_{2m}, v_{2n}\}^T.$$

The velocity equation in local coordinates before and after two particles collide can be written as

$$V = [A]^{-1} \cdot [B] \cdot v = [C] \cdot v \quad (8)$$

Where matrix  $A$  and  $B$  are

$$[A] = \begin{bmatrix} m_1 & 0 & 0 & m_2 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 \\ 0 & 0 & m_1 & 0 & 0 & m_2 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & m_1 & 0 & -\lambda_1 m_1 & 0 & 0 \\ 0 & 0 & m_1 & -\lambda_2 m_2 & 0 & 0 \end{bmatrix},$$

$$[B] = \begin{bmatrix} m_1 & 0 & 0 & m_2 & 0 & 0 \\ 0 & m_1 & 0 & 0 & m_2 & 0 \\ 0 & 0 & m_1 & 0 & 0 & m_2 \\ -e & 0 & 0 & e & 0 & 0 \\ 0 & m_1 & 0 & -\lambda_1 m_1 & 0 & 0 \\ 0 & 0 & m_1 & -\lambda_2 m_2 & 0 & 0 \end{bmatrix}$$

After the adjacent particles interact, one particle will change to a new velocity and position. Then continue colliding with other particle and change its velocity and position again. When the collision energy dissipation of impact damper with particles is calculated, the cycle repeats, and the system energy is continually dissipated. Therefore, after one time step  $\Delta t$ , new position coordinates of particle 1 are

$$\begin{cases} x_{1r} = x_1 + \frac{(x'_{1r} + x'_1) \cdot \Delta t}{2} \\ y_{1r} = y_1 + \frac{(y'_{1r} + y'_1) \cdot \Delta t}{2} \\ z_{1r} = z_1 + \frac{(z'_{1r} + z'_1) \cdot \Delta t}{2} \end{cases} \quad (9)$$

## 2.2. Collision Model between the Particles and the Damper

When the particles move in enclosed space, in addition to the collision among the particles, they are also subjected to the restriction of the damper wall. In this paper, the contact between the particles and the damper wall is equivalent to the mass-spring-damping system.

It is assumed that the mass of particle  $i$  is  $m_i$ , its velocity is  $v_i$  and the included angle between the particle and the damper wall  $xoy$  plane is  $\alpha$  during colliding with the damper wall. The collision process is divided into two stages, i.e. compression stage and recovery stage [14]. The velocity of particle leaving away from the damper wall is  $v_{it}$  after the collision. The schematic representation of collision is given in Fig. 2.

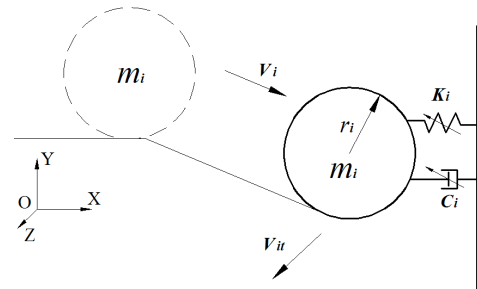


Fig. 2. Collision model of particles and damper wall.

### 1) Compression stage.

The velocity of particles before the collision is divided into  $x'_i$ ,  $y'_i$  and  $z'_i$  respectively in the direction of tangent plane and normal plane. According to the impulse theorem, we obtains

$$\begin{cases} mx'_i{}^c - mx'_i = I_{cx} \\ my'_i{}^c - my'_i = I_{cy} \\ 0 - mz'_i = I_{cz} \end{cases} \quad (10)$$

where  $x'_i{}^c$ ,  $y'_i{}^c$  and  $z'_i{}^c$  are the velocity components of particle in tangent plane and normal plane when the compression stage finishes;  $I_{cx}$  and  $I_{cy}$  are the impulse component in tangent plane in this stage;  $I_{cz}$  is the impulse component in normal direction.

Define the tangential dynamic friction factor as

$$\mu = \frac{\sqrt{I_{cx}^2 + I_{cy}^2}}{I_{cz}}$$

The velocity component  $v_{it}^c$  of particle in tangent plane when the compression stage completes is given by

$$v_{it}^c = \sqrt{(x'_i)^2 + (y'_i)^2} - \mu z'_i \quad (11)$$

2) Recovery stage.

In recovery stage, the collision impulse components in tangent plane and normal direction are  $I_{rx}$ ,  $I_{ry}$  and  $I_{rz}$ , and the impulse equations become

$$\begin{cases} mx'_{it} - mx'^c_i = I_{rx} \\ my'_{it} - my'^c_i = I_{ry} \\ mz'_{it} - 0 = I_{rz} \end{cases} \quad (12)$$

The normal recovery coefficient is given by

$$e = \frac{-z'_{it}}{z'_i}$$

The position of particle unit between the particle and the damper wall after the collision is  $(x_{it}, y_{it}, z_{it})$ . The velocity component  $v_{it}$  of particle in tangent plane when the recovery stage completes is given by

$$v_{it} = \sqrt{(x'_i)^2 + (y'_i)^2} - z'_i(1+e) \cdot \mu \quad (13)$$

After one time step  $\Delta t$ , new position coordinates of particle  $i$  are

$$\begin{cases} x_{it} = x_i + \frac{(x'_i + x'_{it})}{2} \cdot \Delta t \\ y_{it} = y_i + \frac{(y'_i + y'_{it})}{2} \cdot \Delta t, \\ z_{it} = z_{iw} + r_i \end{cases} \quad (14)$$

where  $z_{iw}$  is the normal coordinate of particle unit on the damper wall.

### 2.3. Collision Model of Adjacent particles after Colliding with the Damper Wall

After the particle 1 collides with the damper wall, the collision model of other particles is shown in Fig. 3.

For collision of two particles, the masses of particle 1 and particle 2 are respectively  $m_1$  and  $m_2$ . It is assumed that the velocity of the particle 1 is  $v_1$

and the particle 2 is stationary after the particle 1 collides with the damper wall. And the velocity of the particle 2 is  $v_2$  after the particle 1 collides with the particle 2. Define the recovery coefficient of two particles as  $e_{12}$ . According to the momentum theorem,  $v_2$  is written as

$$v_2 = m_1(1 + e_{12})v_1 / (m_1 + m_2) \quad (15)$$

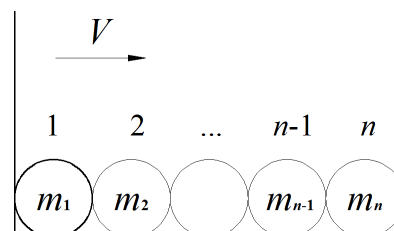


Fig. 3. Collision model of multiple particles.

Similarly, the energy of particle group in the damper is transferred from left to right. After the first particle collides with the damper wall, it collides with the second particle; after the second particle collides with the first particle, it collides with the third particle, and the like, till the rightmost particle. So the velocity after the  $n-1$  particle collides with the  $n$  particle is

$$v_n = \frac{m_{n-1}[1 + e_{(n-1)n}]}{m_n + m_{n-1}} v_{n-1} \quad (16)$$

Through the recursion, we obtained

$$v_n = \prod_{i=2}^n \frac{m_{i-1}[1 + e_{(i-1)i}]}{m_i + m_{i-1}} \cdot v_1 \quad (17)$$

It is assumed that the velocities of damper before and after the first particle collides with the damper are respectively  $V_1$  and  $V_2$ . The recovery coefficient  $e_0$  between the first particle and the damper is given by

$$\frac{0 - V_2}{V_1 - v_1} = e_0 \quad (18)$$

So the velocity  $v_n$  becomes

$$v_n = \prod_{i=2}^n \frac{m_{i-1}[1 + e_{(i-1)i}]}{m_i + m_{i-1}} \cdot [e_0 V_1 + V_2] \quad (19)$$

From the above equation, after the collision between the particle group and the damper, the

velocity  $v_n$  is related to the number of particles, the materials of particles, the velocity before the collision and the nature of damper.

When the motion status of particle unit change, it will return to the above calculation process, enter into the next time step, and calculate the movement increment of particle units again. Through the iterative computation, the real-time tracking of the movement of each particle unit can be done.

## 2.4. Collision Energy Dissipation Factor

For the collision of any particle [3, 14], it is assumed that the velocity before the collision is  $v_{n0}$  and the velocity after the collision is  $v_{nt}$ . According to the law of conservation of momentum, the velocities of  $v_{(n-1)t}$  and  $v_{nt}$  are written as

$$\begin{aligned} v_{(n-1)t} &= v_{(n-1)0} - \frac{m_n(1+e)}{m_n + m_{n-1}} [v_{(n-1)0} - v_{n0}] \\ v_{nt} &= v_{n0} - \frac{m_{n-1}(1+e)}{m_n + m_{n-1}} [v_{(n-1)0} - v_{n0}] \end{aligned} \quad (20)$$

The energy dissipation factor could be calculated by the following equation

$$\eta = \frac{[m_{n-1}v_{(n-1)0}^2 + m_nv_{n0}^2] - [m_{n-1}v_{(n-1)t}^2 + m_nv_{nt}^2]}{2\pi \cdot [m_{n-1}v_{(n-1)0}^2 + m_nv_{n0}^2]} \quad (21)$$

## 3. Collision Energy Dissipation Model Among the Particles

### 3.1. Experimental Equipment

A cube damper is used in experiment, which the material is 40Cr, length  $\times$  Width  $\times$  Height is 70 mm  $\times$  40 mm  $\times$  120 mm, and the wall thickness is 4 mm. The schematic diagram and photographic picture of the impact damper with particles are shown in Fig. 4.

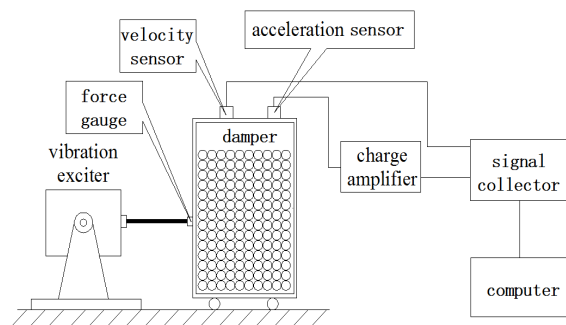


Fig. 4. The experiment system.

The test system is composed of the signal acquisition analyzer of INV3018C, ICP acceleration sensor of INV9822, acceleration and force compound sensor of CL-YD-331 and electromagnetic exciter of JZ-50 etc. The number of sampling points within each vibration cycle was not less than 400 and the experiment was repeated 5 times under every initial condition.

## 3.2. Results and Analysis

In accordance with the theoretical model, the regular smooth sphere particles are used, with 3mm diameter and 90 % filling rate. The test materials are the aluminum oxide, steel and tungsten carbide with the densities of 3.42 g/cm<sup>3</sup>, 7.86 g/cm<sup>3</sup> and 18.3 g/cm<sup>3</sup>.

Fig. 5 compares theoretical and experimental results of the relationship of damper energy dissipation factor vs. acceleration under different densities of particle. From Fig. 5, the energy dissipation factor of particles along with the acceleration is nonlinear. Due to less density, in the same volume, the energy dissipation caused by inelastic collision of aluminum oxide particles is less and its damping effect is also less. With the increase of particle density, the energy dissipation caused by inelastic collision becomes larger. The damping effect of tungsten carbide particle is the most considerable.

It would appear from the data presented in Fig. 5 that the theoretical and experimental results are in good agreement. This proves the rationality for energy dissipation model of impact damper with particles. In addition, the energy dissipation values of the numerical simulation are slightly less than the experimental values. And the reason for such deviation is that the numerical simulation only considered main mechanism of energy dissipation i.e. collision, however, many mechanisms of energy dissipation like the collision, the friction, the noise, the adhesion are included in the experiment.

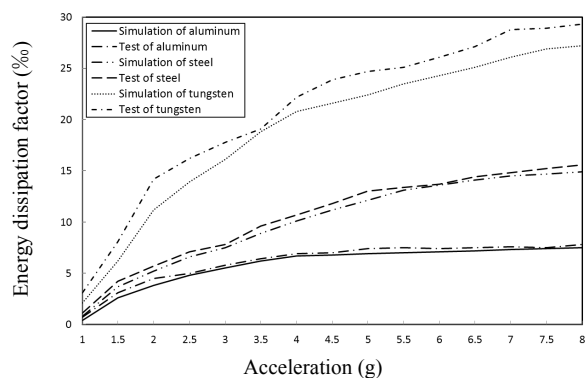


Fig. 5. The effect of acceleration and density of particles on energy dissipation factor.

The effect of acceleration and particle size on the energy dissipation factor is shown in Fig. 6. The test material is the steel. It can be seen from this plot that the energy dissipation factor of particles will almost increase with the increase of the acceleration. With the increase of particle size from 0.5mm to 4mm, the energy dissipation factor becomes larger, however, when the particle size is more than 4mm, it begins decreasing. So for the steel particle, too small or too large particle size will both not obtain good damping effect, and the energy dissipation of 4 mm steel particle is the best in this damper. The reason for such turn is that when the particle size is too small, the adhesion among the particles will reduce the damping effect; when the particle size is too large, the particle number of involved in the collision decrease, which will also reduce the damping effect. Therefore, in the design of dampers, choosing a proper particle size is necessary and the optimal particle size can be determined by this simulation model.

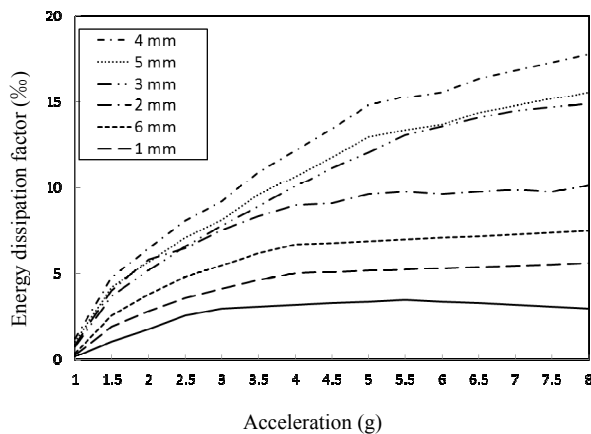


Fig. 6. The effect of acceleration and particle size on energy dissipation factor.

After the collisions of the outer layer of particles and the damper complete, the relationship between the velocity of adjacent particles and the recovery coefficients is shown in Fig. 7. The recovery coefficients of collision among the particles are respectively 0.2, 0.4, 0.6. From the Fig. 7, it can be observed that after colliding with the damper, the movement velocities of the adjacent particles reduce rapidly. The low recovery coefficients among the particles result in a faster reduction of velocity, a larger energy dissipation, and a greater damping effect. When the movement is transferred to the seventh adjacent particles, if the recovery coefficient  $e$  is less than 0.4, the velocity will reduce to about 5% initial velocity after collision. So, the velocity attenuation of particles is very obvious.

The vibration amplitude of damper has great influence on the damping effect of impact damper with particles. The relationship between the velocities of the adjacent particles and the vibration amplitude of the damper after the outer layer of particle collides with the damper is shown in Fig. 8.

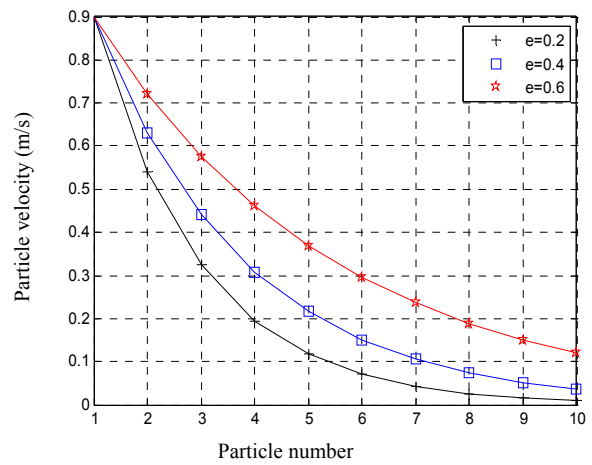


Fig. 7. The velocity distribution after collision under different recovery coefficients.

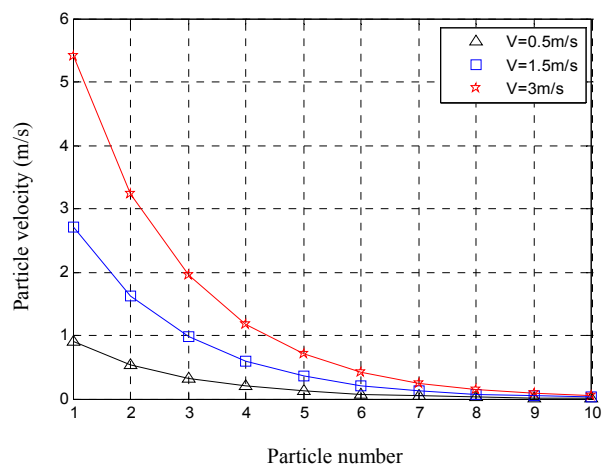


Fig. 8. The velocity distribution after collision under different vibration amplitudes.

The vibration velocities of the damper are taken as  $V = 0.5m/s$ ,  $V = 1.5m/s$ ,  $V = 3m/s$ . It can be seen from this plot that the larger amplitude of vibration velocity of damper will result in a higher velocity of adjacent particles, but a faster velocity attenuation of particles. It indicates that the energy dissipation effect of particle group will greatly increase when the vibration amplitude of the damper increase. When the movement is transferred to the eighth adjacent particle, all the particle velocity can be reduced to about 3% initial velocity after collision.

The energy dissipation distribution vs. the recovery coefficient of particle and the initial velocity of damper is shown in Fig. 9.

It should be noted from this plot that with the initial velocity of damper rising, the energy dissipation of particles increases and the energy attenuation becomes faster at higher velocity. It indicates that with more violent vibration, the number of particles involved in the collision increases and the energy dissipation effect of is enhanced. With the recovery coefficient decreasing, the energy dissipation effect is enhanced in the impact damper with particles.

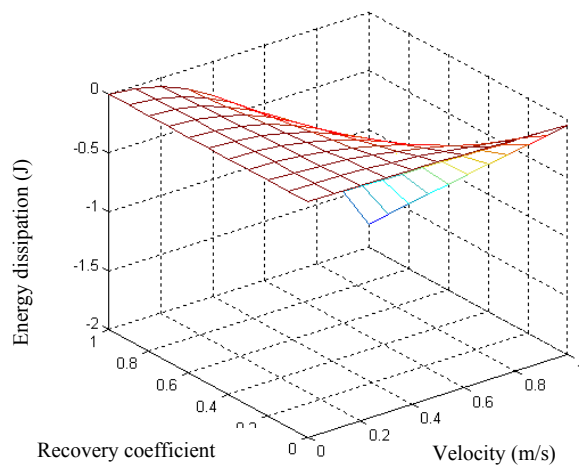


Fig. 9. Variations of energy dissipation with recovery coefficient and initial velocity.

#### 4. Conclusion

This paper establishes an energy dissipation model of impact damper with particles based on the collision theory. Through the experiment, the parameters of particles are further researched into, such as the particle number, the particle density, the particle size, the recovery coefficient of granular material and the initial velocity of damper, which have a great effect on the damping characteristics.

The energy dissipation factor of particles along with the acceleration is nonlinear. The theoretical and experimental results are in good agreement. The energy dissipation values of the numerical simulation are slightly less than the experiment. And the reason for such deviation is that the numerical simulation only considered main mechanism of energy dissipation i.e. collision, however, many mechanisms of energy dissipation like the collision, the friction, the noise, the adhesion are included in the experiment.

With the increase of particle density, the energy dissipation caused by inelastic collision becomes larger. The damping effect of tungsten carbide particle is the most considerable. Too small or too large particle size will both not obtain good damping effect, and the energy dissipation of 4mm steel particle is the best in this damper. Therefore, in the design of dampers, choosing a proper particle size is necessary and the optimal size can be determined by this simulation model.

After colliding with the damper, the movement velocities of the adjacent particles reduce rapidly. The low recovery coefficients among the particles result in a faster reduction of velocity, and a greater damping effect. When the recovery coefficient  $e$  is less than 0.4, the velocity attenuation of particles is very obvious. The larger amplitude of vibration velocity of damper will result in a faster velocity attenuation of particles. The energy dissipation effect of particle group will greatly increase when the vibration amplitude of the damper increase.

The results show that the impact damper with particles can effectively reduce the kinetic energy of structure, have an apparent damping effect. This simulation model will provide the engineering applications with the theoretical guidance and design criteria.

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