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Design and Analysis for Weak Rigidity Combination Planetary Disc of Precision Ball Transmission

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Abstract: According to the structure and drive principle of the precision ball transmission, an innovative structure of weak rigidity combination planetary disc which set planetary motion and clearance adjustment function as a whole was proposed. The performance requirements of the key connection pair in weak rigidity combination planetary disc were analyzed. A structure connected by pins was designed. The analysis model which described the backlash in pin connection pair was established. The calculation formula of backlash was derived. The failure modes and strength of pin connection pair was analyzed. The Monte Carlo simulation and calculation of backlash was carried out through an example programming by the software MATLAB. The results show that the decrease of backlash can be accomplished by changing the structure parameters. The practicability of pin connection pair is verified by the backlash simulation and strength check. The results provide important theory basis for research on the real-time non gap precision ball transmission.

Keywords: Precision ball transmission, Weak rigidity, Combination planetary disc, Connection pair, Backlash.

1. Introduction

As the core mechanism of highly sophisticated mechanical transmission equipment, the performance of precision transmission mechanism is an important index to evaluate the equipment quality. Real time precision transmission is the embodiment of important value and is the target of various highly precision transmission mechanism as well [1]. Using the cycloid ball decelerating meshing pair and the ball constant velocity meshing pair in precision ball transmission, the precision transmission is realized [2, 3]. During the transmission process, the thermal expansion and the friction wear of the two types of balls meshing pair occur in real-time, the wearing clearance and the thermal expansion interference exist real time, which lead to the appearance of abnormal meshing pair or backlash. Currently, most of the related studies on controlling the backlash are manual adjustment regularly [4-6]. Therefore, the wearing clearance and the thermal expansion interference of normal rigidity meshing pair can't be eliminated real-time and the non-gap precision transmission can't be realized. Also, there are researchers proposed rigidity structures to realize no-backlash transmission [7, 8]. But these structures are difficult to machining.

An innovative structure of weak rigidity combination planetary disc which realized the real-time non gap meshing of the two kinds of ball meshing pair was proposed. In the circumferential direction, the structure can realize real-time rigid rotation, and in the axial direction, it can realize real-time micro mobile. Design and analysis for weak rigidity combination planetary disc is the key technology of how to realize the two kinds of motion. Therefore, a structure connected by pins was designed and the backlash as well as the strength was analyzed.

2. Structure and Drive Principle of the Precision Ball Transmission

Fig. 1 shows the structure of the precision ball transmission. The group 1 of steel balls, the cycloid groove of the center disc and weak rigidity combination planetary disc make up the weak rigidity non gap ball decelerating meshing pair; the group 2 of steel balls, the ring groove of the output disc and weak rigidity combination planetary disc make up the weak rigidity non gap ball velocity meshing pair. Through the weak rigidity non gap ball decelerating meshing pair the input motion is turned into low speed, the low speed motion is output by the weak rigidity non gap ball velocity meshing pair, above all the stable drive ratio speed transform is realized.

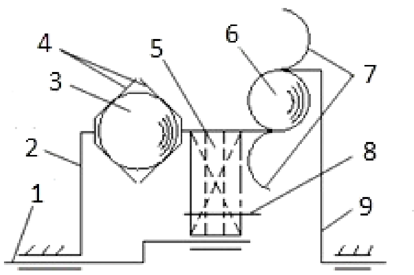


Fig. 1. Structure diagram of precision ball transmission.

Weak rigidity combination planetary disc can realize planetary motion and clearance adjustment. Fig. 2 shows the structure. Weak elastomer is located between the planetary discs. The planetary discs connected by connection part and combine with connection pair can realize circumferential synchronous rigid rotation. With the weak elastomer the structure can realize relative micro mobile in the axial direction. Therefore, the real-time synchronous rigid rotation in the circumferential direction of the weak rigidity combination planetary disc ensures the meshing rule of the surface meshing pair, the thermal expansion and the friction wear are eliminated by the real-time micro mobile in the axial direction.

The key to design for weak rigidity combination planetary disc is how to realize the synchronous rigid rotation in the circumferential direction and the real-time micro mobile in the axial direction using connection pair. A structure connected by pins was designed in this article.

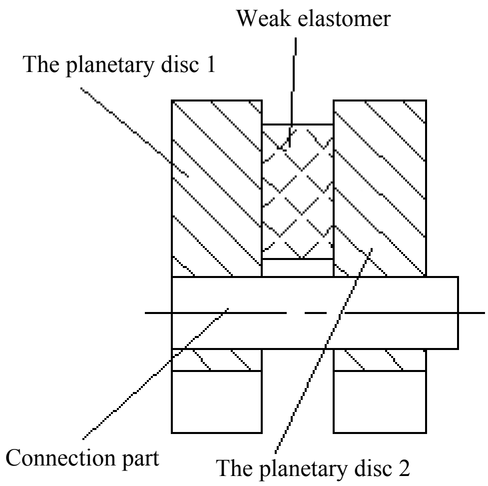


Fig. 2. Structure diagram of weak rigidity combined planetary disc.

3. Design and Analysis of Pin Connection Pair

3.1. Design of the Structure for Pin Connection Pair

Fig. 2 shows the structure of pin connection pair. On the distribution circle of the two planetary discs a certain number of pins shifting pair were designed. As the interference fit between the pin and the planetary disc 1, there is no relative motion between them. As the transition fit or clearance fit between the pin and the planetary disc 2, there are relative motion between the pin and the planetary disc two. The torque between the two discs is transmitted through the pins. The relative micro mobile between the two discs in the axial direction is realized for the disc 1 moving along the bearing and the disc 2 moving along the pin. Thus the clearance is adjusted.

The processing technology and machining accuracy of circular section pin hole is better than any other section shapes. According to material mechanics, the bending strength of circular section pin is better than others. Therefore, we take the cylindrical pin as example to analyze the backlash of weak rigidity combination planetary disc.

The main influences on the backlash of pin connected weak rigidity combination planetary disc are manufacturing error of the two planetary discs and the pins, assembly error and errors caused by temperature change and elastic deformation, etc. [9]. The clearance of pin connection pair is the main reason for backlash. The main sources of the clearance are the clearance of the connection pair which ensures the micro mobile in the axial direction, the manufacturing error of the connection pair and wear. Before analyze the backlash of pin connected weak rigidity combination planetary disc, the analysis model which described the backlash of single pin connection pair was established. The calculation formula of backlash was derived. The influences on backlash were analyzed. After that the analysis model was extended to multiple pins connection pair, which has important significance in reducing backlash and improving transmission accuracy.

3.2. Backlash Analysis of Single Pin Connection Pair

Because of the clearance of the pin connection pair, the relative motion between the pin and pin hole appears. Therefore, the backlash of pin connected weak rigidity combination planetary disc appears. Fig. 3 shows the backlash analysis model of single pin connection pair. The position of dashed line is the initial position of pin. At this time the center of the pin hole B, the center of the pin A and the rotation center of weak rigidity combination planetary disc O three points are collinear. The position of the solid line is the beginning of the pin and pin hole being contact. α is the angle of planetary disc 3 turns from the dashed line position to the solid line position.

According to the definition of backlash that the angle lag of output axis follows the input axis when the input axis counter rotate, 2α is the backlash of single pin connection pair.

In the ΔOAB , α can be obtained using cosine theorem

$$\alpha = \arccos[\frac{R_{xz}^2 + R_{xk}^2 - \delta_3^2}{2R_{xz}R_{xk}}], \quad (1)$$

where R_{xz} is the radius of pins real distribution ($R_{xz} = R_x + \delta_1$, R_x is the radius of theoretical distribution, δ_1 is the error of the radius of pins distribution), R_{xk} is the radius of pin hole real distribution ($R_{xk} = R_x + \delta_2$, δ_2 is the error of the radius of pins hole distribution), δ_3 is the clearance of pin connection pair ($\delta_3 = r_{x2} - r_{x1}$, r_{x1} and r_{x2} is the radius of the pins and pin holes).

Therefore, the backlash of single pin connection pair can be described as below

$$\varphi_1 = 2\alpha = 2\arccos[\frac{R_{xz}^2 + R_{xk}^2 - \delta_3^2}{2R_{xz}R_{xk}}], \quad (2)$$

From Eq.(2), the effects of the backlash of single pin connection pair are the radiuses of pin and pin hole distribution and the clearance of pin connection pair. The clearance changes when there are errors for the radiuses of pin and pin hole, which lead to errors of backlash. The errors of the radiuses of pin and pin hole were considered simultaneously, and the backlash of pin connected weak rigidity combination planetary disc can be described as below

$$\Delta\varphi_1 = \frac{\partial\varphi_1}{\partial R_{xz}}\Delta R_{xz} + \frac{\partial\varphi_1}{\partial R_{xk}}\Delta R_{xk} + \frac{\partial\varphi_1}{\partial\delta_3}\Delta\delta_3,$$

where $\frac{\partial\varphi_1}{\partial R_{xz}}$, $\frac{\partial\varphi_1}{\partial R_{xk}}$ and $\frac{\partial\varphi_1}{\partial\delta_3}$ are the errors transfer coefficient of R_{xz} , R_{xk} and δ_3 , which reflect the influence degree for the parameter errors to backlash errors and can be obtained by calculating the partial derivative of Eq.2

$$\frac{\partial\varphi_1}{\partial R_{xz}} = \frac{-2(R_{xz}^2 - R_{xk}^2 + \delta_3^2)}{R_{xz}\sqrt{(2R_{xz}R_{xk})^2 - (R_{xz}^2 + R_{xk}^2 - \delta_3^2)^2}}, \quad (3)$$

$$\frac{\partial\varphi_1}{\partial R_{xk}} = \frac{-2(R_{xk}^2 - R_{xz}^2 + \delta_3^2)}{R_{xk}\sqrt{(2R_{xz}R_{xk})^2 - (R_{xz}^2 + R_{xk}^2 - \delta_3^2)^2}}, \quad (4)$$

$$\frac{\partial\varphi_1}{\partial\delta_3} = \frac{4\delta_3}{\sqrt{(2R_{xz}R_{xk})^2 - (R_{xz}^2 + R_{xk}^2 - \delta_3^2)^2}}, \quad (5)$$

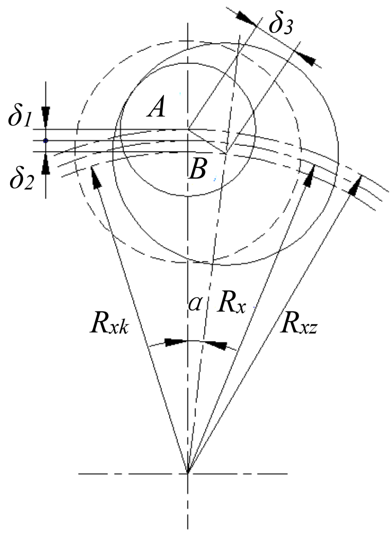
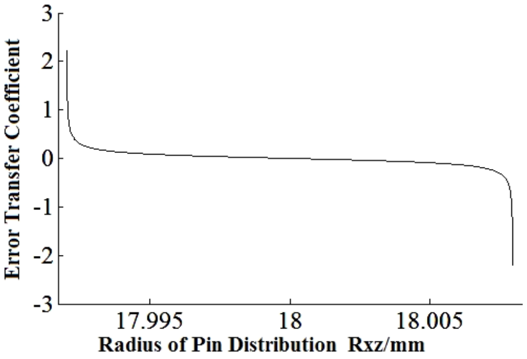
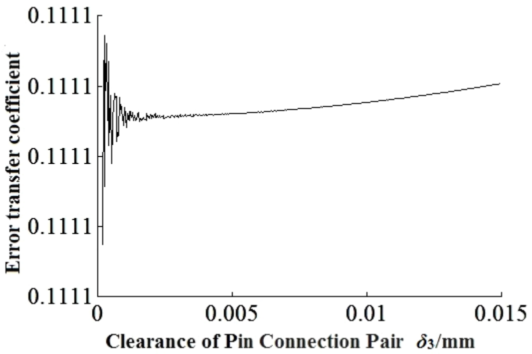


Fig. 3. Backlash analysis model of single pin connection pair.

According to the error independent principle, error transfer coefficient is used to analyze the influence law for the backlash of each parameter. According to Eq.(3), the curve between $\frac{\partial\varphi_1}{\partial R_{xz}}$ and R_{xz} can be obtained and as Fig. 4(a) shows. From which we know if $R_{xz} > R_{xk}$, then $\frac{\partial\varphi_1}{\partial R_{xz}} < 0$, that means if R_{xz} increase the backlash φ_1 will decrease; while if $R_{xz} < R_{xk}$, then $\frac{\partial\varphi_1}{\partial R_{xk}} > 0$, that means if R_{xz} increase φ_1 will increase. As the similar modality of Eq.(4) and Eq.(3), the curve between $\frac{\partial\varphi_1}{\partial R_{xk}}$ and R_{xk} is similar too. From Eq.(5) the curve between $\frac{\partial\varphi_1}{\partial\delta_3}$ and δ_3 can be obtained and as Fig. 4(a) shows. From which we know if $\frac{\partial \varphi_1}{\partial \delta_3}$ is equal to a certain positive value, that means clearance and backlash approximately have linear relationship. The bigger clearance δ_3 is, the more the backlash φ_1 will be.



(a) R_{xk}=18 mm, δ_3=0.008 mm



(b) R_{xz}=R_{xk}=18 mm

Fig. 4. Error transfer coefficient curve of all parameters.

3.3. Backlash of Multiple Pin Connection Pairs

The index error of the structure in machine lead to the distribution error of hole on the two discs in multiple pins connected weak rigidity combination planetary disc. Therefore, clearance distribution error δ4 of pin connection pair should be considered when backlash is analyzed.

The number of pins in weak rigidity combination planetary disc is n, one pin connection pair was chosen as standard marked as pair 0 and its pin marked as *z*0, pin hole marked as *k*0, δ_{40}=0, pin *zi*, pin and hole *ki* have a theoretic angle of 360*i*/*n* with pin *z*0 and pin hole *k*0, the real angle is θ_{z0i} and θ_{k0i}. Let the clearance distribution error of pin connection pair *i* is

$$\delta_{4i} = \theta_{k0i} - \theta_{z0i} \quad (i = 1,2,..., n-1), \qquad (6)$$

Fig. 5 (a) shows the backlash analysis model of weak rigidity combination planetary disc when there are two pins. The pin connection pair 0 is the standard and let pin *z*0 is center. The pin hole center *k*0 and the rotation center of weak rigidity combination planetary disc *O* is collinear (as Fig. 5(a) shows). There is clearance distribution error δ_{41} in the pin connection pair 1. When the planetary disc 3 rotates clockwise, there are two cases.

1) If pin connection pair 0 act only, pin *z*0 and pin hole *k*0 will contact after planetary disc 3 rotates the angle of α_0;

2) If pin connection pair 1 act only, pin *z*1 and pin hole *k*1 will contact after planetary disc 3 rotates the angle of $\alpha_1+\delta_{41}$;

Therefore, when there are two pins, planetary disc 3 rotates the angle of the minimum one in α_0 and $\alpha_1+\delta_{41}$. Similarly available, when planetary disc 3 is counterclockwise rotation the angle needed is $\min(\alpha_0,\alpha_1-\delta_{41})$.

Therefore, when there are two pins, the backlash of weak rigidity combination planetary disc can be calculated as below

$$\varphi_2 = \min(\alpha_0, \alpha_1 + \delta_{41}) + \min(\alpha_0, \alpha_1 - \delta_{41}), \qquad (7)$$

where $\alpha_i = \arccos[\frac{R_{xzi}^2 + R_{xki}^2 - \delta_{3i}^2}{2R_{xzi}R_{xki}}]$, $i = 0,1$;

where α_i is the half of the backlash when pin connection pair *i* act only; R_{xzi} is the radius of real distribution of pin *zi*; R_{xki} is the radius of real distribution of pin hole *ki*; δ_{3i} is the clearance of pin connection pair *i*; min(***X***) is the minimum of ***X***.

When there are three pins, the backlash analysis model of weak rigidity combination planetary disc as Fig. 5(b) shows. Similarly available, the backlash when there are three pins can be calculated as below

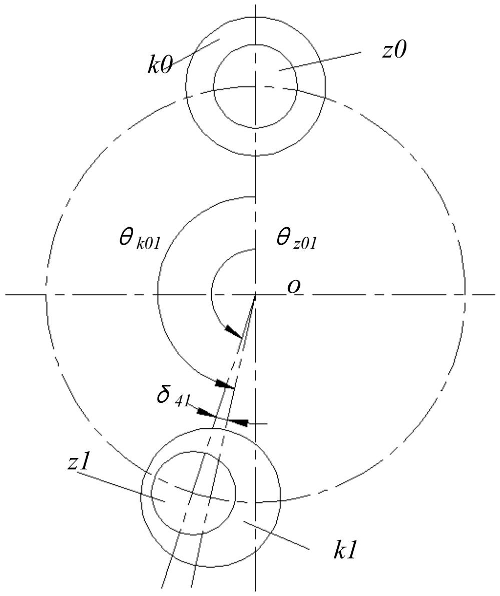
$$\varphi_3 = \min(\alpha_i + \delta_{4i}) + \min(\alpha_i - \delta_{4i}), i = 0,1,2, \qquad (8)$$

Similarly available, when the number of pins is *n*, the backlash can be calculated as below

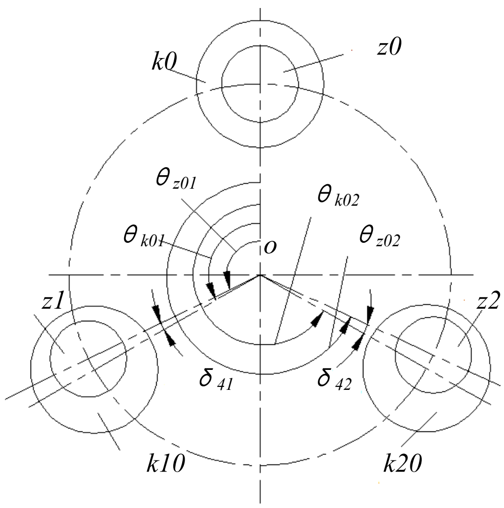
$$\varphi_n = \min(\alpha_i + \delta_{4i}) + \min(\alpha_i - \delta_{4i})\ i = 0,1,..., n-1, \qquad (9)$$

According to Eq.(2), Eq.(7) and Eq.(8), the relationships with different number of pin connection pairs was calculated. Fig. 6 shows the relationships between the theoretical distribution radius of pin and pin hole R_x, the clearance of pin connection pair δ_3, the clearance distribution error δ_4 and backlash of weak rigidity combination planetary disc φ. From which we can know the more pins there were, the smaller backlash is.

Besides, from Fig. 6(a), the larger the theoretical distribution radius R_x is, the smaller backlash is; From Fig. 6(b), the larger the clearance δ_3 is, the larger backlash is; From Fig. 6(c), the larger the error of clearance distribution $|\delta_{4i}|$ is, the smaller backlash is.



(a) two-pin connection pairs



(b) three-pin connection pairs

Fig. 5. Backlash analysis model of multi-pin connection pairs.

3.4. Strength Analysis

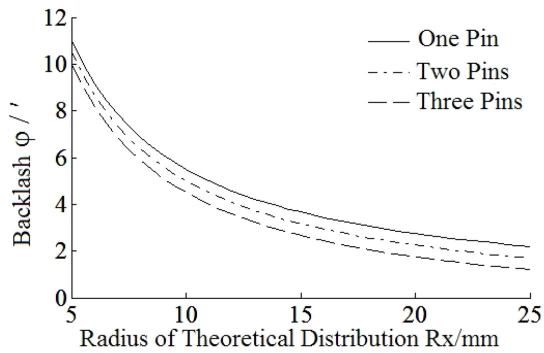
When pin connected weak rigidity combination planetary disc is designed, the first step is to obtain the structure size of weak rigidity combination planetary disc and preliminary obtain the size and number of the pin, then calculate and check the strength. The role of pins is to transfer the torque between the planetary discs and guide the axis move of disc 3 when clearance is adjusted.

Therefore, the main failure modes of pin connection pairs are the crushing or wear of the interface and break of the pin. Therefore, the shear stress and compressive stress of pin connection pair is needed to be calculated. Assume each pin bears averaged force, the check calculation formulas of shear stress and compressive stress for pin connection pair are as follows

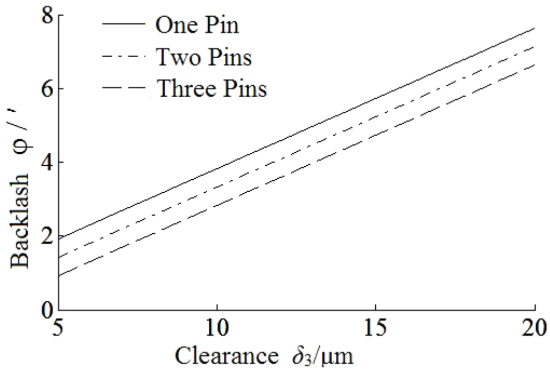
$$\tau = \frac{4T_c}{\pi n R_x d_x^2} \le [\tau], \tag{10}$$

$$\sigma_j = \frac{T_c}{n R_x d_x L} \le [\sigma_j], \tag{11}$$

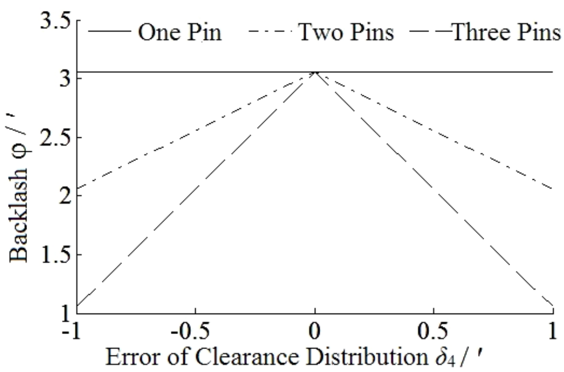
where T_c is the output torque, d_x is the diameter of pins, n is the number of pins, L is the contact width of planetary disc 3 and pins, $[\tau]$ is the allowable shear stress (80 MPa as common), $[\sigma_j]$ is the allowable compressive stress (50 MPa for this structure).



(a) δ_3=0.008 mm, δ_{41}=0.5′, δ_{42}=-0.5′



(b) R_{xz}= R_{xk}=18 mm, δ_{41}=0.5′, δ_{42}=-0.5′



(c) R_{xz}= R_{xk}=18 mm, δ_3=0.008 mm

Fig. 6. Effect law of various parameters on backlash.

4. Example Calculation

Many parameters in mechanical design are not constants but random variables which obey a certain distribution. Therefore, the method of solving the problem usually is numerical simulation. The Monte Carlo is a common way to solve these kinds of engineering problems. Based on the theory of mechanical technology, when a single work piece is produced the dimension errors are even distribution in tolerance zone and normal distribution of batch production. The errors ($\delta_1,\delta_2,\delta_3,\delta_4$) which affect the backlash in pin connected weak rigidity combination planetary disc can be regarded as relative independent normal distribution and different probability distribution in tolerance zone. Based on the 3σ rule, the standard deviation is 1/6 of tolerance zone T [10, 11].

The Monte Carlo simulation and calculation of backlash was carried out through an example programming by the software MATLAB [12]. The concrete steps are as follows

1) According to the structure, dimension and the precision grade of pin holes, the tolerance value δ_i of various errors for the calculation formula of backlash are determined on the basis of relevant standards, and the distributed parameters (μ, σ) for errors are obtained.

2) Determine the sample size N (N should be sufficiently large). Using the random function of rand (1,N), a series of random numbers $\boldsymbol{sjs}_i$ on the closed interval [0,1] as the probability of the errors δ_i are obtained, here $\boldsymbol{sjs}_i$ is a vector of N.

3) According to distributed parameters (μ, σ) for various errors δ_i, using the probability density distribution function of normal distribution icdf('norm',$\boldsymbol{sjs}_i$,μ,σ), the sampling values of errors are obtained.

4) Based on the backlash calculation formula of n pins connected disc, the sampling values (φ_{n1}, φ_{n2},… ,φ_{nN}) of backlashes whose number is N are obtained.

5) Count the sampling values of backlashes whose number is N, and calculate the mean and variance.

The rated power of the motor P_0=0.06 kW, the rated speed n_0=1400r/min, the drive ratio i =7, let the theoretical distribution radius of the center of pin connection pair R_x=18 mm, the diameter of pin d_x=2 mm, Table 1 shows the tolerance and distribution parameters of each error.

Backlashes of 1~3 pins connected weak rigidity combination planetary disc were calculated using MATLAB. Fig. 7 shows the frequency of backlash histogram when the sample size N is 2000. The statistical analysis of simulation result of backlash showed that the mean and variance were 3.0343′ and 1.0721′ when backlash was φ_1 ; and 2.2999′, 0.7010′ when backlash was φ_2 ; and 1.8919′, 0.5848′ when backlash was φ_3 . The calculation results of shear stress and compressive stress for pin connection pair were as follows: when there was one pin τ_1 =45.598 MPa, σ_{j1} =17.906 MPa; when there were two pins τ_2 =22.799 MPa, σ_{j2} =8.953 MPa; when there were three pins τ_3 =15.2 MPa, σ_{j3} =5.969 MPa. All the results are less than the allowable stress and satisfied the strength requirement.

Table 1. Tolerance and distribution parameters of each error.

Errors	Tolerance	Distribution parameters
Error of pin distribution circle δ_1/μm	6	0, 1
Error of pin hole distribution circle δ_2/μm	6	0, 1
Clearance of pin connection pair δ_3/μm	16	8, 2.6667
Error of clearance distribution δ_4/ ′	1	0, 0.1667

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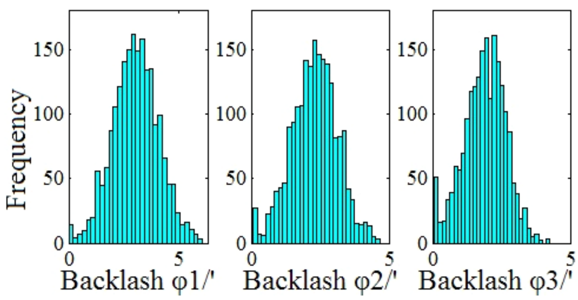

Fig. 7. Backlash simulation of weak rigidity combined planetary disc with 1-3 pin connection.

5. Conclusions

According to the structure and drive principle of the precision ball transmission, an innovative structure of weak rigidity combination planetary disc which realize the real-time non-gap precision transmission through the eliminated in real-time of wearing clearance and thermal expansion interference of ball meshing pairs was proposed. The key pin connection pair of the weak rigidity combination planetary disc was designed. The analysis model that described the backlash was established. The calculation formula of multiple pin connection backlashes was derived. According to analysis, larger radius of pin hole distribution circle and more pins can reduce the backlash of weak rigidity combination planetary disc. The failure analysis and strength calculation of the pin connection pair is useful for the designing of weak rigidity combination planetary disc. Simulation analysis of backlash of weak rigidity combination planetary disc made the distribution of backlash clear and provides measures to reduce backlash, which has great significance for the design and research of the mechanism of real-time non-gap precision ball transmission.

Acknowledgements

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References

[1]. Wang Guobiao, Lai Yinan, Fan Dapeng, et al, Summary of new type precision transmission design and manufacture, *China Mechanical Engineering*, Vol. 21, No. 16, 2010, pp. 1891-1897.

[2]. An Zijun, Qu Zhigang, Wang Guangxin, Research on fatigue strength reliability for the cycloid ball planetary transmission based on fuzzy theory, *China Mechanical Engineering*, Vol. 13, No. 23, 2002, pp. 2010-2012.

[3]. Yang Zuomei, An Zijun, Zhang Peng, Research on undercutting in cycloid ball planetary transmission based on space meshing theory, *Transactions of the Chinese Society for Agricultural Machinery*, Vol. 40, No. 10, 2009, pp. 216-222.

[4]. Wang Guangjian, Jiang Hanjun, Chu Zhigang, Parameters design and simulation of new double crank adjustable backlash precision planetary gears, *Journal of Mechanical Engineering*, Vol. 47, No. 5, 2011, pp. 11-18.

[5]. Chen Bingkui, Mou Huan, Li Chao-yang, et al, New spatial cam sliding tooth planetary gearing, *Journal of Mechanical Engineering*, Vol. 40, No. 1, 2004, pp. 29-32.

[6]. An Zijun, Qu Zhigang, The clearance of engagement pairs and its adjustment mechanism of the cycloidal ball drive, *Journal of Machine Design*, Issue 12, 1996, pp. 33-34, 45.

[7]. Hidetsugu Terada, The development of gearless reducers with rolling balls, *Journal of Mechanical Science and Technology*, Vol. 24, 2010, pp. 189-195.

[8]. Hidetsugu Terada, Kenji Imase, Fundamental analysis of a cycloid ball reducer (5th Report) - Development of a two stage type reduction mechanism, *Japan Society for Precision Engineering*, Vol. 75, No. 12, 2009, pp. 1418-1422.

[9]. Jia Xiaohong, Ji Linhong, Jin Dewen, et al, Modeling of tripod-ball sliding joint with clearance, *Journal of Tsinghua University (Science & Technology)*, Vol. 40, No. 7, 2000, pp. 125-128.

[10]. Wang Taiyong, Xiong Yuedong, Lu Shizhong, et al, Application of Monte Carlo method to dimension and tolerance design, *Transactions of the Chinese Society for Agricultural Machinery*, Vol. 36, No. 5, 2005, pp. 101-104.

[11]. He Xuehong, Zhang Min, Xie Liyang, et al, Accuracy analysis and simulation of 3-TPT parallel machine tool, *China Mechanical Engineering*, Vol. 19, No. 11, 2008, pp. 1336-1339.

[12]. Chen Wenhua, Zhu Haifeng, Fan Xiaoyan, Monte-Carlo simulation analysis of transmission error for gear drive systems, *Chinese Journal of Science Instrument*, Vol. 25, No. 4, 2004, pp. 435-437, 444.