Sensors & Transducers, Vol. 180, Issue 10, October 2014, pp. 215-220

Sensors & Transducers

http://www.sensorsportal.com

A Novel Sub-pixel Measurement Algorithm Based on Mixed the Fractal and Digital Speckle Correlation in Frequency Domain

[1] Zhangfang Hu, [1] Dongdong Huang, [1] Yuan Luo, [2] Yi Zhang, [1] Bing Xi

[1] Key Laboratory of Optical Fiber Communication Technology, Chongqing University of Posts and Telecommunications, No. 2, Chongwen Road, Nan'an District, Chongqing, 400065, China

[2] Research Center of Intelligent System and Robot, Chongqing University of Posts and Telecommunications, No. 2, Chongwen Road, Nan'an District, Chongqing, 400065, China

[1] Tel.: +86-18725875566

E-mail: huzf@cqupt.edu.cn

Received: 19 June 2014 /Accepted: 30 September 2014 /Published: 31 October 2014

Abstract: The digital speckle correlation is a non-contact in-plane displacement measurement method based on machine vision. Motivated by the facts that the low accuracy and large amount of calculation produced by the traditional digital speckle correlation method in spatial domain, we introduce a sub-pixel displacement measurement algorithm which employs a fast interpolation method based on fractal theory and digital speckle correlation in frequency domain. This algorithm can overcome either the blocking effect or the blurring caused by the traditional interpolation methods, and the frequency domain processing also avoids the repeated searching in the correlation recognition of the spatial domain, thus the operation quantity is largely reduced and the information extracting speed is improved. The comparative experiment is given to verify that the proposed algorithm in this paper is effective. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Digital speckle correlation, Sub-pixel displacement, Interpolation algorithm, Fractal, Frequency domain.

1. Introduction

It is well-known that the digital speckle correlation method (DSCM) has nowadays become one of the most commonly adopted optical metrology techniques to quantify surface deformation in a variety of practical fields, since it was originally proposed in the early 1980s [1-5]. The fundamental principle of DSCM lies in comparing gray scale images of an object capture before and after displacement, usually referred to as reference and target images, respectively, based on some specific correlation criterion to get the information on displacements of some points sampled beforehand on the reference image [6, 7]. But this algorithm has a large amount of calculation because of the utilizing of repeated searching, and the accuracy of the measurement can only reach pixel level. To achieve displacement measurement with sub-pixel accuracy, several classes of subset-based sub-pixel registration algorithm [2, 8-10] have been put forward over the past three decades, some of which, e.g., the gradient-based method [2] and the Newton-Raphson (NR) iteration method [8] and the other interpolation methods [11], have already been widely used in a variety of application fields.

Based on an optical flow method developed by Davis and Freeman [12], the gradient-based algorithm that considered the effect of intensity gradients of both images before and after displacement was presented [9]. In practical application, the error is introduced by abandoning the higher order term due to the use of Taylor formula. The coarse-fine (pyramid) search method, iteration and second-order Taylor formula are used to reduce the error, but distinctly increase the computation [13].

Interpolations used in the sub-pixel registration usually include phase correlation interpolation [14], intensity interpolation [15] and correlation function interpolation [16]. The phase correlation interpolation method is suitable for the images seriously distorted, however, the accuracy of which is the lowest among the three [17]. While the intensity interpolation has been less adopted for its immense computation consumption, the correlation function interpolation is generally considered due to its lower computation and better noise-proof feature, nevertheless, an obvious weakness is that the sensitivity is limited by the interpolation step because of the systematic error of the algorithm [18, 19].

Iterative algorithm often requires the calculation of second-order spatial derivatives of the digital images, which further increases computation complexity.

However, the traditional digital speckle correlation method in spatial domain does not avoid the problem of large amount of calculation caused by repeated searching. In this paper, the DSCM is converted into the frequency domain by Fourier transform, so avoids a large number of calculations. Meanwhile, a fast interpolation method based on fractal theory is proposed to improve the measurement accuracy.

2. Fast Sub-pixel Interpolation Based on Fractal

2.1. Fractal and Random Midpoint Method

Fractal theory is a discipline for describing the irregular things and phenomena in the nature. The researching object is a class fractal object of self-similarity and self-affine. Because of many things have explicitly or implicitly fractal characteristics, the fractal can be divided into rules fractal and random fractal. Rules fractal is iteration formation based on certain rules and functions. While random fractal is randomness, it can be better description of natural phenomena.

Mandelbrot applied fractal dimension to quantitative describe the image's similarity between the whole and partial, proposed the concept of Fractional Brownian Motion (FBM). Random midpoint displacement method can use a simple equation to express the interpolation point (x_{mi}, y_{mi})

$$x_{mi} = \left(x_i + x_{i+1}\right)/2 + s \bullet w \bullet rand(), \tag{1}$$

$$y_{mi} = \left(y_i + y_{i+1}\right)/2 + s \bullet w \bullet rand(), \tag{2}$$

where s and w are the parameters indicating the moving direction and the moving distance respectively, $rand()$ is the random variable.

2.2. Fractal Interpolation Algorithm

For an image obtained from nature, the geometrical image follow the variation of fractal geometry, and the distribution of brightness also has the characteristics of fractal. According to the fractal nature of the image, image and the actual scene have the same fractal dimension stayed unchanged even with the different transformation ratios, so fractal interpolation can be used to achieve sub-pixel measurement. The image interpolated usually loses the texture features with using the traditional methods, because of the principle of which based on the fact that it is uniformly continuous change between the data point. A basic feature of fractal is able to demonstrate the fine structure of the object, so fractal interpolation can achieve the better effect in the processing of extracting the sub-pixel displacement.

According to the fractal geometric feature, the new pixel's information is increased between the known pixels. In order to calculate the gray of the new pixel, recursive random midpoint displacement method is adopted as shown in Fig. 1.

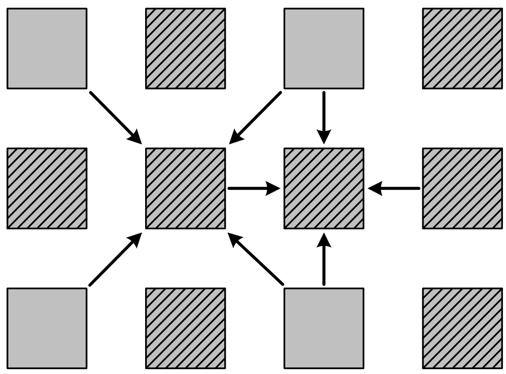

Fig. 1. Schematic of the fractal interpolation.

The recursive formula of the fractal interpolation can be expressed as follows. (i, j) represent the location of the original pixels. Obviously $I(i, j)$ is known when i and j are odd. When i and j are even, the gray of new pixel can be written as

$$\begin{aligned} I(x, y) = [I(i-1, j-1) + I(i+1, j+1) + \\ I(i-1, j+1) + I(i+1, j-1)]/4 + \Delta I \end{aligned} \tag{3}$$

When i is an odd and j is even, or i is even and j is an odd, then

$$I(x,y)=[I(i-1,j)+I(i,j+1)+ I(i,j-1)+I(i+1,j)]/4+\Delta I \quad (4)$$

$$\Delta I=\sqrt{1-2^{2H-2}}H\sigma G\times\left(1/2^{H/2}\right)^{n-1}, \quad (5)$$

where $x,y\subset k+1/2^n$ (k is the positive integer), n is the number of interpolation, H is the fractal parameter, σ is the mean-square deviation of pixels' brightness and G is Gaussian random variable following $N(0,1)$. Traversing image, ΔI multiplied by the factor $2^{-H/2}$ each cycle, and repeating the iteration until achieves the desired spatial resolution. From the theoretical analysis, the measurement accuracy can be achieved $1/2^n$ after n times interpolation.

3. Principle of DSCM in Frequency Domain

The frequency domain processing avoids the repeated searching in the correlation recognition of the spatial domain, thus the operation quantity is largely reduced and the information extracting speed is improved.

First of all, $p(x,y)$ is considered to the sample sub-region of the original digital speckle image, and $p_-(x,y)$ is the other region, so the reference image can be written as

$$I_1(x,y)=p(x,y)+p_-(x,y), \quad (6)$$

Similarly, the image after moving is expressed as

$$I_2(x,y)=q(x,y)+q_-(x,y), \quad (7)$$

where $q(x,y)$ is the target sub-region. In this paper, the motion follows the horizontal or vertical direction in the plane coordinate system. And the gray value of anyone pixel remains the same in reference image and the moved image, because of the displacement is very small compared with the lighting area. So we can acquire the following expression:

$$q(x,y)=p(x+u,y+v), \quad (8)$$

where u and v are the displacement vector in x direction and y direction respectively.

On the one hand, the sample sub-region is converted to frequency domain by Fourier transform, and (w_x,w_y) is used as the coordinate parameters.

$$f(w_x,w_y)=FFT\{p(x,y)\} =\left|f(w_x,w_y)\right|exp\left[j\varphi(w_x,w_y)\right] \quad (9)$$

We can easily find the conjugate expression:

$$f^*(w_x,w_y)=\left|f(w_x,w_y)\right|exp\left[-j\varphi(w_x,w_y)\right] \quad (10)$$

Eq. (10) can be seen as the conjugate filter, $\left|f(w_x,w_y)\right|$ and $\varphi(w_x,w_y)$, respectively, represent the amplitude and the phase.

On the other hand, we can readily acquire the following expression similar to the Eq. (9):

$$\begin{aligned}g(w_x,w_y)&=FFT\{I_2(x,y)\}=FFT\{q(x,y)+q_-(x,y)\}\\&=FFT\{p(x+u,y+v)+q_-(x,y)\}\\&=\left|f(w_x,w_y)\right|exp\left[j2\pi(uw_x+vw_y)\right]+k(w_x,w_y)\end{aligned} \quad (11)$$

with

$$k(w_x,w_y)=FFT\{q_-(x,y)\}, \quad (12)$$

The frequency spectrum of the target image filtered with the conjugate filter, that is, Eq. (11) multiplied by Eq. (10).

$$\begin{aligned}h(w_x,w_y)&=f^*(w_x,w_y)g(w_x,w_y)\\&=\left|f(w_x,w_y)\right|^2 exp\left[j2\pi(uw_x+vw_y)\right]\\&+f^*(w_x,w_y)k(w_x,w_y)\end{aligned} \quad (13)$$

Finally, Eq. (13) is analyzed by using the Fourier transformation. That is

$$\begin{aligned}H(\varepsilon,\eta)&=FFT\{h(w_x,w_y)\}\\&=G(\varepsilon,\eta)\delta(\varepsilon-u,\eta-v)+F(-\varepsilon,-\eta)*K(\varepsilon,\eta)\end{aligned} \quad (14)$$

in which, $*$ represents the convolution and

$$K(\varepsilon,\eta)=FFT\{k(w_x,w_y)\}, \quad (15)$$

We may equivalently rewrite Eq. (14) as

$$H(\varepsilon,\eta)=G(\varepsilon-u,\eta-v)+F(-\varepsilon,-\eta)*K(\varepsilon,\eta), \quad (16)$$

The first item of Eq. (16) shows up as a correlation light spot, and the magnitude and direction of displacement can be extracted according to the location of the light spot. And the second one represents fuzzy convolution. But in the actual experiment, the two sub-regions have the same size and the location are acquired, respectively, from the reference image and the target image because of the fact that the sub-pixel displacement is tiny compared with the size of the sub-region. Additionally, fractal interpolation is used to ensure the sub-pixel precision. For clarity, Table 1 presented a simple step to illustrate how the newly proposed algorithm works.

Table 1. The steps of this algorithm.

Step. 1	Collect sub-region images, respectively, from reference image and target image
Step. 2	Fractal interpolation on the two sub-regions
Step. 3	Make the filter
Step. 4	Filtering the frequency spectrum of the target sub-region with the filter
Step. 5	Acquire the Fourier transformation of the output obtained in Step. 4
Step. 6	Obtain the light spot and extract the displacement according to the location of the light spot

4. Results and Discussion

To verify the feasibility and effectiveness of the proposed algorithm, synthetic speckle images are created. It was hypothesized that the simulated reference image of size 512×512 pixels, and the target image is assumed to undergo in-plane displacement along horizontal and vertical direction, and the value, respectively, is 0.5 and 0.8 pixel, as shown in Fig. 2.

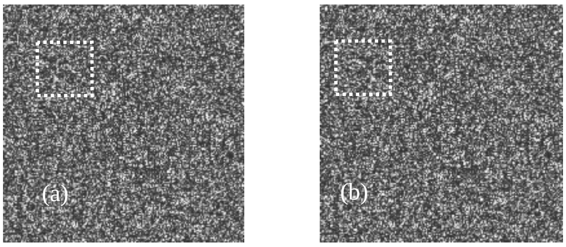

Fig. 2. (a) Simulated reference image, (b) the corresponding deformed image with the motion following horizontal and vertical direction in-plane.

In the Fig. 2, the dotted box part is used as the subsets have the same size and the location in our experiment, Fig. 3 exhibited the influence on the choice of the subset size.

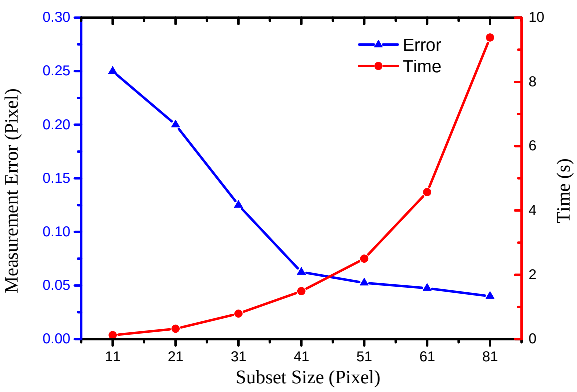

Fig. 3. The choice of the subset size.

In Fig. 3, the true displacement is 0.5 pixels only following the horizontal direction. It could be easily learned that the measurement error decreases along with the increscent subset size, inversely, the elapsed time is proportional to the size of the sub-region. In this paper, the experimental environment as follows: Windows 7, 64-bit operating system, Intel(R) Celeron(R)CPU 2.60 GHz, MATLAB 2012a. Considering the measurement error and the run-time, synthetically, 41×41 is the desired size of the subset, so the size of subset is set to 41×41 pixel. The experimental result of Fig. 2 without taking fractal interpolation as shown in Fig. 4.

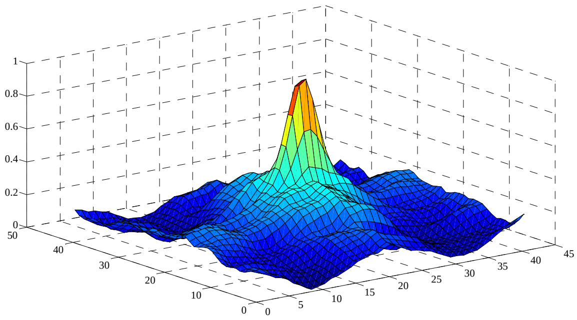

Fig. 4. Measurement result without using fractal interpolation.

It could be learned from Fig. 4 that the peak seems very wide. And finally, the actual measured value is 1 pixel and 1 pixel according to the location of the peak, respectively, in the x direction and y direction. Obviously, the measurement error is unacceptable, so the fractal interpolation algorithm described above is employed to improve the measurement accuracy as shown in Fig. 5.

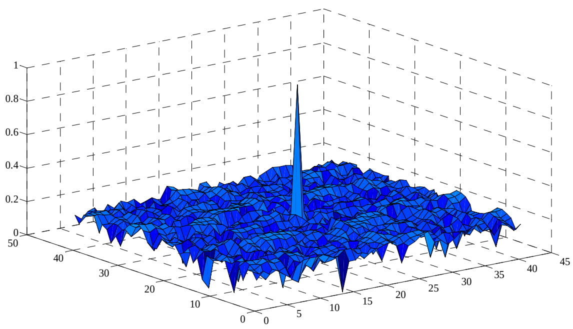

Fig. 5. Measurement result after using fractal interpolation.

Compared with Fig. 4, the peak in Fig. 5 seems more exquisite and sharp. And the actual measured value is 0.4375 pixels and 0.8125 pixels in horizontal direction and vertical direction, meanwhile the measurement error is in the acceptable range.

Another set of experiments indicate the effectiveness of the proposed algorithm via detecting displacement of different sizes as shown in Fig. 6 and Fig. 7.

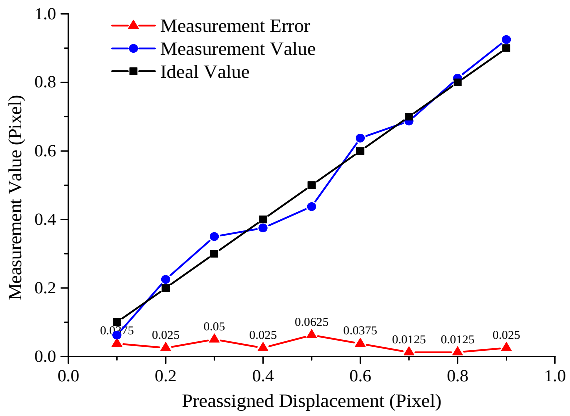

Fig. 6. The measurement results and error curve presented by the proposed algorithm.

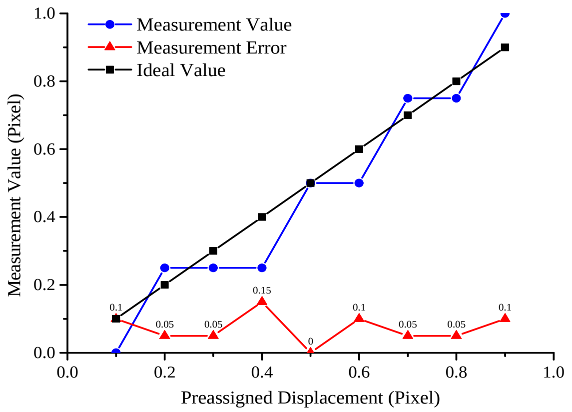

Fig. 7. The measurement results and error curve presented by the bicubic interpolation with the DSCM in spatial domain.

Obviously, in the same coordinate system, the dot line seems smooth in Fig. 6 compared with that in Fig. 7, and overall, the triangle curve in Fig. 6 is more close to the horizontal axis. Data also show that the maximum absolute error is 0.0625 pixels by using the proposed algorithm less than 0.15 extracted by the traditional method in spatial domain. In this paper, the stability of algorithm can be represented by the uncertainty, and the standard deviation of the absolute error indicates the uncertainly. After calculation, the uncertainly of former is 0.034 less than 0.075 calculated by another algorithm. Summary it can be concluded that the stability and effectiveness of the proposed algorithm is superior to the traditional digital speckle correlation method in spatial domain.

5. Conclusions

In this paper, a novel sub-pixel measurement algorithm combining fractal interpolation in frequency domain is proposed. The traditional digital speckle correlation method is converted to the frequency domain via the Fourier transform, and avoids the repeating search in spatial domain. Meanwhile, the fractal interpolation is utilized to ensure the sub-pixel accuracy. Finally, comparative experiments verify the fact that the effectiveness and stability of proposed algorithm is superior to the traditional method.

Acknowledgements

The work is supported by the National Natural Science Foundation of China (Grant NO.51075420) and the Natural Science Foundation of CQ CSTC (CSTC 2012jjB40009).

References

[1]. M. A. Sutton, J. J. Orteu, H. W. Schreier, Image correlation for shape, motion and deformation measurements, *Springer*, 2009.

[2]. Zhang Jun, Jin Guanchang, Ma Shaopeng, Me Libo, Application of an improved subpixel registration algorithm on digital speckle correlation measurement, *Optics & Laser Technology*, Vol. 35, Issue 7, 2003, pp. 533-542.

[3]. M. Fazzini, S. Mistou, O. Dalverny, L. Robert, Study of image characteristics on digital image correlation error assessment, *Optics and Lasers in Engineering*, Vol. 48, Issue 3, 2010, pp. 335-339.

[4]. T. C. Chu, W. F. Ranson, M. A. Sutton, Applications of digital-image-correlation techniques to experimental mechanics, *Experimental Mechanics*, Vol. 25, Issue 3, 1985, pp. 232-244.

[5]. Chen Jinlong, Zhang Xiaochuan, Zhan Nan, Hu Xiaoyan, Deformation measurement across crack using two-step extended digital image correlation method, *Optics and Lasers in Engineering*, Vol. 48, Issue 11, 2010, pp. 1126-1131.

[6]. F. Hild, S. Roux, Digital image correlation: from displacement measurement to identification of elastic properties – a review, *Strain*, Vol. 42, Issue 2, 2006, pp. 69-80.

[7]. Huang Jianyong, Pan Xiaochang, Peng Xiaoling, et al, High-efficiency cell-substrate displacement acquisition via digital image correlation method using basis functions, *Optics and Lasers in Engineering*, Vol. 48, Issue 11, 2010, pp. 1058-1066.

[8]. C. Cofaru, W. Philips, W. Van Paepegem, Improved Newton-Raphson digital image correlation method for full-field displacement and strain calculation, *Applied Optics*, Vol. 49, Issue 33, 2010, pp. 6472-6484.

[9]. Zhou Peng, K. E. Goodson, Subpixel displacement and deformation gradient measurement using digital image/speckle correlation (DISC), *Optical Engineering*, Vol. 40, Issue 8, 2001, pp. 1613-1620.

[10]. Hu Zhenxing, Xie Huimin, Lu Jian, et al, Study of the performance of different subpixel image correlation methods in 3D digital image correlation, *Applied Optics*, Vol. 49, Issue 21, 2010, pp. 4044-4051.

[11]. Luo Yuan, Liu Yuan, A fast interpolation algorithm based on the fractal in the digital speckle sub-pixel

displacement measurement, *Applied Mechanics and Materials*, Vol. 344, 2013, pp. 238-241.
[12]. C. Q. Davis, D. M. Freeman, Statistics of subpixel registration algorithms based on spatiotemporal gradients or block matching. *Optical Engineering*, Vol. 37, Issue 4, 1998, pp. 1290-1298.
[13]. Hu Zhangfang, Huang Dongdong, Luo Yuan, et al, Research on a novel compensation algorithm for MEMS sub-pixel displacement measurement, *Advanced Materials Research*, Vol. 901, 2014, pp. 81-86.
[14]. L. Luu, Z. Wang, M. Vo, et al, Accuracy enhancement of digital image correlation with B-spline interpolation, *Optics Letters*, Vol. 36, Issue 16, 2011, pp. 3070-3072.
[15]. P. Lava, S. Cooreman, S. Coppieters, M. De Strycker, D. Debruyne, Assessment of measuring errors in DIC using deformation fields generated by plastic FEA, *Optics and Lasers in Engineering*, Vol. 47, Issue 7, 2009, pp. 747-753.
[16]. V. N. Dvornychenko, Bounds on (deterministic) correlation functions with application to registration, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 5, Issue 2, 1983, pp. 206-213.
[17]. Tian Qi, Michael N. Huhns, Algorithms for subpixel registration, *Computer Vision, Graphics, and Image Processing*, Vol. 35, Issue 2, 1986, pp. 220-233.
[18]. Yu Qifeng, Lu Hongwei, X. Liu, Image based precise measurement and motion measurement, *Beijing Science Press*, Beijing, 2002.
[19]. H. W. Schreier, J. R. Braasch, M. A. Sutton, Systematic errors in digital image correlation caused by intensity interpolation, *Optical Engineering*, Vol. 39, Issue 11, 2000, pp. 2915-2921.