State Estimation for Sensor Monitoring System with Uncertainty and Disturbance

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Abstract: This paper considers the state estimation problem for the sensor monitoring system which contains system uncertainty and nonlinear disturbance. In the sensor monitoring system, states of each inner sensor node usually contains system uncertainty, and external noise often works as nonlinear item. Besides, information transmission in the system is also time consuming. All mentioned above may arouse in unstable of the monitoring system. In this case, states of sensors could be wrongly sampled. Under this circumstance, a proper mathematical model is proposed and by the use of Lipschitz condition, the nonlinear item is transformed to linear one. In addition, we suppose that all sensor nodes are distributed arranged, no interface occurs with each other. By establishing proper Lyapunov–Krasovskii functional, sufficient conditions are acquired by solving linear matrix inequality to make the error augmented system stable, and the gains of observers are also derived. Finally, an illustrated example is given to show that system observed value tracks system states well, which fully demonstrate the effectiveness of our result.

Keywords: State estimation, Sensor monitoring system, System uncertainty, Nonlinear disturbance, Linear matrix inequality.

1. Introduction

During last decades, states estimation and other related problems are hot issues in the scholar field due to the increasing demand of sensor monitoring system, which is a distributed system composed of sensor nodes [1-3]. Because of its convenience for use and low cost, sensor monitoring system aroused much attention both from theoretical researches and practical occasions, such as in industry, agriculture, military and some other occasions [4]. Sensors collect data from the field and also control the operating parameters. By running a dedicated data collection procedure to achieve field data collection, in addition, control operating parameters are transferred to sensors via data bus to achieve automatic control. However, many problems are resulted in for its special system structure. Information flow between sensor nodes is time consuming, which reflects as time-delay in system model. In addition, outer environment, take temperature, pressure, moist for example, can greatly affect sensors’ working condition, so system uncertainties and disturbances are inevitable when modeling for sensor monitoring system. Take all mentioned above, states estimation is quite important for the stability analysis of sensor monitoring system.
Still now, great effort has been done to deal with states estimation problem [5-7]. Reference [8] studied the spread of subsurface contaminants, proposed a dual estimation strategy for data assimilation into a one-way coupled system by treating the flow and the contaminant models separately. This strategy didn’t only deals with states estimation, but also could be used for parameter estimation. Reference [9] considered a fuzzy-integral model of a fuzzy process. Sufficient conditions were established for the existence of an optimal estimating fuzzy process. Reference [10] researched estimation problem around human environment. A Human motion map was proposed which incorporating human states information into results of conventional SLAM.

Motivated by previous research stated above, our target is to make state estimation for sensor monitoring system, which contains uncertainty and nonlinear disturbance. By establishing proper Lyapunov-Krasovskii functional, sufficient conditions are proposed to make system stable, and observe gains are also acquired.

2. Problem Formulation

Consider a type of sensor monitoring system, which is composed of several sensor nodes, and the dynamic system with uncertainty and disturbance can be described as shown in (1):

\[
\begin{align*}
    x_i(k+1) &= (A + \Delta A)x_i(k) + A_d x_i(k-\tau) + B u(k) + E_g g(x_i(k)) + \Delta z_i(k), \\
    y_i(k) &= C x_i(k)
\end{align*}
\]  

where \( x_i(k) \in \mathbb{R}^n \) denotes the state of \( i \)-th node, \( x_i(k-\tau) \in \mathbb{R}^n \) denotes the state delay of the node, \( g(x_i(k)) \in \mathbb{R}^n \) denotes the nonlinear disturbance, \( u(k) \in \mathbb{R}^{n_u} \) is the system input, \( y_i(k) \in \mathbb{R}^n \) denotes system output, \( \Delta A \) is internal perturbation arising from uncertain factors, \( A, A_d, E_g, B \) and \( C \) are constant matrices with appropriate dimensions.

Here we make the following assumption for system model (1).

**Assumption 1.** Perturbation parameter of the model satisfies:

\[
\Delta A = GD(k)H
\]  

where \( G \) and \( H \) are known constant matrix, \( D(k) \) is time delay uncertain matrix, yet Lebesgue-measurable, and \( D^T(k)D(k) \leq I \).

Next, a state observer for the \( i \)-th node is constructed.

\[
\begin{align*}
    \dot{\hat{x}}_i(k+1) &= A \hat{x}_i(k) + \sum_{j=1}^{N} f_{ij} \hat{y}_j(k), \\
    \dot{\hat{y}}_i(k+1) &= C \hat{x}_i(k), \\
    \end{align*}
\]

where \( \hat{x}_i(k) \) and \( \hat{y}_i(k) \) are the observed value of \( x_i(k) \) and \( y_i(k) \), respectively. \( K_{ji} \in \mathbb{R}^n \) is the gain of observer to be designed.

In this case, the states error and output error are defined as:

\[
\begin{align*}
    e_i(k+1) &= x_i(k) - \hat{x}_i(k), \\
    e_y_i(k+1) &= y_i(k) - \hat{y}_i(k),
\end{align*}
\]

where

\[
\begin{align*}
    e_i(k+1) &= (A - \sum_{j=1}^{N} K_{ji}) e_i(k) + \Delta A x_i(k) + A_d x_i(k-\tau) + \Delta \hat{z}_i(k), \\
    e_y_i(k+1) &= C x_i(k).
\end{align*}
\]

By utilizing the Kronecker product, the error dynamics governed by (5) can be rewritten as:

\[
\begin{align*}
    e(k+1) = (\tilde{A} - K \tilde{C}) e(k) + \Delta \tilde{A} x(k) + \tilde{A}_d x(k-\tau) + \tilde{E}_g g(e_i(k)),
\end{align*}
\]

where \( \tilde{A} = I_N \otimes A, \tilde{C} = I_N \otimes C \), \( \Delta \tilde{A} = I_N \otimes \Delta A, \tilde{A}_d = I_N \otimes A_d, \tilde{E}_g = I_N \otimes E_g \), \( \tilde{E}_f = I_N \otimes E_f \). By employing an augmented vector

\[ Z(k) = \begin{bmatrix} x(k) & \tilde{e}(k) \end{bmatrix}^T \], we have augmented system:

\[
\begin{align*}
    Z(k+1) &= \tilde{A} Z(k) + \Delta \tilde{A} Z(k) + \tilde{A}_d Z(k-\tau) + \tilde{E}_g g(Z(k)) + \tilde{E}_f f(k).
\end{align*}
\]

**Assumption 2.**

State delay has an upper bound, which satisfies \( 0 \leq \tau \leq d \).

**Assumption 3.**

System nonlinear factor \( g(\cdot) \) satisfies the following Lipschitz condition:

\[
\frac{\| g(x_1) - g(x_2) \|}{\| x_1 - x_2 \|} \leq \Sigma
\]

where \( \Sigma \) is the constant matrix with proper dimensions, and \( g(0) = 0 \).
where
\[
\begin{pmatrix}
\bar{A} & 0 \\
0 & \bar{A} - K \bar{C}
\end{pmatrix},
\begin{pmatrix}
\bar{A}_d & 0 \\
\bar{A}_d & 0
\end{pmatrix}
\]
\[
\bar{A} = [\bar{A} \quad 0]
\]
\[
\bar{A}_d = [\bar{A}_d \quad 0]
\]
\[
\Delta A = \begin{bmatrix}
\Delta \bar{A} & 0 \\
0 & \Delta \bar{A}
\end{bmatrix},
\begin{bmatrix}
\bar{E}_g & 0 \\
0 & \bar{E}_g
\end{bmatrix}
\]
\[
\bar{E}_f = \begin{bmatrix}
\bar{E}_f \\
\bar{E}_f
\end{bmatrix}
\]

Furthermore, some important lemmas will be used in this paper are listed below.

**Lemma 1.** For any \(x, y \in \mathbb{R}^n, \mu > 0\), the following equation holds.

\[
2x^T y \leq \mu x^T x + \frac{1}{\mu} y^T y
\]

**Lemma 2.** Let \(Y = Y^T, M, N, D(t)\) be real matrix of proper dimensions, and \(D^T(t)D(t) \leq I\), then inequality \(Y + MDN + (MDN)^T < 0\) holds if there exists a constant \(\varepsilon\), which makes the following inequality holds.

\[
Y + \varepsilon NN^T + \varepsilon^{-1} M^T M < 0
\]

### 3. Main Results

In this part, two sufficient conditions are proposed to make augmented system (7) stable, in addition, the gains of observer can also be acquired.

**Theorem 1.** For the augmented system as shown in (7), it is said to be asymptotically stable if there exists positive definite matrix \(P = \text{diag} \{P_1, P_2\}\), \(Q\) and a constant \(\varepsilon\), make the following inequality holds.

\[
\begin{bmatrix}
\Omega & \Gamma & 0 & 0 & 0 & 0 \\
* -Q & 0 & 0 & 2\bar{A}_d^T P & 0 \\
* * -P & 0 & 0 & 0 \\
* * * -P & 0 & 0 & 0 \\
* * * * -P & 0 & 0 & 0 \\
* * * * * -P & 0 & 0 & 0 \\
* * * * * * -\varepsilon I
\end{bmatrix} < 0
\]

where
\[
\Omega = -P + (1 + d)Q + 4\lambda \sum T + \varepsilon HH^T \\
H = [\bar{H} \quad 0] \\
P = [P_1^T \quad P_2^T]^T \\
\bar{G} = I_n \otimes G
\]

Proof: By constructing the following Lyapunov-Krasovski functional

\[
V(k) = V_1(k) + V_2(k),
\]

where
\[
V_1(k) = Z^T(k)PZ(k) \\
V_2(k) = \sum_{i=k-t}^{k} Z^T(i)QZ(i)
\]

So the forward difference of \(V(k)\) along (7) is:

\[
E \Delta V_1 = E\{Z^T(k + 1)PZ(k + 1) - Z^T(k)PZ(k)\} \\
= Z^T(k)\bar{A}^T P \bar{A} Z(k) \\
+ 2Z^T(k)\bar{A}^T P \Delta \bar{A} Z(k) \\
+ 2Z^T(k)\bar{A}^T P \Delta \bar{A} Z(k) \\
+ 2Z^T(k)\bar{A}^T P \bar{E}_g g(Z(k)) \\
+ Z^T \Delta \bar{A}^T P \Delta \bar{A} Z(k) \\
+ 2Z^T \Delta \bar{A}^T P \bar{A} Z(k - \tau) \\
+ 2Z^T \Delta \bar{A}^T P \bar{E}_g g(Z(k)) \\
+ Z^T \Delta \bar{A}^T P \bar{A} Z(k - \tau) \\
+ 2Z^T \Delta \bar{A}^T P \bar{E}_g g(Z(k)) \\
+ Z^T \Delta \bar{A}^T P \bar{A} Z(k - \tau) \\
+ 2Z^T \Delta \bar{A}^T P \bar{E}_g g(Z(k))
\]

By the use of **Lemma 1**, we can obtain:

\[
2Z^T(k)\bar{A}^T P \bar{A} Z(k - \tau) \leq Z^T(k)\bar{A}^T P \bar{A} Z(k) \\
2Z^T(k)\bar{A}^T P \bar{A} Z(k - \tau) \leq Z^T(k)\bar{A}^T P \bar{A} Z(k)
\]
\[
2Z^T(k)\Delta A^T P\Delta A Z(k) - \tau \\
\leq Z^T(k)\Delta A^T P\Delta A Z(k) \\
+ Z^T(k - \tau)\Delta A^T P\Delta A Z(k - \tau) \\
+ g^T(Z(k))\bar{E}_g P \bar{E}_g g(Z(k)) \\
\leq Z^T(k)\Delta A^T P\Delta A Z(k) \\
+ g^T(Z(k))\bar{E}_g P \bar{E}_g g(Z(k)) \\
(13)
\]

where

\[
X(k) = \begin{bmatrix} Z^T(k) & Z^T(k - \tau) \end{bmatrix}^T \\
Y = \begin{bmatrix} Y_{11} & 0 \\
0 & Y_{22} \end{bmatrix} \\
Y_{11} = 4\bar{A}_f^T P\bar{A} + 4\Delta \bar{A}_f^T P\Delta \bar{A} + 4\lambda \sum_{i=1}^{n} \sum_{j=1}^{m} Z_i Q Z_j \\
- P + (1 + d)Q \\
Y_{22} = 4\bar{A}_f^T P\bar{A}_d - Q \\
\]

According to Schur complement, \( Y < 0 \) equals to:

\[
\begin{bmatrix}
Y_{11} & 2\bar{A}_f^T & 2 \Delta \bar{A}_f^T & 0 \\
* & -Q & 0 & 0 & 2\bar{A}_f^T \\
* & * & -P^{-1} & 0 & 0 \\
* & * & * & -P^{-1} \\
* & * & * & * & -P^{-1}
\end{bmatrix} < 0
\]

where

\[
Y_i = P + (1 + d)Q + 4\lambda \sum_{i=1}^{n} \sum_{j=1}^{m} Z_i Q Z_j \\
\]

According to Lemma 2 and the notation defined in the theorem, result can be easily acquired, so the proof of Theorem 1 is complete.

Theorem 1 only demonstrate the stability of error system (7), however, observer gains can't be acquired, so the following theorem is given.

**Theorem 2.** For the augmented system as shown in (7), it is said to be asymptotically stable if there exists positive definite matrix \( P = \text{diag}\{P_1, P_2\} \), \( Q \) and a constant \( \epsilon \), make the following inequality holds.

\[
\begin{bmatrix}
Y_{11} & 2\bar{A}_f^T & 2 \Delta \bar{A}_f^T & 0 \\
* & -Q & 0 & 0 & 2\bar{A}_f^T \\
* & * & -P^{-1} & 0 & 0 \\
* & * & * & -P^{-1} \\
* & * & * & * & -P^{-1} \\
\end{bmatrix} < 0
\]

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Theorem 1 only demonstrate the stability of error system (7), however, observer gains can’t be acquired, so the following theorem is given.

**Theorem 2.** For the augmented system as shown in (7), it is said to be asymptotically stable if there exists positive definite matrix \( P = \text{diag}\{P_1, P_2\} \), \( Q \) and a constant \( \epsilon \), make the following inequality holds.
where

\[
\Omega = -P + (1 + d)Q + 4\kappa \sum \sum + \epsilon \tilde{H}^T \tilde{H}
\]

\[
\tilde{H} = \begin{bmatrix} \tilde{H} & 0 \end{bmatrix}
\]

\[
\tilde{F} = \begin{bmatrix} F_1^T & F_2^T \end{bmatrix}^T
\]

\[
\tilde{G} = I_x \otimes \tilde{G}
\]

\[
\tilde{F} = 2 \begin{bmatrix} \tilde{A}P & 0 \\ 0 & \tilde{A}P - \tilde{C}^T S \end{bmatrix}
\]

\[\Omega, \tilde{H}, \tilde{F}, \tilde{G}\] have the same meaning as defined in Theorem 1, and the gain of observe is:

\[
K = P_2^{-1} S^T
\]

Proof: Setting \( K^T P_2 = S \), so \( K = P_2^{-1} S^T \), by substituting \( K^T P_2 = S \) into Theorem 1, we can get the result easily.

4. Numerical Example

In this part, a numerical example is given to show the effectiveness of proposed method. Transform the case on [11], parameters in (1) are given as:

\[
A = \begin{bmatrix} 0.2546 & 0.2472 \\ 0.2323 & 0.2268 \end{bmatrix},
\]

\[
A_d = \begin{bmatrix} -0.1434 & 0.0061 \\ -0.0118 & 0.0384 \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0.1669 \\ 0.1174 \end{bmatrix},
\]

\[
E_g = \begin{bmatrix} -0.0130 \\ 0.0742 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 1.1866 & 0.4552 \\ 0.5395 & 0.3164 \end{bmatrix},
\]

\[
G = \begin{bmatrix} 0.0618 & 0.1236 \\ 0 & 0.1422 \end{bmatrix},
\]

\[
D(k) = \begin{bmatrix} 0.2 \sin(0.6k) & 0 \\ 0 & 0.2 \sin(0.6k) \end{bmatrix},
\]

\[
H = \begin{bmatrix} 0.4450 & 0.3214 \\ 0.2596 & -0.4326 \end{bmatrix},
\]

\[
g(x(k)) = \begin{bmatrix} 0.12 \cos x_1(k) \\ 0.41 x_2 \end{bmatrix}^T
\]

\[u(k) = 5e^{-0.01k} \sin(k)\]

The initial states of each sensor nodes are:

\[
x_1 = \begin{bmatrix} 3.2 \\ -3.5 \end{bmatrix},
\]

\[
x_2 = \begin{bmatrix} 2.2 \\ -1.6 \end{bmatrix},
\]

\[
x_3 = \begin{bmatrix} 2.72 \\ 3.25 \end{bmatrix}
\]

Observer gain can be obtained based on Theorem 1 and Theorem 2, we have:

\[
k_{11} = k_{22} = k_{33} = \begin{bmatrix} -0.4067 & 1.3663 \\ -0.3762 & 1.2581 \end{bmatrix}
\]

Simulation results are given in Fig. 1, where red lines denotes for real system state, blue dashed line denotes for observed value. From the result, we can see that observed value tracks system states well, which can fully demonstrate the effectiveness of proposed method. Estimation error is shown in Fig. 2, which is much smaller than the method proposed in [12], compared with Fig. 3.
5. Conclusions

This paper deals with the state estimation problem for a class of sensor monitoring system, which contains system uncertainty and nonlinear disturbance. By establishing proper Lyapunov–Krasovskii functional, sufficient conditions are proposed to guarantee the stability of error system, and the gains of observers are also acquired. Finally, an illustrated example shows the effectiveness of proposed method.

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