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Single Allocation Hub-and-spoke Networks Design Based on Ant Colony Optimization Algorithm

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Abstract: Capacitated single allocation hub-and-spoke networks can be abstracted as a mixed integer linear programming model equation with three variables. Introducing an improved ant colony algorithm, which has six local search operators. Meanwhile, introducing the "Solution Pair" concept to decompose and optimize the composition of the problem, the problem can become more specific and effectively meet the premise and advantages of using ant colony algorithm. Finally, location simulation experiment is made according to Australia Post data to demonstrate this algorithm has good efficiency and stability for solving this problem.

Keywords: Hub-and-spoke network, Ant Colony Optimization Algorithm, Solution pair, Capacitated hub location, Local search.

1. Introduction

Hub-and-spoke networks are usually used to describe this sort of question: a graph contains more than one node and these nodes send and receive the commodities [1]. In one of the subgraphs, all the flow of goods must go through a number of special nodes which will be called hubs in this paper [2]. Hub-and-spoke networks are being increasingly used in postal services, aviation and telecommunications and other fields. The existence of hub nodes makes it possible to re-find the path selection in the process of cargo transportation and lower the cost of network construction as well [3-4]. The capacity of hub-and-spoke networks is often limited in practical applications, thus introducing hub-and-spoke networks with capacity limits is of more practical significance [5-7]. This paper deals with the solution to Capacitated Single Allocation Hub Location Problems (CSAHLP). (Each node can only be allocated to one hub, as shown in Fig. 1).

Ant colony optimization algorithm is optimized on the basis of ant system proposed by the Italian scholars M. Dorigo, V. Maniezz and A. Colorni in the early 1990s [8]. Ant colony optimization algorithm is a new kind of intelligent optimization algorithm which has been used to solve the famous Traveling Salesman Problem [9-11]. In recent years, this algorithm has been more and more used in combinatorial optimization, function optimization, robot path planning, data mining and other fields [12-15]. In this paper, ant colony optimization algorithm will be used to solve CSAHLP problem, using the CSAHLP problem whose optimal variable is $O(n^3)$ for validation.

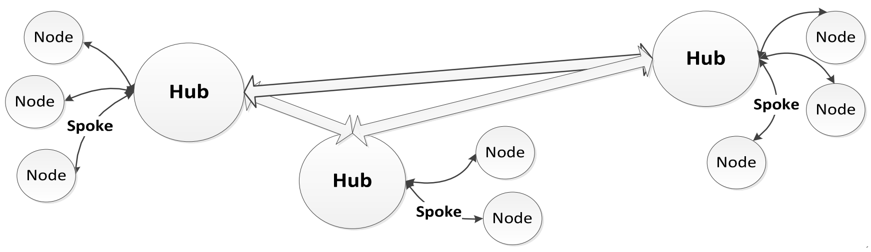

Fig. 1. Single allocation hub-and-spoke network.

2. Design Model for the Capacitated Single Allocation hub-and-spoke Networks

Formulation 1 can be used to describe CSAHLP. Hub node p here is still not sure, and one of the goals of this paper is to determine hub node p. Model equations are as follows:

$$L = min \sum_i \sum_k C_{ik} X_{ik} (\chi O_i + \delta D_i) + \sum_i \sum_k \sum_l \alpha C_{kl} Y^i_{kl} + \sum_k F_k X_{kk}, \quad (1)$$

$$s.t. \sum_k X_{ik} = 1 \; \forall i \in N, \quad (2)$$

$$X_{ij} \le X_{jj} \quad \forall i, j \in N \quad (3)$$

$$\sum_{i \in N} O_i X_{ik} \le \Gamma_k X_{kk} \; \forall k \in N, \quad (4)$$

$$\sum_l Y^i_{kl} - \sum_l Y^i_{lk} = O_i X_{ik} - \sum_j W_{ij} X_{jk} \; \forall i, k \in N, \quad (5)$$

$$X_{ik} \in \{0,1\} \; Y^i_{kl} \ge 0 \quad \forall i, k, l \in N \quad (6)$$

The objective function 1 minimizes the transportation costs of the network. The fees include the fixed costs of hub facilities and the sum of the total transportation costs. We assume that the network has n nodes. $N = \{1,2, \dots, n\}, i \in N$ represents the set of all nodes. We will choose p nodes from it as hubs and any two of them can be used as the origin and destination of the traffic flow (OD). W_{ij} represents the traffic flow from node *i* to node *j*, $O_i = \sum_j W_{ij}$, $D_i = \sum_i W_{ij}$. C_{ij} represents the standards of transportation cost in traffic flow from node *i* to node *j*, α represents the discount factor of the standards of transportation cost of traffic flow from in node *k* to node *l*, generally $\alpha \le 1.0$, χ represents the discount factor of the transportation cost from node to hub, δ represents the discount factor from hub to node. They are all set according to the data from Australia Post (AP). We assume that the fixed construction cost for changing node j to hub is F_j. Y^i_{kl} represent all the traffic flow that go through node *i* and node *l*. X_{ij} represents integer variables {0,1}. When node *i* is the branch node of hub X_{ij}, the value is 1, otherwise, the value is 0. $X_{kk} = 1$ shows that node k is a hub and Γ_k is the collection capacity of hub *k*. Any node will be constrained by single allocation, namely each can only be allocated to one hub (as the constraint conditions shown in equation 2). Meanwhile, *e* is not a hub and can only be allocated to hubs (as the constraint conditions shown in equation 3). The capacity of the hub also has certain constraints (as the constraint conditions shown in equation 4). The flow conservation equality *i* at node *k* is constrained by equation 5, and the demand and supply is decided by the allocation X_{ik} at node *k*.

Due attention should be paid to the equation:

1. A node or hub may have reached their own flow, namely $W_{ii} > 0$.
2. Any two hubs can be directly connected to each other, namely there are no other hubs between hub *k* and hub *l*.
3. Any two nodes belonging to one hub can be simply connected to each other by common hub.
4. Considering the capacity limits, only the lowest overall cost remains to be solved, namely the minimum of objective function.

3. Ant Colony Optimization Algorithm Model

Ant colony optimization algorithm can be seen as a search algorithm framework based on the parametric probability distribution model of solution space. In this algorithm, pheromones are parameters of the model when solving the parametric probability of solution space, thus pheromone model is the parametric probability distribution model. In this model-based search algorithm, feasible solutions can be generated by searching the parametric probability distribution model of solution space. We use the generated solutions to update the parameters of this model and make the search in new models gather in high-quality search space of solutions.

When solving general problems, ant colony optimization algorithm is divided into the following steps:

1. In each iteration, when solving the problem every ant in the ant colony form their own solution as they move until they constitute all the solutions.
2. After every ant completing its solution, the pheromone will be updated according to the composition of the solution. The pheromone update

makes the solution space to optimized region. The moving direction of the next ant, the probability of the direction, and the path the ant choose later are decided by the updated pheromone.

The formula used in the update is:

$$\tau_{\mathrm{gh}}(i,j) \longleftarrow (1-\rho)\cdot\tau_{\mathrm{gh}}(i,j)+\rho\cdot\tau_{\mathrm{g0}}, \tag{7}$$

$\rho \in (0,1)$ is a constant representing the volatilizing factor of local pheromones.

3. Each ant will face the problem of path selection in its moving process. It follows pseudo random choice rule and the following transfer probability formula:

$$s=\begin{cases}\arg\, max_{(i,j)\epsilon J_{gh}(r)}[\tau_{gh}(i,j)]^{\alpha}[\eta_g(i,j)]^{\beta}, & if\ q \le q_0\\ \text{(Formulation 9)} & \text{otherwise}\end{cases} \tag{8}$$

$$P_{\mathrm{gh}}(s)=\begin{cases}\dfrac{[\tau_{\mathrm{gh}}(i,j)]^{\alpha}\cdot[\eta_g(i,j)]^{\beta}}{\sum_{(i,j)\in J_{\mathrm{gh}}(r)}[\tau_{\mathrm{gh}}(i,j)]^{\alpha}[\eta_g(i,j)]^{\beta}}, & if\ \ s\in J_{\mathrm{gh}}(r),\\ 0, & \text{otherwise}\end{cases} \tag{9}$$

where $q_0 \in (0,1)$ is the constant, $q \in (0,1)$ is the random number, $\tau_{gh}(i,j)$ is the pheromone of solution composition (i,j) , $\eta_g(i,j)$ is the heuristic factor of solution composition (i, j), α represents the importance of information quantity the ants accumulated in their moving process, β is the relative strength of heuristic factor. q will have been generated randomly before the next step. If $q \le q_0$, we need to find the maximum solution composition $[\tau_{gh}(i,j)]^{\alpha}\cdot[\eta_g(i,j)]^{\beta}$ from all the rest feasible solution compositions. And $[\tau_{gh}(i,j)]^{\alpha}\cdot[\eta_g(i,j)]^{\beta}$ is the solution composition that will be selected next; if $q > q_0$, we need to select the next solution composition according to the probability calculated in the equation 9.

4. In each iteration, we need to find the ant that has reached the optimal value of objective function and update the global pheromone about solution composition according to it, using the formula as follows:

$$\begin{aligned}&\tau_g(i,j) \longleftarrow (1-\gamma)\cdot\tau_g(i,j)+\gamma\cdot\Delta\tau_g(i,j)\\ &\Delta\tau_g(i,j)=\\ &\begin{cases}L_g^{-1}, & if(i,j)\in the-best-solution\\ 0, & \text{otherwise}\end{cases}.\end{aligned} \tag{10}$$

4. The Strategy of ant Colony Optimization Algorithm for Solving CSAHLP

4.1. The Design of ant Colony Optimization Algorithm

The method Tabu List Search was used to solve the problem TSP in ant colony optimization algorithm. This algorithm obviously contracts allocation of solution space in every step, accelerates convergence and gets optimal solution in finite iterations. This paper introduces the concept of solution pair (SP) when using ant colony optimization algorithm.

In this paper, solution pair is defined like this: In the CSAHLP, node i and its corresponding hub k is a solution pair. Considering the order of solutions, hub node k and hub k itself is also called a solution pair.

With the concept of solution pair, the solutions to CSAHLP are decomposed into discrete solutions pairs in this paper. The example is as follows (as shown in Fig. 2): in general cases, solution compositions i, m deliver freights to *j* via *k*, *l* .

After processing the models of solution pair, we got the following solution pair(as shown in Fig. 3).

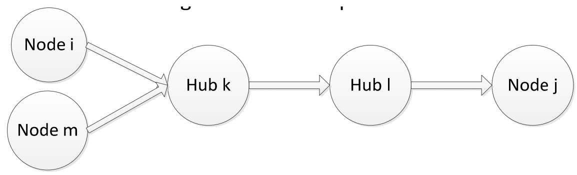

Fig. 2. Solution composition.

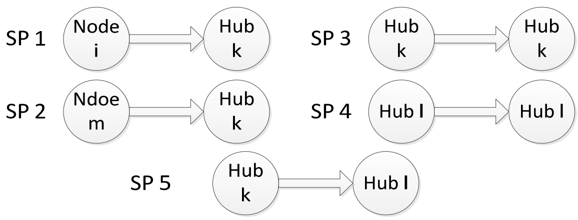

Fig. 3. Solution Pair.

It's not difficult to see that in CSAHLP the number of solution pairs equals that of cities. Ant colony optimization algorithm will turn the problem into deterministic problem after the solutions are decomposed into solution pairs.

The steps of ant colony optimization algorithm for solving CSAHLP are as follows:

1. Initialization.

1) Confirming the number g of ants in each iteration ($1 \le g \le \sqrt{1.44n}$, n represents the number of cities).

2) Setting the maximum iteration number called MaxiItcount. MaxiItcount is general constraint condition helping the program avoid getting into endless loop when falling into local optimal solutions. Randomly ranking n cities, the serial numbers of each city are: $k_1, k_2, \dots, k_n$. After randomization, it forms arrays to g*MaxiItcount in total, like $k_{j1}, k_{j2}, \dots\ k_{jn}$ $(j \in \mathrm{random}(n))$, ensuring that every ant will get different collating sequence, also ensuring the diversity and uniqueness of solution.

3) When initializing CSAHLP, we calculate the distance $\mathrm{distance}(k_i, k_j)$ between two cities first and then initialize the pheromone $\mathrm{m\_dTrial}(k_i, k_j)$ between two cities. According to the ACS theory of M. Dorigo [16], the initial non-zero pheromone between any two points is very small. The purpose is to facilitate the operation of local pheromone updating. Initializing every city capacity $\mathrm{Cap}(k_i)$ and logistics quantity $\mathrm{Weight}(k_i)$ that every city needs to distribute.

2. Finding solution pairs.

Each ant chooses a randomized sequence of cities and begins to build solution pair from the first city. In every city, ant k_j uses pseudo random choice rule to calculate and uses equation 8, 9 to find the city which has the strongest relationship with it to form k_{sp}. k_{sp} is labeled as $\mathrm{comba}[\,k_j] = k_{j_{sp}}$with array link. After that, the $\mathrm{comba}[k_{j_{sp}}]$ of $\mathrm{k_{sp}}$ needs to be labeled as $\mathrm{comba}[k_{j_{sp}}] = k_{j_{sp}}$ in case that $\mathrm{k_{sp}}$ points to other cities later, namely hubs won't form any solution pair with unknown nodes any more. After finding solution pair, $\mathrm{Cap}(k_{sp})$ minus $\mathrm{Weight}(k_j)$ is Cap(k_{sp}), namely Cap(k_{sp})=Cap(k_{sp})-Weight(k_j). It reflects the constraints of capacity limit. Every ant builds their own solution following these steps.

3. Calculating the value of objective function.

After each ant completing crawling all the nodes, we will deal with solutions. We then calculate the value L of objective function according to equation 11 and update the local pheromone of L's solution according to L.

$$L = min \sum_i \sum_k C_{ik} X_{ik} (\chi O_i + \delta D_i) \sum_i \sum_k \sum_l \alpha C_{kl} Y_{kl}^{i} + \sum_k F_k X_{kk}, \tag{11}$$

4. Global pheromone updating.

In each MaxiItcount, the ant who has the minimal value L of objective function will update the global pheromone according to its own solution pair.

5. Calculating the optimal solution in finite MaxiItcount and solution saving solution.

4.2. Local Search Strategy

Ant colony algorithm needs to be combined with local search strategy. This paper introduces local search algorithm to accelerate the convergence speed. In this paper, with reference to the six kinds of local search operator proposed by literature [16-18] to calculate with ant colony algorithm. We define the concept of group before introducing the six kinds of local search operator. Group refers to node group containing a hub node and the nodes allocated to the hub. Each hub node is allocated to itself. Here are the six kinds of local search operator:

1. Resetting the hub: changing the hub node in any of the groups into another randomly selected node in the group. The transformation is applied to those groups containing at least one node.

2. Resetting the node: allocating a node in any of the groups to another randomly selected group, especially when a group contains only one hub node. Allocating this node to other group will reduce the number of groups or hubs.

3. Setting new hubs: setting a randomly selected node as hub node to establish a new group containing only one node.

4. Combining groups: allocating all the nodes in a group to a hub node in another randomly selected group and combining these two groups as a whole.

5. Splitting groups: allocating part of the nodes in a group to another randomly selected node to split them in two groups.

6. Switching node: switching the nodes in two groups and allocating these nodes to the hub node in each other's group.

All these local search operator cannot violate weight limit in operation. And as to every ant colony system, the global update of its own ant colony algorithm should come after completing local search. The operation of the six local search operators first follows criterion [17]. As long as a local search operator generates more reasonable objective function values, it will update solutions. If there are more than one search operators to generate more reasonable objective function values, then we randomly choose the result of a search operator to update solutions.

5. Experimental Result

This paper verifies this method using data from Australia Post (AP) [18]. Through repeated simulation operation, we got the following solutions (Table 1, Table 2): It is relatively obvious that the ant colony algorithm for solving CSAHLP cannot avoid the inherent shortcoming of falling into local optimums.

Compared with TSP, in CSAHLP, the production of solution pairs and the diversity of path selection make it easy to fall into local optimums. Combined with six local search operators to avoid falling into local optimums, the data below 25TT are all optimal solutions generated after several operations and are the same as proven optimal solutions.

This algorithm adds some methods of combining groups in the process of design to reduce the number of solutions.

In the experimental result, we can see the allocation of every node and the confirmation hub nodes. Compared with the proven optimal results, its deviation is 0, well proving the feasibility and reliability of the solution.

In solving practical engineering, combined with optimal solutions, we find that the solutions of this paper generate too many hub nodes, and optimal solutions were gotten after using the method of merging groups to merge some hub nodes randomly.

Table 1. Calculation results.

Problem	*Hubs*	*Allocation*
Problem 10LL	2, 3, 6	2 3 2 3 6 3 6 6 6 6
Problem 10LT	0,3,4,9	0 3 4 3 4 3 9 9 9 9
Problem 10TL	3, 4, 9	4 3 4 3 4 3 9 9 9 9
Problem 10TT	3, 4, 9	4 3 4 3 4 3 9 9 9 9
Problem 20LL	6,13	6 6 6 6 6 6 6 6 13 6 6 6 13 13 13 13 13 13 13 13
Problem 20LT	9,13	9 9 9 9 9 9 9 9 9 9 9 9 9 13 13 9 9 13 13 13
Problem 20TL	6,18	6 6 6 6 6 6 6 6 18 6 6 6 18 18 18 18 18 18 18 18
Problem 20TT	0, 9,18	0 0 9 9 9 9 9 9 9 9 9 9 9 18 9 9 18 18 18 18
Problem 25LL	7,17	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 17 17 17 17 17 17 17 17 17 17
Problem 25LT	8,15,24	8 8 8 8 8 15 8 8 8 8 15 15 8 8 8 15 15 24 24 24 15 15 24 24 24
Problem 25TL	8,22	8 8 8 8 8 22 8 8 8 8 22 22 8 8 8 22 22 22 22 22 22 22 22 22 22
Problem 25TT	8,15,24	8 8 8 8 8 15 8 8 8 8 15 15 8 8 8 15 15 24 24 24 15 15 24 24 24

Table 2. Comparison of results.

Problem	*Ant Colony Optimization Algorithm*		*Hybrid simulated annealing algorithm*		*Lagrange relaxation algorithm*	
	calculation time (s)	**average deviation (%)**	**calculation time (s)**	**average deviation (%)**	**calculation time (s)**	**average deviation (%)**
Problem 10LL	0.03	0.00	0.05	0.00	0.04	0.00
Problem 10LT	0.03	0.00	0.06	0.00	0.05	0.00
Problem 10TL	0.03	0.00	0.06	0.00	0.06	0.00
Problem 10TT	0.03	0.00	0.07	0.00	0.07	0.00
Problem 20LL	0.06	0.00	0.20	0.00	0.42	0.00
Problem 20LT	0.08	0.00	0.13	0.00	1.51	0.51
Problem 20TL	0.08	0.00	0.12	0.00	0.19	0.00
Problem 20TT	0.09	0.00	0.21	0.00	0.21	0.00
Problem 25LL	0.35	0.00	0.36	0.00	1.95	0.01
Problem 25LT	0.43	0.00	0.42	0.01	2.34	0.04
Problem 25TL	0.50	0.00	0.43	0.02	1.85	0.03
Problem 25TT	1.02	0.01	0.44	0.03	2.43	0.31

6. Conclusion

This paper takes the advantage of ant colony optimization algorithm in solving combinatorial optimization problems. The final solutions are built in the form of solution pairs and six kinds of local search algorithms are inserted into parallel computing ant colony groups to enhance the search ability of ant colony algorithm on optimal solutions. Example simulation experiment is conducted according to the AP database as well. The experimental result shows that: ant colony optimization algorithm can be better used in CSAHLP indeed, which solves a lot of complex location problems that are hard to calculate. Ant colony optimization algorithm itself is a probability selection algorithm that has the speed and convenience that other fixed algorithms don't have. Combined with some other mature algorithms, it will be of wide use in the near future.

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