Application of PCA-LINMAP Coupling Algorithm and Model in Determining Weight of Influence Factors of Surface Deformation

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Abstract: Based on analyzing the limitations and shortcomings of the traditional method of empowerment, the PCA-LINMAP coupling model which uses to determine the weight of influence factors of surface deformation is established by principal component analysis and linear programming. This method completely considers the difference and the merits order of the samples, and overcomes the one-sidedness of the traditional method of empowerment in determining weight of index. The index system can reflect the object to be evaluated comprehensively and objectively, at the same time, provide an ideal way for improving the scientific of evaluation. Examples show that the method is scientific and reasonable; it also has extensive application value.

Keywords: Surface subsidence, PCA-LINMAP coupling algorithm, Influence factors, Weight.

1. Introduction
The surface subsidence deformation which is caused by underground mining is a complicated mechanics process. The factor set that influences the surface subsidence deformation is a mutually affiliation and restriction complex system (Yu and Zhang, 2004; He et al., 1994; Guo and Zhang, 2007; Xia et al., 2008; Xia 2008). In addition, there is great difference between factors which influence the surface subsidence deformation. The prediction and evaluation are much objective and exact if strictly consider every detailed factor, but more factors will bring about much inconvenience on practical prediction and evaluation. So accurately predict the impacts of each factor, there is an important significance not only to the research of mineral resources exploitation, but also to the mining disasters control, the ecological environment of the mining area improving and the sustainable development strategy implementing and so on.

The comprehensive analysis of multiple indicators system is the effective tool to improve the overall evaluation. The same model of this system analysis can be used in multi-objective decision, multi-factor analysis and system quality evaluation and so on, according to the difference of the meaning and purpose. And to solve this kind of problem which needs to reveal weight of each target (indexes or factors) in the whole system. In this sense, to determine the index weights is an important part of comprehensive system analysis. In addition, once the index weights are identified, the key factors or the key indicators which affects the comprehensive evaluation effect of the system will be known it also provides the theoretical basis for system research and
improvement (Guo et al., 2013; Jin et al., 2012; Qin, 2003; Zhang and Wang 2008).

The basic idea of PCA-LINMAP coupling algorithm is beginning from the given sample-index data. The sample merit order and the principal component which significantly reflect the sample differences can be found by means of PCA, then the weights of each index can be gotten through inputting the ordered pair to LINMAP (Liu and Zhang, 2012; Zhou 2006; Chen et al., 2004).

2. PCA-LINMAP Coupling Weighting Algorithm and Model

PCA-LINMAP coupling model consists of two basic sub-models, i.e. PCA model and LINMAP model. The specific application process is deriving the sample merit order from the initial decision matrix by PCA sub-model, and then, the weights of each index can be gotten according to the order pair of samples by LINMAP sub-model.

Where PCA is the abbreviation of principal component analysis, LINMAP is the abbreviation of linear programming techniques for multi-dimensional analysis preference.

2.1. The Basic Principle and Algorithm of PCA Sub-model

The principal component analysis method in multivariate statistical analysis is widely used to evaluate (comprehensive evaluation) and sort in engineering calculation field, because of the simplicity of its theory and the objective in weighting and so on. Its basic principle is to find few irrelevant (or independent) comprehensive indexes as basic indexes to evaluate the subjects from more indexes. To achieve the purpose of revealing the relationship among variables and simplifying the data and analyzing total data characteristics, the basic algorithm is following as (Qin 2003).

Step 1: Collecting p dimension random vectors, the n samples \( x=(x_1, x_2, \ldots, x_p)^T \) \((i=1,2,\ldots,n; n>p)\) will construct sample array

\[
X = \begin{bmatrix}
X_1^T \\
X_2^T \\
\vdots \\
X_n^T
\end{bmatrix} = 
\begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1p} \\
x_{21} & x_{22} & \cdots & x_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{np}
\end{bmatrix}, \tag{1}
\]

Step 2: Making the following transform form the elements of the sample matrix X

\[
y_{ij} = \begin{cases} x_{ij}, & \text{The positive matrix} \\
-x_{ij}, & \text{The inverse matrix}
\end{cases}
\]

Step 3: Making standard transform of the elements of matrix Y

\[
z_j = \frac{(y_{ij} - \bar{y}_j)}{s_j}, \quad i=1,2,\ldots,n, \quad j=1,2,\ldots,p, \tag{2}
\]

where \(\bar{y}_j = \frac{\sum_{i=1}^n y_{ij}}{n}, \quad s_j^2 = \frac{\sum_{i=1}^n (y_{ij} - \bar{y}_j)^2}{n-1} \)

Step 4: Calculating of sample correlation matrix from the standardized matrix Z

\[
R = \left[ r_{ij} \right]_{p \times p} = \frac{Z^T Z}{n-1}, \tag{4}
\]

where \(r_{ij} = \frac{\sum_{k=1}^n z_{ik} \cdot z_{kj}}{n-1}, \quad i,j=1,2,\ldots,p \).

Step 5: Solving the characteristic equation of the sample correlation coefficient R

\[
|R - \lambda I_p| = 0, \tag{5}
\]

The sorting result of eigenvalues is following \(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0\).

Step 6: Making use of the principle \(\sum_{j=1}^k \lambda_j \geq 0.85\) to confirm the principal components, letting utilization ratio of information is more than 85%. For each of the \(\lambda_j, \quad j=1,2,\ldots,m\), solving the function \(Rb = \lambda_j b\), so the unit eigenvectors can be gained.

\[
b_j = \frac{b_j}{|b_j|}. \tag{6}
\]

Step 7: Finding out the m principal component of \(z_i = (z_{i1}, z_{i2}, \ldots, z_{ip})^T, \quad i=1,2,\ldots,n \), \(u_{ij} = z_i^T b_j^T \), \(j=1,2,\ldots,m\).
So the decision matrix can be conclude.
\[
U = \begin{bmatrix}
    u_1^T \\
    u_2^T \\
    \vdots \\
    u_n^T
\end{bmatrix} = \begin{bmatrix}
    u_{11} & u_{12} & \cdots & u_{1m} \\
    u_{21} & u_{22} & \cdots & u_{2m} \\
    \vdots & \vdots & \cdots & \vdots \\
    u_{n1} & u_{n2} & \cdots & u_{nm}
\end{bmatrix},
\]
(7)

where \( u_i \) presents the principal component vector belonging to the i-th sample, \( i = 1,2,\ldots,n \).

Step 8 Reducing the dimension system to a one-dimensional system by selecting the appropriate principal component value function model.

2.2. The Basic Principle of LINMAP Sub-model

The standardization decision matrix \([z_{ij}]_{10p}\) indicates that there are \( n \) sample points at \( p \) dimension index space. Now assuming \((z_1^*, z_2^*, \cdots, z_p^*)^T\) expresses the ideal point of the most preference sample index space of the decision maker, so the weighted Euclid distance square \( S_j \) between any sample point \((z_{i1}, z_{i2}, \cdots, z_{ip})^T\) of index space and ideal point is following as:
\[
S_j = \sum_{i=1}^{p} w_j (z_{ij} - z_j^*)^2, i = 1,2,\cdots,p,
\]
(8)

where \( w_j (j = 1,2,\cdots,p) \) presents the weight square of the i-th index.

Then, according to the preference order to PCA of the decision maker, the ascending sequence of \( S_j \) can be obtained if converting \( S_j \) from formula (8), where \( w_j \) and \( z_j^* \) are unknown numbers in formula (8), the purpose of LINMAP method is to solve \( w_j \).

2.3. The Basic Algorithm of LINMAP Sub-model

Definition 1. Let the set of the sample order pair \((k,l)\) as
\[
Q = \{(k,l)\} \text{the } k \text{-th sample is better than the } l \text{-th sample},
\]
(9)

If \( S_k \geq S_l \), because the priority of formula (8) and the order pair is consistent, so the inconsistency degree is zero; if \( S_k \leq S_l \), because the priority of formula (8) and the order pair is inconsistent, the inconsistency degree depends on \( S_k - S_l \). So,

\[
\text{Definition 2. If the inconsistency degree between samples sorting by formula (8) and the order pair } (k,l) \text{ was recorded as } (S_k - S_l), \text{ so,}
\]
\[
(S_k - S_l) = \begin{cases}
0 & S_k \geq S_l \\
S_k - S_l & S_k < S_l
\end{cases},
\]
(10)

The total inconsistency degree can be gotten through adding the inconsistency degree of all the order pairs \((k,l)\) in \( Q \), it is called inconsistency degree B, that is
\[
B = \sum_{(k,l) \in Q} (S_k - S_l),
\]
(11)

So the consistency G can be defined by the similar method,
\[
G = \sum_{(k,l) \in Q} (S_k - S_l),
\]
(12)

where
\[
(S_k - S_l) = \begin{cases}
[S_k - S_l] & S_k \geq S_l \\
0 & S_k < S_l
\end{cases} = \text{max} \{0,(S_k - S_l)\},
\]
(13)

Lets \( h = G - B \), \( h \) is obviously a non-negative integer when the sorting according to formula (8) and the order pairs \((k,l)\) is consistent.

Furthermore,
\[
G - B = \sum_{(k,l) \in Q} (S_k - S_l)^T - \sum_{(k,l) \in Q} (S_l - S_k)^T
\]
\[
= \sum_{(k,l) \in Q} [(S_k - S_l)^T - (S_l - S_k)^T]
\]
\[
= \sum_{(k,l) \in Q} (S_k - S_l),
\]
(14)

Let \( \lambda_{kl} = \text{max} \{0,(S_k - S_l)\} \), so the square \( w_j \) of index weight can be obtained through the linear programming problem, following as
\[
\min \sum_{(k,l) \in Q} \lambda_{kl},
\]
(15)

subject to
\[
\sum_{(k,l) \in Q} (S_k - S_l) + \lambda_{kl} \geq 0, \forall (k,l) \in Q
\]

and
\[
\lambda_{kl} \geq 0, \forall (k,l) \in Q
\]
(16)
Combining formula (1) and $S_i$, the LINMAP sub-model can be gotten, following as

$$\min _{(k,j) \in Q} \lambda _{kj},$$

$$\sum _{j=1} ^p w_j (z_j^2 - z_j^2) - 2 \sum _{j=1} ^p v_j (z_j - z_j) + \lambda _{kj} \geq 0,$$

s.t.

$$\sum _{j=1} ^p w_j \sum _{u=1} ^p (z_{uj} - z_{uj}) - 2 \sum _{j=1} ^p v_j \sum _{(k,j) \in Q} (z_j - z_j) = h,$$

$$w_j > 0, j = 1, 2, \cdots, p,$$

$$\lambda _{kj} \geq 0, \forall (k,j) \in Q,$$

$$v_j = w_j, \text{unrestraint}$$

(17)

(18)

3. Engineering Application

As described in the introduction, due to the diversity of surface subsidene deformation factors, also there are interaction effects among the factors. So determining the comprehensive influence of surface subsidene deformation factors and the influence difference of each factor to surface subsidene deformation, not only has a great theoretical significance, but also has important guiding significance to the production practice.

The surface deformation system is a complex open system, the factors influencing the deformation are multi-faceted. Generally speaking, it can be divided into geological factors and engineering factors (Xia et al., 2008; Xia 2008; Wu et al., 2011; Xia 2005). At the same time, each category can be further divided into specific factors. According to previous research results, the geological factors affecting surface deformation is classified into the following categories:

- $x_1$: durability coefficient (MPa);
- $x_2$: mining depth (m);
- $x_3$: mining thickness (m);
- $x_4$: tilt angle ($^\circ$);
- $x_5$: topsoil thickness (m)

According to 208 typical observation station data of surface movement in the literature (Yu and Zhang, 2004; He et al., 1994), screening out 8 measured data as the sample, see Table 1.

The eigenvalue and the eigenvector of correlation coefficient matrix can be gained through inputting the observation data into PCA model, see Table 2.

So the principal component analysis result (merit evaluation and sorting) can be gotten by PCA model, see Table 3.

<table>
<thead>
<tr>
<th>Number of stations</th>
<th>Durability coefficient $x_1$</th>
<th>Mining depth $x_2$</th>
<th>Mining thickness $x_3$</th>
<th>Tilt angle $x_4$</th>
<th>Topsoil thickness $x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.2</td>
<td>60</td>
<td>2.1</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>3.74</td>
<td>42</td>
<td>1.45</td>
<td>10.5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>175</td>
<td>1.6</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>6.0</td>
<td>280</td>
<td>2.4</td>
<td>12</td>
<td>17.4</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>98.5</td>
<td>2.0</td>
<td>16</td>
<td>64</td>
</tr>
<tr>
<td>6</td>
<td>2.5</td>
<td>181</td>
<td>1.94</td>
<td>9.5</td>
<td>110</td>
</tr>
<tr>
<td>7</td>
<td>1.3</td>
<td>325</td>
<td>8.2</td>
<td>4.3</td>
<td>197</td>
</tr>
<tr>
<td>8</td>
<td>5.0</td>
<td>130</td>
<td>1.9</td>
<td>37</td>
<td>34.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Evaluation groups</th>
<th>Eigenvectors (Principle component part)</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of index</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.4083</td>
<td>-0.6809</td>
<td>-0.1502</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.4076</td>
<td>-0.6623</td>
<td>-0.2254</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5077</td>
<td>-0.0166</td>
<td>-0.3636</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.3686</td>
<td>0.2549</td>
<td>-0.8684</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.5229</td>
<td>0.1805</td>
<td>-0.2008</td>
<td></td>
</tr>
<tr>
<td>Eigenvalue</td>
<td></td>
<td>3.2513</td>
<td>0.9210</td>
<td>0.6563</td>
</tr>
<tr>
<td>Contribution ratio</td>
<td></td>
<td>0.6503</td>
<td>0.1842</td>
<td>0.1313</td>
</tr>
<tr>
<td>Cumulative contrib.</td>
<td></td>
<td>0.6503</td>
<td>0.8345</td>
<td>0.9658</td>
</tr>
</tbody>
</table>
Table 3. The results of principal component analysis (the rank program).

<table>
<thead>
<tr>
<th>Number of stations</th>
<th>The evaluation value of the first principal component</th>
<th>The evaluation value of the second principal component</th>
<th>The evaluation value of the third principal component</th>
<th>Comprehensive evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.9022</td>
<td>3.1886</td>
<td>-2.2404</td>
<td>-2.2444, 7</td>
</tr>
<tr>
<td>2</td>
<td>-2.6899</td>
<td>1.6735</td>
<td>3.2406</td>
<td>-1.0155, 6</td>
</tr>
<tr>
<td>3</td>
<td>-1.0039</td>
<td>-1.1608</td>
<td>1.4476</td>
<td>-0.6766, 5</td>
</tr>
<tr>
<td>4</td>
<td>0.2752</td>
<td>-4.5459</td>
<td>0.4980</td>
<td>-0.5930, 4</td>
</tr>
<tr>
<td>5</td>
<td>0.7487</td>
<td>1.5033</td>
<td>0.6895</td>
<td>-0.1194, 3</td>
</tr>
<tr>
<td>6</td>
<td>2.6272</td>
<td>0.0971</td>
<td>1.4280</td>
<td>1.9139, 2</td>
</tr>
<tr>
<td>7</td>
<td>9.3858</td>
<td>-2.1475</td>
<td>-0.1356</td>
<td>5.6902, 1</td>
</tr>
<tr>
<td>8</td>
<td>-3.9435</td>
<td>1.3917</td>
<td>-4.9277</td>
<td>-2.9551, 8</td>
</tr>
</tbody>
</table>

The merit order $O$ is following as

\[ O = \{\text{observation station 7, observation station 6, observation station 5, observation station 4, observation station 3, observation station 2, observation station 1, observation station 8}\} \]

Then the order set $Q$ of observation stations is following as

\[ Q = \{(7,6),(7,5),(7,4),(7,3), (7,2),(7,1),(6,5),(6,4),\]
\[ (6,3),(6,2),(6,1),(5,4), (5,3),(5,2),(5,1),(4,3), (4,2),(4,1),(3,2),(3,1), (2,1),(1,8)\} \]

Putting $Q$ into LINMAP model (18), the weight square vector $w$ of the five indexes can be found out by the simplex method (Su 2004; Zhang 2010), following as

\[ w = (0.3473 \ 0.7859 \ 0.5193 \ 0.4806 \ 0.4265)^T \]

So the five index weight vector $\bar{w}$ as

\[ \bar{w} = (0.5893 \ 0.8865 \ 0.7206 \ 0.6932 \ 0.6531)^T \]

normalization $\rightarrow$

\[ (0.1663 \ 0.2502 \ 0.2304 \ 0.1957 \ 0.1843)^T \]

4. Conclusion

The conclusions were gotten due to the above studying:

1) The PCA-LINMAP coupling algorithm and model is used firstly in prediction of surface subsidence deformation, it overcomes the disadvantage of traditional statistical method by the sample size, provides a fast and effective method for the engineering prediction and evaluation.

2) The method is proved to be feasible by the instance, also the affecting difference of each factor can be differentiated by making use of this method to evaluate the surface subsidence deformation. In the same time, these results of method provide the basis for the comprehensive evaluation and prediction of the surface subsidence deformation.

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