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The Low-frequency Compensation of the Vibration Sensor's Amplitude-frequency Characteristics

Lifeng Pan
School of Electrical & Information Engineering, Hunan International Economics University,
Changsha, 410205, China
Tel.: 86-0731-88760386
E-mail: matlab_wjf@126.com

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Abstract: The hydropower generating units' vibration parameter is an important indicator to monitor its roll-stabilization. To measure the low-frequency vibration of the large and middle scale of hydropower generating units, according to frequency characteristic compensation principle of the vibration sensor, a low-frequency compensating circuit was designed to extend the frequency characteristic to the low frequency region. The mathematical model of the magnetoelectric dromometer vibration sensor and the design of the compensation circuit were detailed. The amplitude-frequency characteristics of the sensor and its mathematical model were comparatively analyzed before and after compensation respectively. The experimental result shows that the design could extend the amplitude-frequency characteristics to the low frequency region, and have good sensitivity and linearity. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Sensor, Vibration, Amplitude-frequency characteristics, Compensating circuit, Transfer function, Sensitivity.

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1. Introduction

Hydro accident showed vibration fault [1]. Medium-sized hydro group transfer frequency is low, about 1-2 Hz, and unit water vortex-induced vibration which hydro draft Tube produces is lower, about 1/5 to 1/3 of rotation frequency. In addition, once the accident occurred during hydropower units operation, which load shedding and load rejection transition process, vibration signal frequency will be lower [2]. In magnetic vibration velocity sensors, smaller natural frequency, the greater the volume. Considering the ease of installation of the sensor, the magnetic vibration velocity sensor is widely used in engineering, and its natural frequency limit of is about 2.5 Hz, but also the measurement frequency is higher and 2 to 3 times than the natural frequency of vibration sensor [3]. If the sensor is without compensation, and the direct the measurement is operated, the measurement is not accurate. How to design compensation circuit, and the vibration frequency characteristics is allowed to expand the low-frequency or to meet the requirements of the low-frequency test, it is the focus of the study.

2. Establish and Validate of the Vibration Sensor Model

2.1. Operating Principle of the Oscillation Sensor

Magnetic vibration velocity sensors have been widely applied in low-frequency sensor, because of these advantages which their output signal is large, the follow-up circuit is simple, anti-jamming capability is strong. As shown in Fig. 1, the vibration sensor is fixed with a permanent magnet inside, and in the outer casing, the magnetic circuit of the magnet is enclosed within the housing. Two circular spring upper and lower is fixed between the magnet and the housing. They support the coil, and are surrounded by the coil with mass m and without touching the magnet. When the external action of the vibration sensor, the relative motion is made between the magnet and the coil, the coil is cutting magnetic induction line, and is generating a voltage signal.

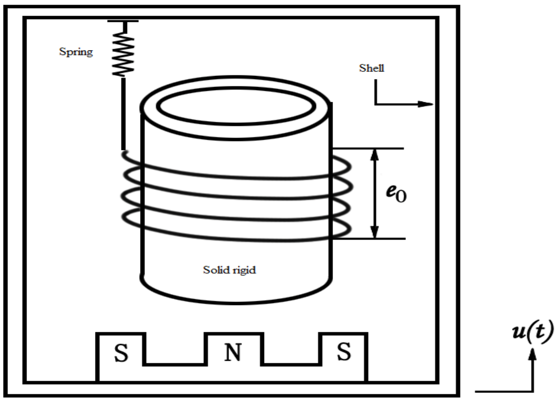

Fig. 1. Magnetic vibration sensor structure diagram.

2.2. Model Building

It is an inertial sensor, the mechanical model can be simplified as a single system with the degree of freedom, which is composed of three-part system of a spring with elastic coefficient k, inertial mass m and damping C [4] as shown in Fig. 2.

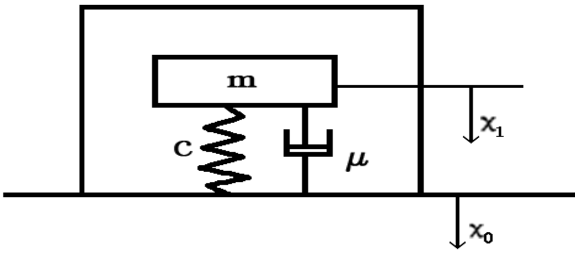

Fig. 2. Mechanical model.

The motion equation is

$$m x_1'' + \mu(x_1' - x_0') + c(x_1 - x_0) = F(t), \tag{1}$$

where m is the inertial body mass (coil);

C is the spring stiffness;

μ is the Damping;

x_0 is the absolute displacement of the base housing;

x_1 is the absolute displacement of the inertial body;

x_0' is the speed of the base housing;

x_1' is the speed of the inertia body;

x_1'' is the acceleration of the inertial body;

$F(t)$ is the forces acting on the inertial body;

Take $F(t) = 0$;

$\omega_0^2 = c/m$, ω_0 is the characteristic angular frequency;

$2\xi_0\omega_0 = \mu/m$, ξ_0 is the damping ratio;

$x_r = x_1 - x_0$, Mass displacement is relative to vibration sensor housing. (1) by the Laplace transform as follows:

$$H(s) = \frac{X_r(s)}{X_0(s)} = \frac{s^2}{s^2 + 2\varsigma_0\omega_0 s + \omega_0^2} \tag{2}$$

Vibration sensors coil is are cutting magnetic induction line, the output voltage is:

$$u_0 = Blx_r' \tag{3}$$

By Laplace transform as follows:

$$U_0(s) = BlsX_r(s) = k_0 s X_r(s), \tag{4}$$

which B is the magnetic field strength;

l is the length of the coil which is cutting magnetic field lines;

k_0 is the sensor sensitivity coefficient.

By the Formula (2), (4), a vibration sensor output response:

$$\begin{aligned} U_0(s) &= k_0 H(s) s X_0(s) \\ &= \frac{-k_0 s^2}{s^2 + 2\xi_0\omega_0 s + \omega_0^2} s X_0(s) \end{aligned} \tag{5}$$

Vibration sensor transfer function is:

$$G_1(s) = \frac{-k_0 s^2}{s^2 + 2\xi_0\omega_0 s + \omega_0^2} \tag{6}$$

In CDJ-Z2.5C vibration sensor as a reference, to establish the mathematical model. Natural frequency f_0=2.5±10 % Hz, take value 2.5. Damping is $\xi_0 = 0.7 \pm 10\%$, take value 0.75. $\omega_0^2 = (2\pi f_0)^2$ =246.74, $2\xi_0\omega_0 =$ 23.56. If k_0=1800, the Formula (6) can be written as:

$$G_1(s) = \frac{-1800 s^2}{s^2 + 23.56 s + 246.74} \tag{7}$$

Matlab simulating curve with the amplitude-frequency characteristic, as shown in Fig. 3, curve B is the simulation results.

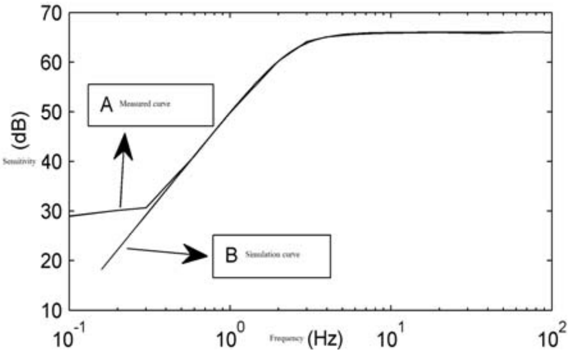

Fig. 3. Amplitude-frequency characteristic curve.

2.3. Verify Model

Equipment used in the experiment: one CDJ-Z2.5C vibration sensor, an integrated vibration tester. The vibration sensor is put vertically on the vibration generator, the output of the sensor is directly connected to the tester's input. Set the output vibration level, vibration level is 0.5 cm/s. Table 1 shows the raw data which is collected by the vibration.

Table 1. Original vibration data.

0.5 Vibration level data (before compensation)			
Frequency (Hz)	**Voltage (mV)**	**the measured vibration level (cm/s)**	**Sensitivity (mV/cm·s^{-1})**
0.1	0.01288	0.465	28
0.2	0.01088	0.485	32
0.3	0.01641	0.483	34
0.4	0.02689	0.483	56
0.6	0.05101	0.452	113
0.8	0.10522	0.522	201
1	0.15651	0.507	308
2	0.53186	0.519	1023
4	0.87278	0.487	1793
6	2.47287	1.292	1913
8	1.47976	0.759	1949
10	1.1035	0.562	1962
20	0.90408	0.453	1993
40	0.92791	0.476	1948
60	0.96076	0.479	2007
80	0.96471	0.482	2001
100	0.96373	0.484	1992

Note: Sensitivity = voltage / vibration level, the unit is mV/cm·s^{-1}.

In Table 1, the sensitivity is the voltage which is normalized by dividing the vibration level, unit is mV/cm·s^{-1}. The measured amplitude-frequency characteristic curve is curve A which is shown in Fig. 3. Comparison of the curves A, B in Fig. 3, it can be seen that the simulation and measured curve are same when frequency is larger than 0.3 Hz, almost exactly. The vibration frequency is less than 0.3 Hz, output of the sensor itself is small, and because of the floor and all noise, the relative output is high. The lower the frequency, the larger the error. Comprehensive assessment is that model is feasible.

3. Design of Compensation Aspects

3.1. Compensation Principle

In order to extend the frequency response of the low-frequency magnetic vibration velocity sensors, the circuit compensation method is required. There are two compensation forms in the circuit compensates, which are feedback compensation and series compensation, and the feedback compensation is with lower natural frequency, while the damping ratio is reduced, stability is poor, but also it is easy to produce oscillations [5]. It is used in series compensation. The sensor output voltage is in series with a compensation network C(s), so that the pole of the original transfer function G1(s) is eliminated by the C(s) zero point, C(s)'s pole becomes the pole of the transfer function G(s) after passing compensation, so the low-frequency output characteristics of the sensor can been changed by changing the compensation network's poles. The transfer function of the compensation aspects were given in Literature [5-8], equation (8) below, this transfer function can be decomposed by three parallel filtering part, namely all-pass, low-pass, band-pass filtering links.

$$\begin{aligned} C(s) &= \frac{s^2+2\xi_0\omega_0 s+\omega_0^2}{s^2+2\xi_1\omega_1 s+\omega_1^2} \\ &= 1+\frac{\omega_0^2-\omega_1^2}{s^2+2\xi_1\omega_1 s+\omega_1^2}+\frac{2(\xi_0\omega_0-\xi_1\omega_1)s}{s^2+2\xi_1\omega_1 s+\omega_1^2}, \end{aligned} \tag{8}$$

where ξ_1 is the damping ratio of compensation aspects, which reflects the frequency characteristic in the oscillation links, and does not directly reflect the frequency response of the filter circuit. So in this paper, the damping ratio ξ_1 is replaced by the equivalent quality factor Q, Q is a parameter which reflects the frequency response of the filter circuit. As shown in Formula (9):

$$C(s)=\frac{s^2+2\xi_0\omega_0 s+\omega_0^2}{s^2+\frac{\omega_1}{Q}s+\omega_1^2}, \tag{9}$$

where $\omega_1 = 2\pi f_1$ is the characteristic angular frequency which is compensated. After the compensation, transfer function G(s) of the vibration sensor is:

$$G(s) = G_1(s) \bullet C(s) = \frac{-k_0 s^2}{s^2 + \frac{\omega_1}{Q} s + \omega_1^2} \quad (10)$$

Compensation circuit C(s) is designed, so that $\omega_1 < \omega_0$, and Q is the best equivalent quality factor. Compensated system maintains the original vibration sensor high mechanical properties unchanged, and the natural frequency depends entirely on the compensation circuit which is connected in series, thus in compensated sensor system, the original sensor volume is small, and performance characteristics is good, but also low-frequency output characteristics is improved.

3.2. Vibration Sensor Compensation Circuit Design

To meet the measurement requirements of hydroelectric generating low-frequency vibration, the natural frequency of the sensor is needed to expand from 2.5 Hz to the low-frequency 0.2 Hz, to maintain the best quality factor. That is $f_1 = 0.2$ Hz, Q = 0.7. The compensation link C(s) is divided into:

$$C(s) = 1 + \frac{k_1 A_{VF} \omega_1^2}{s^2 + \frac{\omega_1}{Q} s + \omega_1^2} + \frac{k_2 A_{VF} \omega_1 s}{s^2 + \frac{\omega_1}{Q} s + \omega_1^2}, \quad (11)$$

in the formula,

$$k_1 = \frac{\omega_0^2 - \omega_1^2}{\omega_1^2 A_{VF}}, k_2 = \frac{2\xi_0 \omega_0 - \omega_1 / Q}{\omega_1 A_{VF}}$$

Among them, A_{VF} for the same phase scaling circuit voltage gain.

By Formula (11), it can be seen that the compensation aspect is the parallel together with all-pass, low-pass, band-pass three aspects, the structure is shown in Fig. 4.

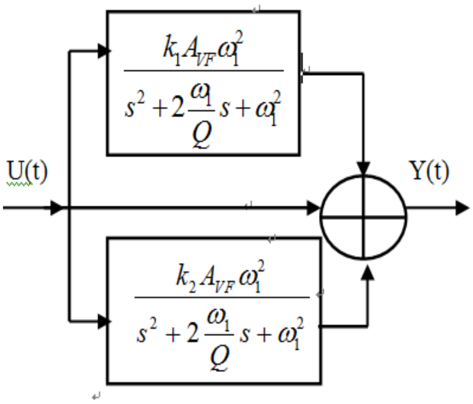

Fig. 4. Compensation link structure figure.

The ratio between these three gains is adjusted, zerosof the compensation link and poles of the original system are made offset, allowing the system to achieve the desired frequency characteristics. Because of the integrated operational amplifier module features itself, little DC offset will been produced, after amplification, this can't be ignored, so after the filter circuit, the DC offset is removed in series with a first-order high-pass filter circuit. The cutoff frequency should be much less than 0.1 Hz. Fig. 5 is compensation circuit wiring diagram.

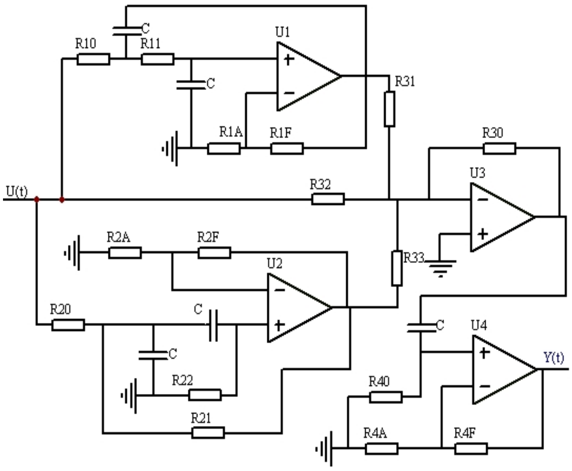

Fig. 5. Compensation circuit wiring diagram.

Wherein: R_{10}, R_{11}, R_{1A}, R_{1F}, C, U_1 constitutes a second-order low-pass filter circuit; R_{20}, R_{21}, R_{22}, R_{2A}, R_{2F}, C, U_2 constitutes a second order band-pass filter circuit; $\mathbf{R_{32}}$ is an all-pass circuit; R_{30}, R_{31}, R_{32}, R_{33}, U_3 constitute an addition circuit; R_{40}, R_{4A}, R_{4F}, C, U4 form a first order high pass filter. To reduce system circuit noise, each resistor in the circuit should be trillion level and below, so the capacitance value C = 10 uF to simplify the calculation, so that $R_{10}=R_{11}=R_{20}=R_{22}=R$; $R_{22}=2R$; $R_{30}=R_{32}=100$ kΩ. The design requirements were: $R_{1A}=R_{2A}=R_A$; $R_{1F}=R_{2F}=R_F$. And $V_{AF} = 1 + \frac{R_F}{R_A} = \frac{3Q-1}{Q}$, $R_A \parallel R_F = 2R$, $\omega = 2\pi f = 1/RC$, $R_{31} = R_{30}/k_1$, $R_{33} = R_{30}/k_2$, so the parameter values of the circuit is about R = 80 kΩ, R_A = 440 kΩ, R_F = 250 KΩ, R_{31} = 1 kΩ, R_{33} = 9 kΩ. For a first-order high-pass filter, parameter is not strictly required to achieve the blocking effect. Due to the low signal to noise ratio of low frequency sensor output voltage, low-frequency amplitude of the original system is bigger than the theoretical value. R_{20}, R_{21}, R_{22} resistance can be increased, so that the amplitude-frequency characteristic of the band rejection filter circuit is translated to the high frequency. In order to increase the output voltage of the sensor system, the voltage drop of the circuit is compensated. The measured data is showed in Table 2 after compensation.

Table 2. Measured data after compensated.

0.5 vibration level data (after compensation)			
Frequency (Hz)	**Voltage (mV)**	**Measured vibration level (cm/s)**	**Sensitivity (mV/cm·s^{-1})**
0.1	472	0.6	788
0.2	800	0.531	1508
0.3	856	0.510	1679
0.4	796	0.462	1725
0.6	864	0.480	1789
0.8	1031	0.563	1833
1	916	0.496	1846
2	970	0.504	1924
4	907	0.467	1942
6	2516	1.295	1942
8	1472	0.758	1943
10	1473	0.760	1943
20	885	0.455	1945
40	916	0.481	1905
60	943	0.487	1937
80	942	0.484	1946
100	936	0.487	1922

4. Analysis

4.1. Development of Low-frequency

The amplitude-frequency characteristic curve before and after the system compensation can be drawn from Tables 1 and 2, as shown in Fig. 6. The curves A, B is respectively the amplitude-frequency characteristic curve before and after the system compensation [9-11]. The linearity relative errors are shown in Table 3. And generally, the-3dB frequency corresponding about 70 % of the flat area is the natural frequency of the system. For example, the compensation is not increased in circuit, if frequency is equal to 20Hz, the sensitivity is 1993, and its 70 % is 1395. From the data in the table, the natural frequencies are showed between 2-3 Hz, about 2.5 Hz, and it is the actual match. After adding the compensation circuit, if frequency is equal to 20 Hz, the sensitivity is 1945 and its 70 % is 1361, its natural frequency should be between 0.1-0.2 Hz which is seen from Table 2, approximately 0.18 Hz. It can be seen in Fig. 6 that curve A natural frequency is about 2.5 Hz, the natural frequency of curve B is about 0.18 Hz, thus the system is expanded to 13.8 times of low frequency, that is 0.18 Hz, the design requirements are meet.

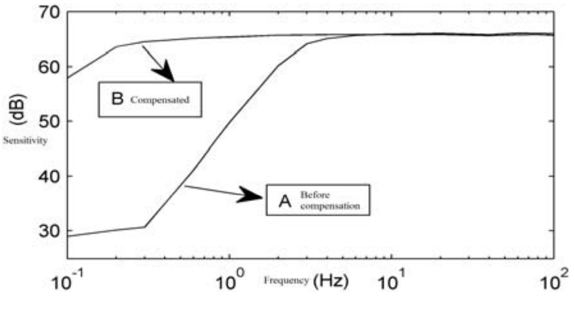

Fig. 6. Compensated amplitude-frequency characteristic curve.

Table 3. Linearity relative error.

Frequency (Hz)	Before compensation	After compensation
0.1	-0.986	-0.595
0.2	-0.984	-0.225
0.3	-0.983	-0.137
0.4	0.972	-0.113
0.6	-0.943	-0.080
0.8	-0.899	-0.058
1	-0.845	0.051
2	-0.487	-0.011
4	-0.100	-0.002
6	-0..040	-0.002
8	-0.022	-0.001
10	-0.016	-0.001
20	0	0
40	-0.023	-0.020
60	-0.007	-0.004
80	-0.004	0.0005
100	-0.0005	-0.012

4.3. Sensitivity

Vibration data 0.5 cm/s is for a standard vibration level, the relative error of the sensitivity is calculated by a vibration data acquisition another 0.8 cm/s vibration level, so the sensitivity stability of the compensated system is the assessed. Table 4 shows that the relative error is 0.005 or less in the sensitivity, a sensitivity of the system remains stable after compensation.

Table 4. Sensitivity relative error.

Frequency (Hz)	0.5 cm/s	0.8 cm/s	Relative error
0.1	788	793	-0.006
0.2	1508	1506	0.001
0.3	1679	1687	-0.005
0.4	1725	1727	-0.001
0.6	1789	1782	0.004
0.8	1833	1825	0.004
1	1846	1850	-0.002
2	1907	1906	0.0005
4	1942	1946	-0.002
6	1942	1940	0.001
8	1943	1941	0.001
10	1943	1940	0.002
20	1945	1942	0.002
40	1905	1895	-0.005
60	1937	1936	0.0005
80	1946	1949	-0.002
100	1922	1922	0

5. Conclusion and Outlook

In the design of the compensation aspects, the equivalent quality factor is cited by Q, so that the compensation circuit design of the magnetic vibration velocity sensor is simple and straight victory. The low-frequency system is developed to low-frequency after compensated, there are sensitivity, linearity, more comprehensive analysis, these are indicating that the sensor system performance is good after compensating, the design of the compensation aspects is proved reasonably practicable.

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