Rigid Rotor Dynamic Balancing with the Influence Coefficient Method

1 Xiang XU, 2 Ping Ping FAN

1 The State Key Laboratory of Fluid Power transmission and Control Zhejiang Provincial Key Laboratory of Advanced Manufacturing Technology, Institute of Manufacturing Engineering Zhejiang University, Hangzhou 310027, China
2 Mechanical Engineering Department, School of Engineering & Science, Stevens Institute of Technology, Castle Point on Hudson, Hoboken NJ 07030-5991 USA

1 Tel.: 18258853297, 2 +1.201.216.5000
E-mail: xuxiang218@126.com

Received: 22 August 2013 /Accepted: 25 September 2013 /Published: 31 October 2013

Abstract: On the base of the causes of the rotor's unbalanced vibration, the rigid rotor mathematic model was established to analyze the vibration form of a rigid rotor under unbalance inertia force. The principle of the influence coefficient method had been presented, and the principle of the cross-correlation method to extract the fundamental frequency signal of unbalance vibration was introduced. The process of conducting 1-plane and 2-plane dynamic balancing were described in detail. The method was found to be effective and practical. Finally through examples testify, it verified the feasibility of the rigid rotor dynamic balancing with the influence coefficient method. Copyright © 2013 IFSA.

Keywords: Influence coefficient method, Cross-correlation method, Rigid rotor, Dynamic balance.

1. Introduction

The rotor is the core component of rotary machines. When a rotor rotating, its centroid axis may deviate from its spin axis, for machining error or alignment error, etc.; and this could lead to the unbalance of a rotor. The main excitation source of rotary machines is the periodic vibration caused by such unbalance. Nowadays, the main methods used to balance the rotor are methods based on amplitude or phase, modal balancing technique and the influence coefficient method. The method this paper adopted is the influence coefficient method, which does not need that much information of the rotor system, and it can collect and process data automatically.

The key point of the rotor dynamic balancing is how to extract the fundamental frequency signal from the mixed signal collected by sensors. Researchers often apply fast Fourier transform (FFT) method, discrete Fourier transform (DFT) method, or cross correlation method to extract the desired signal. The cross correlation method has been adopted here, for it has a good depression on DC component restraint, high-order harmonic signal and random noise [1-2].
2. Principle of the Influence Coefficient Method

The influence coefficient method is based on the premise of the linear bearing system, using effect coefficient to express the linear relationship between the unbalance of the rotor and the vibration that caused by the rotor’s uneven quality. Assuming the rotor in the balanced speed of $\omega$, the i-th measuring point original unbalance is $A_{i0}$. After adding the trying weight $Q$ on the j-th correction plane, measure the i-th measuring point vibration $A_{ij}$ in the same measuring point.

In that way the influence coefficient is

$$\alpha = \frac{A_{ij} - A_{i0}}{m_j},$$ (1)

Set there are N balanced speeds in the rotor balancing system, measurement points and K correction planes. The unbalance and original vibration of the rotor are $Q$ and $A_0$. Then the influence coefficient matrix

$$\alpha_{i,j} = \begin{bmatrix}
\alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,K} \\
\alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,K} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{N,1} & \alpha_{N,2} & \cdots & \alpha_{N,K}
\end{bmatrix},$$ (2)

According to the principle of influence coefficient method, the correction vector $\overline{Q}$ must meet the equation (3).

$$\overline{\alpha} \cdot \overline{Q} + \overline{A}_0 = 0,$$ (3)

2.1. Calculate the Influence Coefficient

The way to apply influence coefficient method to correct the unbalance of a rigid rotor is regarding the rotor and the support system as a closed system. The input of the system is the trial weight on the correction planes; the output is the variation of the fundamental frequency signal of the rigid rotor under the same condition, i.e., neither the rotate speed nor the measure method changes. The transitive relationship of input and output is the influence coefficient of the system.

For 1-plane dynamic balancing, according to the equation (2) the influence coefficient is a plural $\alpha$. The process of calculating influence coefficient as follows.

1) Locate a phase reference point on the rotor; rotate the rotor at a balance speed. The amplitude and phase of fundamental frequency vibration vector $\overline{A}_0$.

$$\overline{A}_0 = A_0 \cos \theta_0 + j A_0 \sin \theta_0,$$ (4)

2) Add trial weight $m_1$ on correction plane I, and measure the amplitude and phase of fundamental frequency vibration vector $\overline{A}_1$ at the same speed.

$$\overline{A}_1 = A_1 \cos \theta_1 + j A_1 \sin \theta_1,$$ (5)

3) Use the following functions to calculate the influence coefficient.

$$\alpha = \frac{A_1 - A_0}{m_1},$$ (6)

4) Apply equation (7) to calculate the correction vector $\overline{Q}_1$ that needed to add on the correction plane.

$$\overline{\alpha} \cdot \overline{Q}_1 + \overline{A}_0 = 0,$$ (7)

For 2-plane dynamic balancing (Fig.1), according to the equation (2) the influence coefficient matrix is a $2 \times 2$ plural matrix.

The process to set up the matrix is

1) Locate a phase reference point on the rotor; rotate the rotor at a balance speed. The amplitude and phase of fundamental frequency vibration vector $\overline{A}_0$ and the amplitude on correction plane.

$$\overline{A}_0 = A_0 \cos \theta_0 + j A_0 \sin \theta_0,$$ (4)

2) Add trial weight $m_1$ on correction plane I, and measure the amplitude and phase of fundamental frequency vibration vector $\overline{A}_1$ and the
amplitude and phase $B_i$ and $\phi_i$ of fundamental frequency vibration vector $\vec{B}_i$ at the same speed.

$$\begin{align*}
\vec{A}_i &= A_i \cos \theta_i + jA_i \sin \theta_i, \\
\vec{B}_i &= B_i \cos \phi_i + jB_i \sin \phi_i,
\end{align*}$$  

(9)

3) Unload $m_1$, and add trial weight $m_2$ on correction plane II. Measure $\vec{A}_2$ and $\vec{B}_2$ using the same method.

$$\begin{align*}
\vec{A}_2 &= A_2 \cos \theta_2 + jA_2 \sin \theta_2, \\
\vec{B}_2 &= B_2 \cos \phi_2 + jB_2 \sin \phi_2,
\end{align*}$$  

(10)

4) Use the following functions to calculate the influence coefficient matrix.

$$\alpha = \begin{bmatrix} \frac{\vec{A}_2 - \vec{A}_1}{m_1 - m_2} & \frac{\vec{A}_2 - \vec{A}_1}{m_1 - m_2} \\ \frac{\vec{B}_2 - \vec{B}_1}{m_1 - m_2} & \frac{\vec{B}_2 - \vec{B}_1}{m_1 - m_2} \end{bmatrix},$$  

(11)

5) Apply equation (12) to calculate the correction vector $\vec{Q}_1$ and $\vec{Q}_2$ that needed to add on the correction plane I and II.

$$\begin{align*}
\alpha_{11} \cdot \vec{Q}_1 + \alpha_{12} \cdot \vec{Q}_2 + \vec{A}_1 = 0, \\
\alpha_{21} \cdot \vec{Q}_1 + \alpha_{22} \cdot \vec{Q}_2 + \vec{B}_2 = 0.
\end{align*}$$  

(12)

### 2.2. Foundation of Choosing 1-plane and 2-plane Dynamic Balance

The 1-plane and 2-plane dynamic balance are most commonly used in the production practice. Table 1 shows the Foundation of choosing 1-plane and 2-plane dynamic balance [4].

### 3. The Mechanics Principle of Rigid Rotor

When a rotor rotates at a constant speed, the numerous particle inertia force constitute the inertial force system, which can be simplified as a principal vector $\vec{R}$ and a principal moment $\vec{H}$ by theoretical mechanics simplifying rule of force system. The goal of balance of the rotor can be achieved by adding the corresponding balance weight on the rotor.

Set a rotor $A$ rotates around the Z axis at speed of $w$. Establish a coordinate system as shown in Fig. 2 [5, 6].

According to d’Alembert’s principle of a particle system, the principal vector $\vec{R}$ and principal moment $\vec{H}$ can be calculated as the equations (14).

$$\begin{align*}
\sum F_x &= R_x + \int \rho \omega^2 \cdot x \sqrt{x^2 + y^2} \, dm = 0, \\
\sum F_y &= R_y + \int \rho \omega^2 \cdot y \sqrt{x^2 + y^2} \, dm = 0, \\
\sum M_x &= H_x + R_x \cdot z = 0, \\
\sum M_y &= H_y + R_y \cdot z = 0, \\
\vec{R} &= \sqrt{R_x^2 + R_y^2}, \\
\vec{H} &= \sqrt{H_x^2 + H_y^2},
\end{align*}$$  

(13)

(14)

To balance the rotor, the principal vector $R_{add}$ and principal moment $H_{add}$ that caused by the correction mass must meet the equation (15).
\[
\begin{align*}
R + R_{\text{add}} &= 0 \\
H + H_{\text{add}} &= 0,
\end{align*}
\] 
(15)

3.1. Principle of 1-plane Dynamic Balancing

For the thin plate parts such as gear, etc. can be regarded as a small, thin, circular slice. The inertial force system can be regarded as concurrent forces, which can be simplified as a resultant \( F_r \) as shown in equation (16) and Fig. 3 [7].

\[
F_r = \sum F_i,
\] 
(16)

\[\begin{array}{c}
F_1 = \sum F_{1i} \\
F_{II} = \sum F_{2i}
\end{array}\] 
(18)

To balance the rotor, the correction mass must meet the equation (17) and the position must be opposite to:

\[
F_r - m_o\omega^2 e = 0,
\] 
(17)

3.2. Principle of 2-plane Dynamic Balancing

For the rotor whose length is greater than its diameter, it can be regarded as a composition of numerous small, thin, circular slices. (Fig. 4(a)) Assume \( F_i \) is the exciting force of each slice, which is caused by its unbalance; \( F_i \) passes the slice center and is perpendicular to the spin axis. \( F_i \) can be decomposed into two parallel forces \( F_{i1} \) and \( F_{i2} \) on two correction planes. Guide by this method, two coplanar force systems are concurrent on correction planes, and they could be presented as

\[
\begin{align*}
I_{1} &= \sum_{J} F_{1j} \\
II_{2} &= \sum_{JJ} F_{2j},
\end{align*}
\]

4. Dynamic Response of the Rigid Rotor

For the thin plate parts, the model of a rigid rotor can be simplified as an equivalence of the right and left dampers and springs with damping and stiffness coefficient respectively \( c \) and \( k \). (Fig. 5) [8].

\[
M \ddot{x} + c \dot{x} + kx = m\omega^2 e\sin(\omega t + \theta),
\] 
(19)

\[
\begin{align*}
x(t) &= \frac{m\omega^2 e}{M \sqrt{\left(k - \omega^2 M\right)^2 + \omega^2 e^2}} \sin(\omega t + \theta + \varphi) \\
\varphi &= \arctan \left( \frac{c\omega}{M(\omega^2 - \omega^2 e^2)} \right)
\end{align*}
\]
(20)

where \( \omega_n \) is the natural frequency of the rotor.

For the rotor whose length is greater than its diameter, the model of a rigid rotor can be simplified as an equivalence of the right and left dampers and springs with damping and stiffness coefficient respectively \( c_1, c_2 \) and \( k_1, k_2 \). (Fig. 6) [8].

Set the unbalanced force on correction planes I and II are \( T_I \) and \( T_{II} \); the dynamic functions of the rotating rotor are
Fig. 6. A simplified model of a rigid rotor for 2-plane dynamic balancing rotor.

\[
\begin{align*}
M\ddot{x} + c_1 \dot{x} + c_2 x + k_1 x_1 + k_2 x_2 \\
= T_1 \omega^2 \sin(\omega t + \phi_1) + T_2 \omega^2 \sin(\omega t + \phi_2) \\
J \dot{\theta} + l_c x_1 - l_c x_1 + k_1 x_1 - k_2 x_2 \\
= -T_1 \omega^2 a_1 \sin(\omega t + \phi_1) + T_2 \omega^2 a_2 \sin(\omega t + \phi_2)
\end{align*}
\]

(21)

where

\( x_1 = x - l_c \theta, \quad x_2 = x + l_c \theta \).

Applying the principle of linear superposition, the system response of the rigid rotor unbalanced force should be the superposition of the responses under \( T_1 \) and \( T_2 \) separately.

When the system is excited by \( T_1 \) independently, the kinematics formulation is:

\[
\begin{bmatrix}
M & 0 \\
0 & J
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
c_1 + c_2 & c_1 l_2 - c_2 l_1 \\
c_2 l_1 - c_2 l_2 & c_2 l_2^2 + c_1 l_2^2
\end{bmatrix}
\begin{bmatrix}
x \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
k_1 + k_2 & k_1 l_2 - k_2 l_1 \\
k_2 l_1 - k_2 l_2 & k_2 l_2^2 + k_1 l_2^2
\end{bmatrix}
\begin{bmatrix}
x \\
\theta
\end{bmatrix}
= \begin{bmatrix}
T_1 \omega^2 \sin(\omega t + \phi_1) \\
-T_2 \omega^2 a_1 \sin(\omega t + \phi_1)
\end{bmatrix}
\]

(22)

When the system is excited by \( T_2 \) independently, the kinematics formulation could be written as

\[
\begin{bmatrix}
M & 0 \\
0 & J
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
c_1 + c_2 & c_1 l_2 - c_2 l_1 \\
c_2 l_1 - c_2 l_2 & c_2 l_2^2 + c_1 l_2^2
\end{bmatrix}
\begin{bmatrix}
x \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
k_1 + k_2 & k_1 l_2 - k_2 l_1 \\
k_2 l_1 - k_2 l_2 & k_2 l_2^2 + k_1 l_2^2
\end{bmatrix}
\begin{bmatrix}
x \\
\theta
\end{bmatrix}
= \begin{bmatrix}
T_2 \omega^2 \sin(\omega t + \phi_2) \\
T_2 \omega^2 a_2 \sin(\omega t + \phi_2)
\end{bmatrix}
\]

(23)

5. Analysis of the Vibration Signal

Based on the former analysis, it is vital to measure amplitudes and phases, and to extract the effective fundamental vibration signal.

5.1. Acquisition of the Reference Signal

In order to get the reference signal, paste reflective material on the coupling between the motor and the rigid rotor. The photoelectric sensor would receive a pulse signal when the rotor rotates a circle, which could be used to catch the reference signal [9, 10]. The test principle is shown as Fig. 7.

Set the frequency of the collected signal is \( f_c \).

Then the rotating frequency \( f = n f_c \), which is the fundamental frequency of the rotor vibration signal.

5.2. Extract the Fundamental Frequency Signal with Cross-Correlation Method

The initial vibration signal caught by sensors not only contains the fundamental frequency component but also includes DC component, high-order harmonic, random noise, etc.

Set the initial vibration signal as

\[
x(t) = C + N(t) + A \sin(2\pi f_c t + \phi_0) \\
+ \sum_{i=1}^{n} A_i \sin(2\pi f_i t + \phi_i)
\]

(25)

where \( C \) represents DC component; \( N(t) \) represents random noise; \( A_0 \sin(2\pi f_c t + \phi_0) \) is the fundamental frequency component.

The principle of cross-correlation analysis is shown as following Fig. 8 [11].
Assume the orthogonal sine and cosine function that excited by reference signal is \( v(t) \) and \( z(t) \)

\[
\begin{align*}
\left\{ \begin{array}{l}
v(t) = \sin 2\pi f_c t \\
z(t) = \cos 2\pi f_c t
\end{array} \right.
\end{align*}
\]

Multiply \( x(t) \) with \( v(t) \) and \( z(t) \), respectively.

\[
\begin{align*}
x(t) \cdot v(t) &= C + N(t) + \sum_{n=1}^{N} A_n \sin(2\pi f_n t + \phi_n) \sin 2\pi f_c t + A_n \cos(2\pi f_n t + \phi_n) \cos 2\pi f_c t \\
x(t) \cdot z(t) &= C + N(t) + \sum_{n=1}^{N} A_n \cos(2\pi f_n t + \phi_n) \cos 2\pi f_c t + A_n \sin(2\pi f_n t + \phi_n) \sin 2\pi f_c t
\end{align*}
\]

Deduce from equation (27), when \( f_c \to 0 \)

\[
\begin{align*}
x(t) \cdot v(t) &= A_n \cos \phi_n / 2 \\
x(t) \cdot z(t) &= A_n \sin \phi_n / 2
\end{align*}
\]

Thus, \( v(t) \) and \( z(t) \) is cross correlated with \( x(t) \). Besides, when apply cut-off frequency \( f_c \to 0 \), the effect of DC component, high-order harmonic, and random noise could be ignored.

Set

\[
n(t) = C + N(t) + \sum_{i=1}^{N} A_i \sin(2\pi f_i t + \phi_i)
\]

Similarly, when cut-off frequency \( f_c \to 0 \)

\[
\begin{align*}
R_v(0) &= \int_0^T \sin 2\pi f_c t \cdot A_i \sin(2\pi f_i t + \phi_i) dt \\
&\quad + \int_0^T \sin 2\pi f_c t \cdot n(t) dt = \frac{A_T}{2} \cos \phi_i
\end{align*}
\]

Get

\[
\begin{align*}
A_i &= \frac{2\sqrt{R_v^2(0) + R_z^2(0)}}{T} \\
\phi_i &= \arctan \left( \frac{R_z(0)}{R_v(0)} \right)
\end{align*}
\]

When \( R_v(0) < 0 \),

\[
\phi = \pi + \arctan \left( \frac{R_z(0)}{R_v(0)} \right)
\]

When \( R_v(0) > 0 \) and \( R_z(0) < 0 \),

\[
\phi = 2\pi + \arctan \left( \frac{R_z(0)}{R_v(0)} \right)
\]
In general, the fundamental frequency signal is
\[ Z(t) = A_0 \sin(2\pi f_0 t + \phi), \quad (33) \]

6. Examples Testify

The initial data of the example are taken from reference [12]. The 1-plane and 2-plane dynamic balance calculation results are respectively shown in Table 2 and Table 3. The experimental apparatus can be simplified as Fig. 9 and Fig. 10.

As shown in Table 2 and Table 3 the result calculated by MATLAB is exactly consistent with what is shown in reference [12]. The theoretical calculation is in complete accord with the experimental data, which verified the feasibility of the correctness of the above analysis.

**Table 2. Results of 1-plane dynamic balance experiment.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial vibration amplitude of correction plane I /\textmu m</td>
<td>289.63</td>
</tr>
<tr>
<td>Initial vibration phase of correction plane I /\degree</td>
<td>113</td>
</tr>
<tr>
<td>Add trial weight on plane I /g</td>
<td>300</td>
</tr>
<tr>
<td>Phase of trial weight on plane I</td>
<td>330</td>
</tr>
<tr>
<td>Vibration amplitude of plane I after adding trial weight on plane I /\textmu m</td>
<td>179.48</td>
</tr>
<tr>
<td>Vibration phase of plane I after adding trial weight on plane I /\degree</td>
<td>89</td>
</tr>
<tr>
<td>The correction weight on plane I /g</td>
<td>597.87</td>
</tr>
<tr>
<td>The correction phase on plane I /\degree</td>
<td>-60.153</td>
</tr>
</tbody>
</table>

**Table 3. Results of 2-plane dynamic balance experiment.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial vibration amplitude of correction plane I /\textmu m</td>
<td>7.2</td>
</tr>
<tr>
<td>Initial vibration phase of correction plane I /\degree</td>
<td>238</td>
</tr>
<tr>
<td>Initial vibration amplitude of correction plane II /\textmu m</td>
<td>13.5</td>
</tr>
<tr>
<td>Initial vibration phase of correction plane II /\degree</td>
<td>296</td>
</tr>
<tr>
<td>Add trial weight on plane I /g</td>
<td>2.5</td>
</tr>
<tr>
<td>Phase of trial weight on plane I</td>
<td>0</td>
</tr>
<tr>
<td>Vibration amplitude of plane I after adding trial weight on plane I /\textmu m</td>
<td>4.9</td>
</tr>
<tr>
<td>Vibration phase of plane I after adding trial weight on plane I /\degree</td>
<td>114</td>
</tr>
<tr>
<td>Vibration amplitude of plane II after adding trial weight on plane I /\textmu m</td>
<td>9.2</td>
</tr>
<tr>
<td>Vibration phase of plane II after adding trial weight on plane I /\degree</td>
<td>347</td>
</tr>
<tr>
<td>Add trial weight on plane II /g</td>
<td>2.5</td>
</tr>
<tr>
<td>Phase of trial weight on plane II /\degree</td>
<td>0</td>
</tr>
<tr>
<td>Vibration amplitude of Plane I after adding trial weight on plane II /\textmu m</td>
<td>4.0</td>
</tr>
<tr>
<td>Vibration phase of Plane I after adding trial weight on plane II /\degree</td>
<td>79</td>
</tr>
<tr>
<td>Vibration amplitude of Plane II after adding trial weight on plane II /\textmu m</td>
<td>12.0</td>
</tr>
<tr>
<td>Vibration phase of Plane II after adding trial weight on plane II /\degree</td>
<td>292</td>
</tr>
<tr>
<td>The correction weight on plane I /g</td>
<td>2.9514</td>
</tr>
<tr>
<td>The correction phase on plane I /\degree</td>
<td>50.189</td>
</tr>
<tr>
<td>The correction weight on plane II /g</td>
<td>2.8441</td>
</tr>
<tr>
<td>The correction phase on plane II /\degree</td>
<td>-81.884</td>
</tr>
</tbody>
</table>

7. Conclusion

Through theoretical analyses and examples testify we can draw some conclusions.

1) On the base of the causes of the rotor's unbalanced vibration the mechanical model of rigid rotors were established, which verified that rotation rate. Combining with the correlate analysis, the fundamental frequency signal of unbalanced vibration were extracted. The Correction vector can be calculated by Two-plane Correction with the Influence Coefficient Method which was proved to be feasible to eliminate the unbalance vibration of the rigid rotor theoretically.

2) From an experimental point of view, the examples testify verified the feasibility of the rigid rotor dynamic balancing with the influence coefficient method.

The dynamic balance of rigid rotor with the influence coefficient method can meet the requirements of property and accuracy for dynamic balance testing, which can be applied in the engineering widely.

Reference

[7]. Ye Nengan, Yu Rusheng, Dynamic balance principle and dynamic balancing machine, Wu Han, *Huazhong University of Science and Technology Press*, 1985, pp. 2-8.

2013 Copyright ©, International Frequency Sensor Association (IFSA). All rights reserved. (http://www.sensorsportal.com)