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A Multi-Gradients Algorithm for the Channel Equalizer of the Parallel Structure

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Abstract: With the development of science and technology, the information exchange rate on communication network is increasing gradually. The data throughput of the conventional structure equalizer is limited by the running speed of the electronic device, and result in that information exchange rate in the network is restricted. To achieve a high throughput of the transmission data, the parallel structure of the channel equalizer is designed, and the corresponding algorithms are studied. The paper proposes a Multi-Gradients Algorithm architecture based on a modified version of the standard Bussgang algorithm for the parallel structure equalizer. The convergence characteristics of the algorithm can be improved while the data throughput of the parallel equalizer is maintained. A Multi-Gradients constant modulus Algorithm and A Multi-Gradients multi-modulus Algorithm are given by using Multi-Gradients Algorithm architecture on constant algorithm and multi-modulus. Simulation results demonstrate that using MG-CMA and MG-MMA improve the performance of the channel equalizer by increasing the convergence rate and enhance the data throughput. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Channel equalization, Inter-symbol interference, Parallel structure, Throughput, Convergence rate.

1. Introduction

In modern high data rate communication systems, such as wireless local-area networks, digital subscriber lines and satellite digital transmission system, and digital cable television, the receiver employs an equalizer to cancel the inter-symbol interference (ISI) introduced by the channel [1]. While traditionally a training sequence or a pilot signal is employed to adapt the equalizer settings, in the past decade there has been a growing interest in blind equalization schemes, which do not require a training sequence [2]. These classical blind equalizers were capable of recovering the true power of transmitted data upon convergence and were classified as Bussgang-type [3].

In Bussgang-type algorithms, the estimate of transmission signal on the equalizer is obtained through the linear weighting of interval time delay signals, the working mechanism result in the fact that the maximum data throughput of the class Bussgang equalizer is limited by the running speed of the electronic device.

In this paper, the parallel structure of the channel equalizer is researched to achieve a high throughput of the transmission data the adaptive digital filters.

However, the Parallel filtering structure reduces the utilization rate of the transmission data and is leads to a decreased convergence rate of the algorithm in comparison with the standard Bussgang algorithm.

We, therefore, propose a Multi-Gradients Algorithm architecture based on a modified version of the standard Bussgang algorithm for the parallel structure equalizer. The convergence characteristics of the algorithm can be improved while the data throughput of the parallel equalizer is maintained.

2. System Analysis

Consider a based-band linear time-invariant single-input-single-output discrete-time system with a channel impulse response of $\{h_i(k)\}$. Assume that the transmitted symbols $\{s(k)\}$ is independently and identically-distributed (i.i.d.), and takes values of (square) QAM constellations with equal probability. The received signals with additive white Gaussian noise are expressed as:

$$x(k) = \left[\sum\nolimits_i h_i(k)s(k-i)\right] + v(k) \tag{1}$$

The structure of the inter-symbols interference channel in (1) is shown in Fig. 1.

The output of the equalizer is written as:

$$y(k) = \sum\nolimits_i w_i(k)x(k-i) \tag{2}$$

The standard structure of the channel equalizer in (2) is shown in Fig. 2.

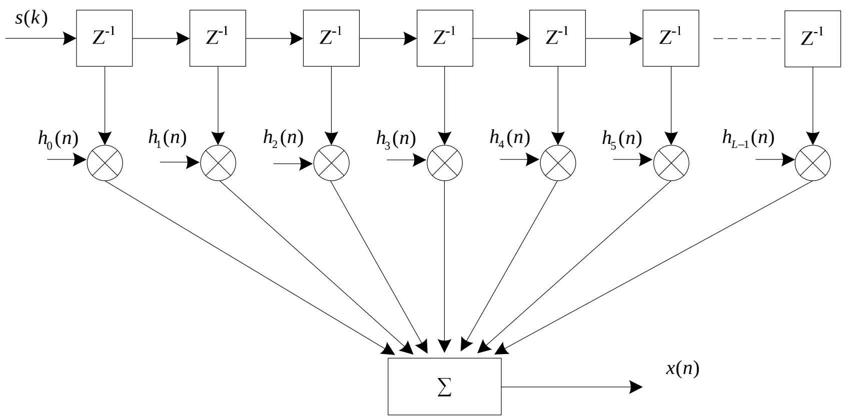

Fig. 1. The structure of the inter-symbols interference channel.

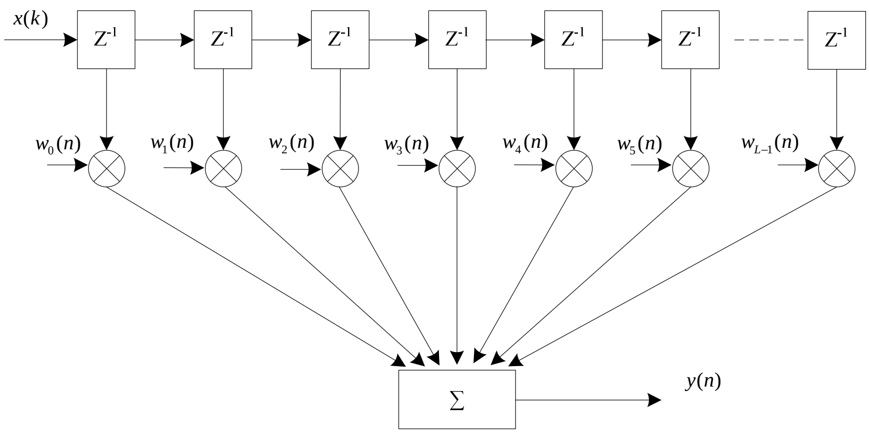

Fig. 2. The standard structure of the channel equalizer.

For the convenience of analysis, omitting the noise in the system, the (1) is expressed as a vector form as below:

$$x(k) = \boldsymbol{h}_k^{\mathrm{T}} \cdot \boldsymbol{s}_k \, , \tag{3}$$

where $\boldsymbol{h}_k$ denote $[h_0(k), h_1(k), \ldots, h_{N-1}(k)]$, $\boldsymbol{s}_k$ denote $[s_0(k), s_1(k-1), \ldots, s(k-\mathrm{N}-1)]$ the (1) is expressed as a vector form as below:

$$y(k) = \boldsymbol{w}_k^{\mathrm{T}} \cdot \boldsymbol{x}_k \, , \tag{4}$$

where $\boldsymbol{h}_k$ denote $[w_0(k), w_1(k), \ldots, w_{L-1}(k)]^{\mathrm{T}}$, $\boldsymbol{s}_k$ denote $[s_0(k), s_1(k-1), \ldots, s(k-\mathrm{N}-1)]^{\mathrm{T}}$.

In (4), $\boldsymbol{x}_k$ are composed of the delay component of the received signals, the relationship with the transmitted signal is expressed as:

$$\boldsymbol{x}_k = \boldsymbol{H}_k \cdot \overline{\boldsymbol{s}}_k\,, \tag{5}$$

where $\boldsymbol{H}_k$ denote a Toeplitz matrix generated by the vector of the channel impulse response.

Substituting (5) into (4), we can obtain

$$y(k) = \boldsymbol{w}_k^{\mathrm{T}} \cdot \boldsymbol{H}_k \cdot \overline{\boldsymbol{s}}_k \tag{6}$$

Substituting $\boldsymbol{c}^{\boldsymbol{T}}_k = \boldsymbol{w}^{\boldsymbol{T}}_k \boldsymbol{H}_k$ into (6), we can obtain

$$y(k) = \boldsymbol{c}_k^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_k \tag{7}$$

where $\boldsymbol{c}^{\boldsymbol{T}}_k$ is the impulse response of the combined channel-equalizer system.

When the equalization algorithm has achieved the ideal state, the combined response can be expressed as:

$$\boldsymbol{c}_k = [0, \cdots, 0, \exp(j\cdot\theta), 0, \cdots, 0]^{\mathrm{T}} \tag{8}$$

At this time, the equalizer output signals can be expressed as:

$$y(k) = s(k-l)\cdot \exp(j\cdot\theta) \tag{9}$$

The above process obtain the conclusion the realization condition of the channel equalization is that the numerical characterization of the impulse response of the combined channel-equalizer system meet (8).

3. The Parallel Equalizer

The increasing of the data throughput of the equalizer can realize the processing of the high-speed data flows by low speed processor. Parallel processing is an effective way to increase the data throughput, but the inter-symbol interference phenomenon is the mutual influence between the symbols of the transmission, simple parallel structure will break the serial relations of the transmission symbols so that the equalizer generates a larger steady-state error that can’t be removed.

This part through the analysis of channel equalization will study the control method of the high speed data flow for the parallel receiving, and the parallel equalizer structure of the high data throughput will is given.

In the parallel signal processing, the running rate of the processor is different with the sample rate of the equalization algorithm, the time variable of the processor and the sample is set as k and n, respectively.

In the dual line of the parallel structure equalizer, two time variables have the following relationship:

$$n = \begin{cases} 2k+1 \\ 2k+2 \end{cases} \quad k = 0, 1, \cdots, \tag{10}$$

By (9), the signals of the equalizer output can be expressed respectively after k time.

$$\begin{cases} y(k) = \boldsymbol{c}_k^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_k \\ y(k+1) = \boldsymbol{c}_{k+1}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_{k+1} \\ y(k+2) = \boldsymbol{c}_{k+2}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_{k+2} \\ y(k+3) = \boldsymbol{c}_{k+3}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_{k+3} \\ \vdots \end{cases} \tag{11}$$

When the equalization algorithm has converged before k times, by (9) shows that the output signals of the equalizer is

$$\begin{cases} y(k) = s(k-m)\cdot \exp(j\cdot\theta) \\ y(k+1) = s(k+1-m)\cdot \exp(j\cdot\theta) \\ y(k+2) = s(k+2-m)\cdot \exp(j\cdot\theta) \\ y(k+3) = s(k+3-m)\cdot \exp(j\cdot\theta) \\ \vdots \end{cases}, \tag{12}$$

by double equalizer realizing the function of (12), then the first equalizer relation between input and output signal can be equivalently written as:

$$\begin{cases} y_1(n) = \boldsymbol{c}_{1,n}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_k \\ y_1(n+1) = \boldsymbol{c}_{1,n+1}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_{k+2} \\ \vdots \end{cases}, \tag{13}$$

then the second equalizer relation between input and output signal can be equivalently written as:

$$\begin{cases} y_1(n) = \boldsymbol{c}_{1,n}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_k \\ y_1(n+1) = \boldsymbol{c}_{1,n+1}^{\mathrm{T}} \cdot \overline{\boldsymbol{s}}_{k+2} \\ \vdots \end{cases} \tag{14}$$

According to Equation (5) on the relationship between the transmitting symbols and the received signal, and Equation (6) on the relationship between the relationship of the impulse response of the equalizer and combined channel-equalizer, the Equation (13) can be written as equalizer forms:

$$\begin{cases} y_1(n) = \boldsymbol{w}_{1,n}^{\mathrm{T}} \cdot \boldsymbol{x}_k \\ y_1(n+1) = \boldsymbol{w}_{1,n+1}^{\mathrm{T}} \cdot \boldsymbol{x}_{k+2} \\ \vdots \end{cases} \tag{15}$$

The Equation (13) can be written as equalizer forms:

$$\begin{cases} y_2(n) = \boldsymbol{w}_{2,n}^{\mathrm{T}} \cdot \boldsymbol{x}_{k+1} \\ y_2(n+1) = \boldsymbol{w}_{2,n+1}^{\mathrm{T}} \cdot \boldsymbol{x}_{k+3} \\ \vdots \end{cases} \tag{16}$$

By the Equation (15) and Equation (16), the general form of dual parallel equalizer can be expressed as:

$$\begin{cases} y_1(n) = \boldsymbol{w}_{1,n}^{\mathrm{T}} \cdot \boldsymbol{x}_{2k+1} \\ y_2(n) = \boldsymbol{w}_{2,n}^{\mathrm{T}} \cdot \boldsymbol{x}_{2k+2} \end{cases} \quad k = 0,1,\cdots \tag{17}$$

At this time, the estimation signals of the equalizer output is

$$\begin{cases} y_1(n) = s(2k+1-l)\exp(j\theta) \\ y_2(n) = s(2k+2-l)\exp(j\theta) \end{cases} \quad k = 0,1,\cdots \tag{18}$$

So far, we have obtained the parallel structure of equalizer, as shown in Fig. 3 and Fig. 4. This structure can enhance the data throughput of the equalizer, and realize the processing of the high-speed data flows by the low speed equalizer.

4. The Multi-Gradients Algorithm

From Equation (17), the parallel equalizer of the double lines is shown as follows:

$$\begin{cases} y_1(n) = \boldsymbol{w}_{1,n}^{\mathrm{T}} \cdot \boldsymbol{x}_{2k+1} \\ y_2(n) = \boldsymbol{w}_{2,n}^{\mathrm{T}} \cdot \boldsymbol{x}_{2k+2} \end{cases} \quad k = 0,1,\cdots \tag{19}$$

The error function of the constant modulus equalization algorithm [4] is

$$\begin{aligned} e_{1,\mathrm{CMA}}(n) = &\left[y_{1,R}(n) + j \cdot y_{1,I}(n) \right] \\ &\left[\left| y_{1,R}(n) \right|^2 + \left| y_{1,I}(n) \right|^2 - R_C \right] \end{aligned} \tag{20}$$

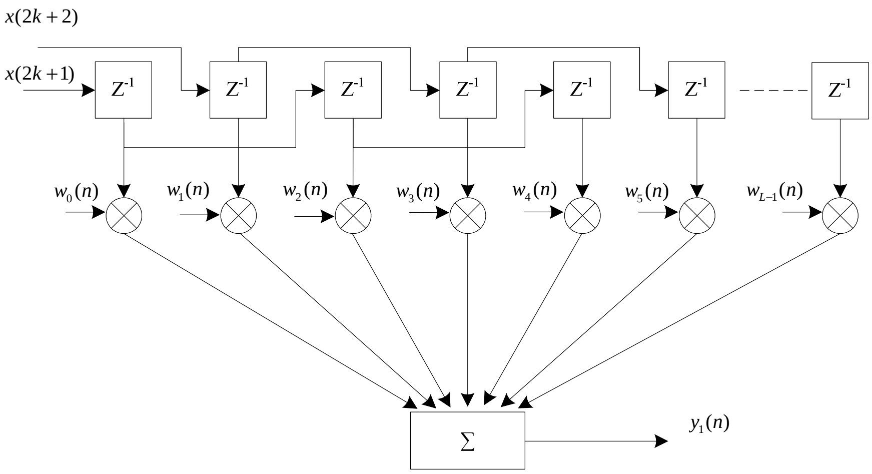

Fig. 3. The parallel structure of the equalizer for the first lines.

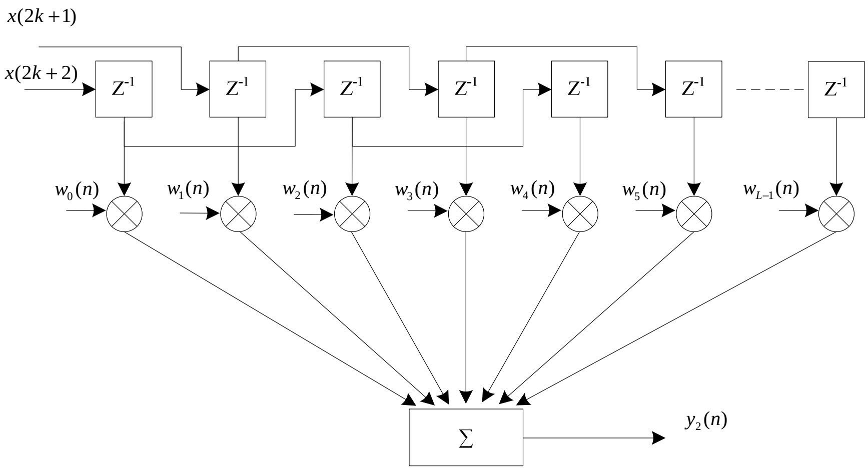

Fig. 4. The parallel structure of the equalizer for the second lines.

The error function of the constant modulus equalization algorithm [4] is

$$e_{1,\mathrm{CMA}}(n)=\left[y_{1,R}(n)+j\cdot y_{1,I}(n)\right]\left[\left|y_{1,R}(n)\right|^2+\left|y_{1,I}(n)\right|^2-R_C\right] \tag{20}$$

The error function of the multi-modulus equalization algorithm [5] is

$$e_{1,\mathrm{MMA}}(n)=y_{1,R}(n)[|\,y_{1,R}(n)\,|^2-R_R]+j\cdot y_{1,I}(k)\cdot[|\,y_{1,I}(n)\,|^2-R_I] \tag{21}$$

When the first equalizer is set as the object of the control, the tap coefficients updating equation of the parallel structure equalizer is obtained as:

$$\begin{cases}\boldsymbol{w}_{1,n+1}=\boldsymbol{w}_{1,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\\ \boldsymbol{w}_{2,n+1}=\boldsymbol{w}_{2,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\end{cases}, \tag{22}$$

where $e_1(n)$ is to be set $e_{1,CMA}(n)$ in (20) for the constant modulus algorithms, $e_1(n)$ is to be set $e_{1,MMA}(n)$ in (20) for the constant modulus algorithms.

When the second equalizer is set as the object of the control, the tap coefficients updating equation of the parallel structure equalizer is obtained as:

$$\begin{cases}\boldsymbol{w}_{1,n+1}=\boldsymbol{w}_{1,n}-\mu\cdot e_2(n)\cdot\boldsymbol{x}^*_{2k+2}\\ \boldsymbol{w}_{2,n+1}=\boldsymbol{w}_{2,n}-\mu\cdot e_2(n)\cdot\boldsymbol{x}^*_{2k+2}\end{cases} \tag{23}$$

where $e_2(n)$ is to be set $e_{2,CMA}(n)$ in (20) for the constant modulus algorithms, $e_2(n)$ is to be set $e_{2,MMA}(n)$ in (21) for the constant modulus algorithms.

When two equalizers are set to the object of the control with the same time, and the tap coefficients are updated by itself error function, respectively, the update equation of the parallel equalizer is expressed as:

$$\begin{cases}\boldsymbol{w}_{1,n+1}=\boldsymbol{w}_{1,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\\ \boldsymbol{w}_{2,n+1}=\boldsymbol{w}_{2,n}-\mu\cdot e_2(n)\cdot\boldsymbol{x}^*_{2k+2}\end{cases} \tag{24}$$

From Equation (24) we can see, two error functions at the same time are used to control the tap coefficient of the equalizer, but the tap coefficient update of two equalizers is independent, and the utilization ratio of the signal did not improve, and increase the hardware cost of the algorithm implement.

The double channel equalizer is extended to the general form, and then the parallel structure of the channel equalizer for *r* lines is get as:

$$\begin{cases}y_1(n)=\boldsymbol{w}^{\mathrm{T}}_{1,n}\cdot\boldsymbol{x}_{2k+1}\\ y_2(n)=\boldsymbol{w}^{\mathrm{T}}_{2,n}\cdot\boldsymbol{x}_{2k+2}\\ \vdots\\ y_r(n)=\boldsymbol{w}^{\mathrm{T}}_{r,n}\cdot\boldsymbol{x}_{2k+r}\end{cases}\quad k=0,1,\cdots \tag{25}$$

The double channel equalizer is extended to the general form, and then the parallel structure of the channel equalizer for *r* lines is get as:

$$\begin{cases}\boldsymbol{w}_{1,n+1}=\boldsymbol{w}_{1,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\\ \boldsymbol{w}_{2,n+1}=\boldsymbol{w}_{2,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\\ \vdots\\ \boldsymbol{w}_{r,n+1}=\boldsymbol{w}_{r,n}-\mu\cdot e_1(n)\cdot\boldsymbol{x}^*_{2k+1}\end{cases} \tag{26}$$

The relationship between the processor time variable and the sample time variable is expressed as:

$$n=\begin{cases}2k+1\\ 2k+2\\ \vdots\\ 2k+r\end{cases}\quad k=0,1,\cdots \tag{27}$$

To enhance the convergence rate of the algorithm and improve the utilization ratio of the received signals for the parallel equalizer, the cost function of the Multi-Gradient algorithm is defined as:

$$J(n)=J_1(n,2k+1)+J_2(n,2k+2) \tag{28}$$

The implement method of the Multi-Gradient algorithm is realized by the stochastic gradient-based adaptive approach, and then the conjugate gradient of $J(n)$ with respect to $\boldsymbol{w}_k$ is

$$\nabla_w J(n)=\frac{\partial J_1(n,2k+1)}{\partial\boldsymbol{w}^*_n}+\frac{\partial J_2(n,2k+2)}{\partial\boldsymbol{w}^*_n} \tag{29}$$

The tap coefficient update equation of the algorithm is get as:

$$\begin{cases}\boldsymbol{w}_{1,n+1}=\boldsymbol{w}_{1,n}-\mu\cdot e_{\mathrm{MG}}(n)\\ \boldsymbol{w}_{2,n+1}=\boldsymbol{w}_{2,n}-\mu\cdot e_{\mathrm{MG}}(n)\end{cases} \tag{30}$$

In Equation (30), the error function of the algorithm is given as:

$$e_{\mathrm{MG}}(n)=[e_1(n)\boldsymbol{x}^*_{2k+1}+e_2(n)\boldsymbol{x}^*_{2k+2}] \tag{31}$$

By Equation (28) to (31), the Multi-Gradient algorithm of the parallel equalizer for double lines can is extended to the general form of *r* lines, and then the parallel structure of the channel equalizer for *r* lines is get as:

$$J\left(n\right)=\sum\nolimits_{i=1}^{r}J_{i}\left(n,2k+i\right) \tag{32}$$

The conjugate gradient of J(n) with respect to wk for r lines is

$$\nabla_{w}J\left(n\right)=\sum_{i=1}^{r}\frac{\partial J_{i}\left(n,2k+i\right)}{\partial \boldsymbol{w}_{n}^{*}} \tag{33}$$

The tap coefficient update equation of the algorithm is given as:

$$\begin{cases}\boldsymbol{w}_{1,n+m+1}=\boldsymbol{w}_{1,n+m}-\mu\cdot e_{\mathrm{MG}}(n)\\ \qquad\vdots\\ \boldsymbol{w}_{r,n+m+1}=\boldsymbol{w}_{r,n+m}-\mu\cdot e_{\mathrm{MG}}(n)\end{cases} \tag{34}$$

The error function of the algorithm is given as:

$$e_{\mathrm{MG}}(n)=\sum\nolimits_{i=1}^{r}e_{i}(n)\cdot\boldsymbol{x}_{2k+i}^{*} \tag{35}$$

5. Simulation

5.1. The Condition of the Simulation

The complex base-band model of the communication system is used to test the performance of the algorithm, the transmission channel is modeled as the filter of the finite impulse response. The real part of the channel filter is given as:

$$\begin{aligned}\mathbf{h}_{\mathrm{R}}=[&0.0091\quad -0.0247\quad 0.0538\\ &-0.1256\quad 0.2407\quad 0.7046\\ &0.2407\quad -0.1256\quad 0.0538\\ &-0.0247\quad 0.0091]\end{aligned} \tag{36}$$

The image part of the channel filter is given as:

$$\begin{aligned}\mathbf{h}_{\mathrm{I}}=[&0.0014\quad 0.0044\quad -0.0032\\ &-0.0154\quad 0.2506\quad -0.4669\\ &0.2506\quad -0.0154\quad -0.0032\\ &0.0044\quad 0.0014]\end{aligned} \tag{37}$$

From Equation (36) and (37), the tap coefficient of the channel filter is expressed as $h=h_R+jh_I$.

In the simulation, the residual ISI equation is used to evaluate the overall performance of the equalizer, and is defined as:

$$\mathrm{ISI(dB)}=\frac{10\log[\sum_{k}|c(k)|^{2}-\max(|c(k)|^{2})]}{\max(|c(k)|^{2})}, \tag{38}$$

where $c(k) = h(k)*w(k)$ denote the impulse response of the combine channel-equalizer systems.

5.2. The Simulation

The Fig. 5 shows that the ISI performance curve of the equalization algorithm is simulated by the 16-quadrature amplitude modulation systems with additive white noise.

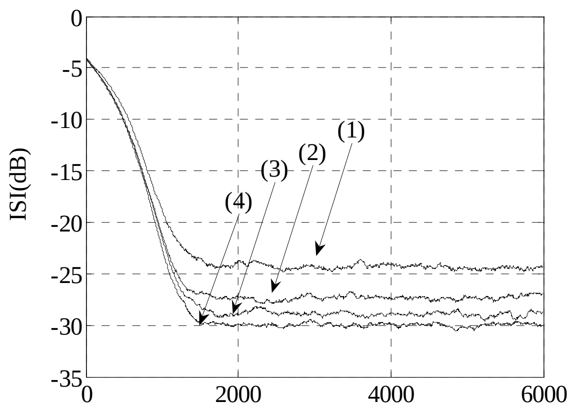

Fig. 5. The ISI performance comparison of 16-QAM signals with SNR=25.

In Fig. 5, (1) denotes the performance curve of the ISI using the parallel equalizer of the dual lines for the conventional constant modulus algorithm. The (2), (3) and (4) curve denote the performance curve of the ISI for the equalizer structure of the dual lines, the three lines and the four lines, respectively, using the Multi-Gradients multi-modulus Algorithm. Corresponding to the step size of the algorithm is 5×10^{-5}, 3×10^{-5}, 2×10^{-5} and 1.5×10^{-5}, respectively. In the figure, the (1), (2), (3) and (4) curve in the 2000 iterations converges to -24 dB -28 dB, -29 dB and -30 dB, respectively.

The Fig. 6 shows that the ISI performance curve of the equalization algorithm is simulated by the 16-Amplitude Phase Shift Keying modulation systems with additive white noise.

In Fig. 6, (1) denotes the performance curve of the ISI using the parallel equalizer of the dual lines for the conventional constant modulus algorithm. The (2), (3) and (4) curve denote the performance curve of the ISI for the equalizer structure of the dual lines, the three lines and the four lines, respectively, using the Multi-Gradients constant modulus Algorithm. Corresponding to the step size of the algorithm is 2.5×10^{-4}, 1.4×10^{-4}, 2×10^{-4}, 1×10^{-4} and 0.75×10^{-4}, respectively. In the figure, the (1), (2), (3) and (4) curve in the 2000 iterations converges to -24 dB -28 dB, -29 dB and -30 dB, respectively.

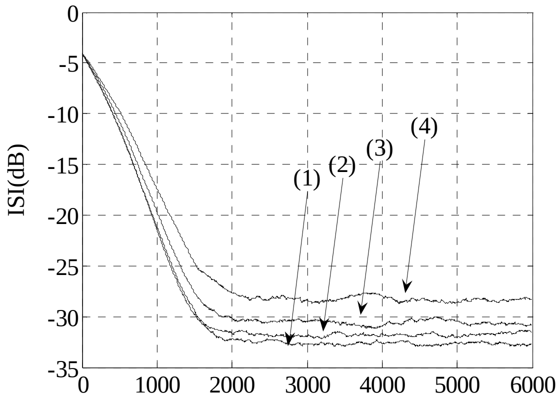

Fig. 6. The ISI performance comparison of 16-APSK signals with SNR=25.

From the Fig. 5 and Fig. 6, it can be seen that due to the enhancement on the utilization rate of the transmission signal, the equalization performance of the parallel equalizer is improved, using same number iteration to obtain the lower steady-state error with each other.

7. Conclusions

In this paper we have designed a parallel structure of channel equalizer to achieve a high throughput of the transmission data, and Multi-Gradients Algorithm architecture has be proposed for the structure equalizer. The parallel equalizer based on the Multi-Gradients Algorithm enabled us to simultaneously achieve good convergence characteristics, and high throughput characteristics.

References

[1]. B. F. Bassel, S. I. Rohit, Analysis and Compensation for Nonlinear Interference of Two High-Order Modulation Carriers over Satellite Link, *IEEE Transactions on Communications*, Vol. 58, No. 6, 2010, pp. 1824-1833.

[2]. S. V. Vaerenbergh, J. Vía, I. Santamaría, Blind Identification of SIMO Wiener Systems Based on Kernel Canonical Correlation Analysis, *IEEE Transactions on Signal Processing,* Vol. 61, Issue 9, 2013, pp. 2219-2230.

[3]. S. Abrar, A. K. Nandi, Adaptive Solution for Blind Equalization and Carrier-Phase Recovery of Square-QAM, *IEEE Signal Processing Letters*, Vol. 17, No. 9, 2010, pp. 791-794.

[4]. J. T. Yuan, T. C. Lin, Equalization and Carrier Phase Recovery of CMA and MMA in Blind Adaptive Receivers, *IEEE Transactions on Signal Processing*, Vol. 58, No. 6, 2010, pp. 3206-3217.

[5]. J. T. Yuan, K. D. Tsai, Analysis of the multimodulus blind equalization algorithm in QAM communication systems, *IEEE Trans. on Com.*, Vol. 53, Issue 9, 2005, pp. 1427-1431.