Modeling and Application of Vehicular Cyber Physical System Based Petri Nets

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Abstract: Mobile cyber physical system (MCPS) has been a hot research area, where mobile nodes can mobile, and communicate with each other. As a typical MCPS, vehicular cyber physical system (VCPS) plays an important role in intelligent transportation, especially in collision avoidance. There is no, however, a formal modeling and analysis method for VCPS. In the paper, the modeling method based Petri nets (PN) is presented. Furthermore, the behavior expression analysis method is also presented which can deal with arbitrary distribution timed transitions. Finally, a case is introduced to verify the effectiveness about proposed method, and the results show that VCPS can greatly reduce the reaction time of vehicles behind when emergent accident occurs and then enhance the traffic safety. Copyright © 2014 IFSA Publishing, S. L.

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1. Introduction

In the traditional concepts of computing systems and physical systems, the cyber space separates from the physical world [1]. With the development of embedded computing, sensor monitoring, wireless communication, data processing and other technologies, the physical processes, computing processes and communication processes can be highly integrated which leads to the Cyber-physical system (CPS). CPS represents a system which tightly integrates computation, communication and physical processes [2]. In CPS, through dynamic sensing of physical environments and resources, real-time reliable information transmission, comprehensive computing processing of data, with feedback cycle, the physical process is effectively controlled. Moreover, the whole process is completed automatically with people in the auxiliary position.

CPS involves computer science, network communication and control theory and multiple disciplines which is widely applied to real systems including transportation, aerospace, telemedicine, power control and many large infrastructure systems [3-5], and other local or fine systems such as precision agriculture, electronic endoscope. The impact of CPS will far bypass the IT revolution at 20th, and it will change the interactive mode between human and physical world just as Internet has changed it among humans.

As one sub-domain of CPS, Mobile Cyber Physical Systems (MCPS) are gaining importance as
key enablers of emerging applications. In MCPS, the autonomously mobile components move over time and can communicate with each other. As a kind of MCPS, Vehicular Cyber Physical Systems (VCPS) can significantly enhance road safety, driving experience and enrich on-road entrainments [6]. It has attracted a lot of research interests in the past few decades [7].

Although VCPS enables a large number of interesting applications, collision avoidance is the major goal of VCPS, and it has the most significant importance [8-9]. It is reported [10] that vehicle collision is typically due to the delay in a driver’s response time when its leader suddenly decelerates. VCPS will greatly help to reduce collisions. When a collision occurs, the emergency message can be quickly notified to the other subsequent drivers in the vehicle chain that can’t typically see the initial deceleration event but could avoid an accident if drivers receive an early warning and then have enough time to react in the emergency situation. The example is shown in Fig. 1.

![Fig. 1. Emergency messages transmission in VCPS.](image)

Although the performance of collision avoidance in VCPS has been widely dealt with, the efforts are mainly based on simulation. The formal model about emergency message transmission scheme not only helps to evaluate the safety application performance, but also construct the functional relations between different parameters and overall performance. As a formal modeling tool, Petri Nets (PN) is very suitable for discrete event dynamic system, especially for description of sequential, parallel, conflict and synchronization relations [11]. Thus, the system modeling and evaluation technologies based on PN are widely proposed. Different types of PN (such as Object-Oriented PN, Colored PN, Stochastic PN and Changeable Structure PN) adapts to different system requirements respectively.

In the paper, the formal model of safety application of VCPS based on PN is constructed. Furthermore, to evaluate the PN model with different timed transitions, the behavior expressions analysis method is adopted which can not only deal with PN with arbitrary distribution time transitions, but also conveniently obtain function relationship for evaluation. Moreover, a case is presented, and the modeling and performance evaluation results are also presented and discussed.

2. PN and Performance Evaluation

Because there are different timed actions in collision avoidance such as message forward, drivers’ brake reaction, the Extended Stochastic Petri Nets (ESPN) is adopted which supports non-exponential distribution transitions. Furthermore, the analysis method based on behavior expression is also introduced to evaluate the performance of ESPN.

### 2.1. Extended Stochastic Petri Nets (ESPN)

An ESPN is a seven-tuple $(P, T, I, O, H, m, F)$, where [12],

\[ P = \{p_1, p_2, \ldots, p_n\}, \quad n > 0, \quad \text{is a finite set of places}; \]

\[ T = \{t_1, t_2, \ldots, t_s\}, \quad s > 0, \quad \text{is a finite set of transitions with} \]

\[ I : P \times T \rightarrow N, \quad \text{and is an input function, where} \]

\[ N = \{0, 1, 2, \ldots\}; \]

\[ O : P \times T \rightarrow N, \quad \text{and is an output function}; \]

\[ H : P \times T \rightarrow N, \quad \text{and is an inhibitor function}; \]

\[ m : P \times T \rightarrow N^*, \quad \text{and is a marking whose $i$th component is the number of token in the $i$th place.} \]

An initial marking is denoted by $m_0$;

\[ F : T \rightarrow R, \quad \text{is a firing time delay with an extended distribution function}. \]

### 2.2. Analysis Method Based Behavior Expression

A behavior expression is compound or power series. It can depict the bounded PN, or some unbounded PN (with existence of expression). According to the expressions and by means of the following theorems, we can obtain the transfer function $W$ of PN, and then analysis method of moment generating function can be used to deal with the quality analysis of arbitrary distribution PN.

**Theorem 1** [13]. Set $\alpha$ is a monomial, $\alpha = t_1t_2\cdots t_q$, then $W_{\alpha}(s) = \prod_{i=1}^{q} W_{t_i}(s)$.

**Theorem 2** [14]. Set $\alpha$ is a standard polynomial, $\alpha = \alpha_1 + \alpha_2 + \cdots + \alpha_s$, then,

\[ W_{\alpha}(s) = \sum_{i=1}^{s} W_{\alpha_i}(s) \quad (1) \]

**Theorem 3** [13]. Set $\alpha = (\alpha')^*$, then,

\[ W_{\alpha}(s) = \frac{1}{1 - W'_{\alpha}(s)} \quad (2) \]

The specific steps are as follows [15]:

1. Generate the behavior expression of ESPN, and convert the polynomial into the standard polynomial.
2) Figure out firing probability and moment generating function according to the distribution parameter and the behavior expression structures. Next, base the transfer function definition, the transfer function of each event can be obtained.

3) Remark the behavior expression to differentiate the same events with different transfer function in expressions according to the second step of the calculation results.

4) Compute the transfer function of remarked behavior expressions according to Theorem 1 to 3.

5) Evaluate performance according to the above results and methods of moment generating function, obtain the quantitative analysis results.

3. Modeling

A car highway platoon example is presented to explain the modeling method which is similar to that of Ref. [8] and shown in Fig. 2.

![Fig. 2. Collision avoidance of VCPS.](image)

Without loss of generality, the vehicles are travelling at a speed of 90 km/hr (25 m/s). If there is no VCPS, when an accident of V0 happens, V1 recognizes it and takes $t_r$ time to act and starts an emergency brake. The $t_r$ time is dependent on many factors such as distractions of drivers, vehicle types, and can be one to several seconds. Next, V2 recognizes it and do the same thing. Finally, when Vn-1 has emergency brake, Vn initiates a deceleration and stops after continuing moving an amount of distance (Stopping sight distance). If there is VCPS, the $t_r$ is much shorter which is mostly dependent on the wireless latency and usually less than 100 ms. Thus, when an accident takes place, much less time is taken to react and stop.

3.1. Emergency Message Analysis

Based on the above scenes, the reaction time of drivers without VCPS or wireless latency with VCPS is modeled as timed transitions of PN with firing rate $\lambda$ which is shown in Fig. 3. The timed transitions represent the emergency message relay among vehicles. For Non-VCPS, it means the reaction time of drivers and usually one to three seconds; for VCPS, it means the wireless latency between vehicle nodes and usually 100 ms.

![Fig. 3. Emergency message relay.](image)

3.2. Emergency Brake

The vehicle nodes of VCPS have two states: move and stop. When vehicle node $V_n$ receives the emergency message, it initiates the emergency brake and its states accordingly turns from move to stop which is modeled as Fig. 4.

![Fig. 4. Emergency brake.](image)

The firing rate of transition $t_{n+1}$ needs more considerations which is approximated by an exponential distributed random value. When vehicle node $V_n$ is traveling at the speed $v$ and suddenly makes an emergency brake, the deceleration rate is $a$, then it will take $\frac{v}{a}$ seconds to completely stop. Thus, the firing rate of transition $t_{n+1}$ can be modeled as $\frac{a}{v}$.

4. Case Analysis

4.1. ESPN Model

We consider a scene with 9 vehicles in VCPS, that means $n = 9$. The whole ESPN model is shown in Fig. 5.

When an accident happens suddenly, the first vehicle initiates emergent alert message which is forwarded backward one by one until it reaches the final vehicle node $V_9$ in VCPS. Next, vehicle node $V_9$ brakes and decelerates till completely stops.
The meanings of places are described in Table 1, and the firing rates are listed in Table 2. \( t_1 \) to \( t_8 \) are determined timed transitions with rate \( \lambda \); \( t_9 \) are exponential distribution with firing rate \( \frac{a}{v} \).

**Table 1. Meaning of places.**

<table>
<thead>
<tr>
<th>Place</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 \ldots p_9 )</td>
<td>Vehicle nodes</td>
</tr>
<tr>
<td>( p_{10} )</td>
<td>Move state</td>
</tr>
<tr>
<td>( p_{11} )</td>
<td>Stop state</td>
</tr>
</tbody>
</table>

**Table 2. Meaning of transitions.**

<table>
<thead>
<tr>
<th>Transition</th>
<th>Meaning</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 \ldots t_8 )</td>
<td>Emergency message forward</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>( t_9 )</td>
<td>Brake</td>
<td>( \frac{a}{v} )</td>
</tr>
<tr>
<td>( t_{10} )</td>
<td>Return</td>
<td>Immediate</td>
</tr>
</tbody>
</table>

\[ W_{t_1} = \cdots = W_{t_8} = e^{\lambda s} \] (3)

\[ W_{t_9} = \frac{\lambda_{t_9}}{\lambda_{t_9} - s} = \frac{a/v}{a/v - s} = \frac{a}{a - vs} \] (4)

\[ W_{t_{10}} = 1 \] (5)

There is no need to remark \( \delta \), for this does not exist different transfer function of the same event for transitions. According to Theorem 1-3, compute the transfer function of \( \delta \):

\[ W_\delta(s) = e^{\lambda s} e^{\frac{a}{v} s} e^{\frac{a}{v} s} e^{\frac{a}{v} s} e^{\frac{a}{v} s} e^{\frac{a}{v} s} e^{\frac{a}{v} s} \frac{a}{a - vs} 1 \] (6)

Then, the mean time from emergency event initiation to completely stop of vehicle \( V_n \) is:

\[ T = \frac{\partial}{\partial s} W_\delta(s) \bigg|_{s=0} = \]

\[ = a \frac{8 \lambda e^{\lambda s} (a - vs) \frac{\partial}{\partial s} (\lambda s + ve^{\lambda s})}{(a - vs)^2} \bigg|_{s=0} = \]

\[ = a \left( 8 \lambda e^{\lambda s} (a - vs) + ve^{\lambda s} \right) \bigg|_{s=0} = 8 \lambda + \frac{v}{a} \] (7)

Let \( s = 0 \) in \( W_1(s) \), except for \( t_9 \), and now:

\[ T_1 = \frac{\partial}{\partial s} W_1(s) \bigg|_{s=0} = \frac{av}{(a - vs)^2} \bigg|_{s=0} = \frac{v}{a} \] (8)

is just the time when vehicle \( V_n \) begins to brake to completely stop.

Let \( s = 0 \) in \( W_2(s) \), for \( t_9 \), and now:

\[ T_2 = \frac{\partial}{\partial s} W_2(s) \bigg|_{s=0} = 8 \lambda e^{8 \lambda s} \bigg|_{s=0} = 8 \lambda \] (9)

So, from emergent accident occurring to vehicle \( V_n \) completely stop, it has travelled the distance

\[ D = T_2 * v + \frac{v T_1}{2} = 8 \lambda v + \frac{v^2}{2a} \] (10)

Furthermore, the steady state probability

\[ P(M_0) = \frac{T_1}{T} = \frac{a}{8 \lambda + \frac{v}{a}} \] (11)

**5. Results and Discussion**

It is shown in Fig. 6 that with the vehicle speed is increasing, the probability of braking state for vehicle \( V_n \) is also growing. However, with VCPS the probability is much higher than that without VCPS. It is because without VCPS, much time is used in the emergent message transmissions between vehicles, and with VCPS, the time of message transmission to vehicle \( V_n \) is much less. Vehicle \( V_n \) consumes greater proportional time on taking collision avoidance actions.
Consider another scene, the vehicles are traveling at a speed of 90 km/hr (25 m/s), and the deceleration speed is 7.5 m/s². Then vehicle V_n traveling distance is shown in Fig. 7 after the accident happens.

![Travel distance after accident.](image)

Fig. 7. Travel distance after accident.

It can be shown in Fig. 7 that the collision avoidance function of VCPS has great impact on travel distance, and then on traffic security. Without VCPS, the drivers' reaction time is usually much longer, and it normally is 2.5 s which is dependent on many factors. Moreover, it can be shown in Fig. 6 vehicle V_n will have travel 541.67 m before it completely stops since the accident takes place. With VCPS, however, the accident event transmission time between vehicles nodes is usually less than 0.1 S. So, the travel distance is 61.67 which is much shorter.

It is shown in Fig. 8 that with VCPS, when vehicle V_n moves faster, the travel distance increases relatively slowly. On the contrary, without VCPS, when the speed increases, the travel distance to completely stop since an emergent accident happens grows greatly. Therefore, with VCPS a vehicle can be stop quicker and safer to avoid collision with previous vehicles.

It is shown in Fig. 9 that when the deceleration rate is increasing, the travel distance is decreasing for both scenes with VCPS and without VCPS. However, with VCPS, the travel distance is greatly less than that without VCPS at all decelerating rates.

It is shown in Fig. 10 that VCPS has great advantages on collision avoidance, especially for a fleet with relatively more vehicles. With VCPS, the travel distance is increasing much more slowly, however, without VCPS, it increases rapidly with the growth of vehicles. With VCPS, vehicles behind accident far away can also quickly get the emergent messages and take correct actions on time which can avoid possible collisions.

![Travel distance with vehicle speed v.](image)

Fig. 8. Travel distance with vehicle speed v.

![Travel distance with decelerating rate](image)

Fig. 9. Travel distance with decelerating rate.

![Travel distance with the number of vehicles](image)

Fig. 10. Travel distance with the number of vehicles.

6. Conclusions

As one of the representative MCPS, VCPS plays a crucial role on intelligent transportation, safety, entertainment and other applications in the future. Collision avoidance, among which is the most important application of VCPS. In the paper, the formal modeling method based ESPN of VCPS is presented which can be applied to collision avoidance. Furthermore, the behavior expressions analysis method is also presented to evaluate the ESPN model which can deal with arbitrary distribution timed transitions. Finally, a case is presented to
verify the effectiveness of modeling and performance evaluation method of VCPS. Moreover, it shows that when a sudden accident happens, vehicles with VCPS can generate the emergency message to vehicles nodes behind it which can greatly reduce the travel distance for vehicles on potentially dangerous zone.

We only consider communications between vehicles. The roadside infrastructure communication support should be also dealt with which can help to mitigate unreliable cases of communications between vehicles and enhance the VCPS reliability. So, accordingly, the novelty model should be constructed for analysis of VCPS which will be our future work.

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References