Non-fragile Robust $L_2$–$L_\infty$ Filtering for Uncertain Neutral Fuzzy Stochastic Time-Delay Systems

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Abstract: This paper investigates the problem of non-fragile $L_2$–$L_\infty$ filter design for a kind of Takagi-Sugeno (T-S) fuzzy stochastic system with neutral delay and parameter uncertainties. The addressed problem is the design of a non-fragile fuzzy filter such that the error system is robustly stochastic stable and the filtering error system can tolerate some level of the gain variations in the filter. Based on the Lyapunov-Krasovskii functional we chose, the delay-dependent conditions are obtained. The resulting filters can ensure that the error system is both stochastically stable and the peak value of the estimation error is bounded by a prescribed level for all possible bounded energy disturbances. Finally, an illustrative example is given to substantiate the effectiveness of the proposed method. Copyright © 2014 IFSA Publishing, S. L.

Keywords: Stochastic systems; Non-fragile filter design; T-S fuzzy systems; $L_2$–$L_\infty$ performance; Neutral delay.

1. Instructions

The non-fragile control problem has attracted considerable attention during the past decades. It has been proven that the robust controller could even make the closed-loop unstable without considering the relatively small uncertainties included in the controller [10]. Such controller is called fragile. Therefore, how to design a non-fragile controller which could insensitive to some level of uncertainties became a hot topic in the control theory. A great number of studies have investigated the non-fragile control problem, such as [26, 32, 33] and the references therein. At the same time, as a dual problem of the controller design, the non-fragile filtering approach have been developed to solve the signal and state estimation problem, such as Kalman filtering [28] and $H_\infty$ filtering [14].

In recent years, the neutral delay differential equation has been used to model many physical systems. Such as distributed networks, heat exchanges, microwave oscillators, population ecology [7, 9, 35]. It can be seen in [31] that a partial element equivalent circuit could be represented as a neutral delay differential system. Based on this, studying the control and filtering problem of neutral delay systems are of theoretical and practical importance, and has attracted a rapid growing interest in the past decades [1, 11, 15, 17]. Meanwhile, since the stochastic modeling approach comes to play an important role in many real-world systems, a great number of results have been proposed in literatures. The stability and control problem of the stochastic system have been addressed in [2, 20, 27]. And the robust filtering problem of the stochastic system has been investigated in [12]. The $H_\infty$ and $L_2$–$L_\infty$ filtering results for stochastic time-delay systems have been...
proposed in [4] and [6, 23], respectively. In particular, when the neutral delay appears in the stochastic system, the $H_\infty$ filtering problem has been studied in [5].

At the same time, the T-S fuzzy model [22] has been firstly discussed in the truck trailer system in [21], and its applications to linearize the nonlinear systems have been studied over the past decades. As an efficient approach to describe a large class of nonlinear systems, the T-S fuzzy model has attracted much attention. For example, the stability and control problem have been studied in [25, 29, 34] and the reference therein. Notice that the neutral delay system could also be represented in fuzzy models, [30] designs the guaranteed cost controller for the fuzzy neutral delay system. Different kinds of non-fragile controllers are proposed in [13] and [18] for fuzzy neutral delay system. Moreover, the non-fragile $H_\infty$ filtering problem has been investigated in [3] for the discrete-time fuzzy system. And the filter for fuzzy stochastic systems is design in [24]. In particular, when the neutral delay appears in fuzzy stochastic system, the stability criterion has been addressed in [19]. Despite these efforts, there is room for further improvement. Yet, how to design a non-fragile filter to further correspond with the practice of the fuzzy stochastic system remains an important and challenging problem.

As the discussion above, the T-S fuzzy model could deal with complex nonlinear systems effectively. Motivated by this fact, the T-S fuzzy model will be employed to deal with the filtering problem of neutral delay stochastic system. First, a neutral delay stochastic system is proposed in fuzzy models in this work. Then, using the Lyapunov functional method, the delay-dependent conditions for the existence of non-fragile energy-to-peak filtering design is fully investigated. The resulting filters can ensure that the error systems is stochastic mean-square stable with a prescribed $L_2$-$L_\infty$ attenuation level. Finally, an example is given to show the effectiveness of the proposed method.

2. Problem Formulation and Preliminaries

Consider a class of neutral delay stochastic system with time-varying parameter uncertainties described by T-S fuzzy models:

\begin{equation}
\begin{aligned}
   &dx(t) + Dx(t-\tau(t)) = \left[ \Xi \Delta \tilde{X}(t) x(t) + (\Xi \Delta \tilde{X}(t)) (x(t-\tau(t)) + b(t) + \Delta \tilde{b}(t) v(t)) \right]dt \\
   &d\gamma(t) = \left[ \Gamma \xi(t) + \Gamma\xi(x(t-\tau(t)) + E v(t)) \right]dt \\
   &z(t) = L x(t) 
\end{aligned}
\end{equation}

where $x(t) \in \mathbb{R}^n$ is the state; $\varphi(t) \in \mathbb{R}^n$ is a given real-value initial function on $[-h,0]$; $\omega(t) \in \mathbb{R}^n$ is a scalar zero mean Gaussian white noise process with unit covariance; $\gamma(t) \in \mathbb{R}^m$ is the measured output; $z(t) \in \mathbb{R}^p$ is a signal to be estimated; $v(t) \in \mathbb{R}^q$ is the input noise signal which belongs to $L_2(0,\infty)$; $\tau(t)$ is a continuous differentiable function representing the time-varying delay in both $x(t)$ and $d\gamma(t)$, which is assumed to satisfy $0 \leq \tau(t) < h$ for all $t \geq 0$. And all $\Xi$ expressions in (1) have the same style as in $\tilde{X} = \sum_{i=1}^r h_i(s(t)) A_i$, where

\begin{equation}
\sum_{i=1}^r h_i(s(t)) = 1, \quad h_i(s(t)) \geq 0, \quad i = 1, 2, \ldots, r.
\end{equation}

In the above neutral delay T-S fuzzy stochastic system, $A_1, A_2, B_1, H_1, H_2, C_1, C_2, E$ and $L_i$ are known constant matrices with appropriate dimensions. $\Delta A_i, \Delta B_i, \Delta H_i$ and $\Delta H_0$ represent the unknown time-varying parameter uncertainties and are assumed to satisfy the following conditions:

\begin{equation}
\begin{aligned}
   &\begin{bmatrix} \Delta A_i(t) \Delta B_i(t) \end{bmatrix} = M_i F_i(t) [N_1 \quad N_2 \quad N_3] \\
   &\begin{bmatrix} \Delta H_i(t) \Delta H_0 \end{bmatrix} = M_i F_i(t) [N_1 \quad N_2]
\end{aligned}
\end{equation}

where $M_i, M_i, N_1, N_2$ and $N_3$ are known real constant matrices and the unknown time-varying matrix function satisfying $F_i(t)^T F_i(t) \leq I, \forall t$.

Suppose the full order non-fragile filters are proposed to estimate the signal $z(t)$:

\begin{equation}
\begin{aligned}
   \dot{x}(t) &= (\tilde{X} + \Delta \tilde{X}(t)) x(t) dt + (\tilde{B} + \Delta \tilde{b}(t)) v(t) dt \\
   \tilde{z}(t) &= L \tilde{x}(t)
\end{aligned}
\end{equation}

in which, the fuzzy rules and the matrices $\Xi$ have the same representations as in (1)-(4). $\Delta A_i(t)$ and $\Delta B_i(t)$ represent the unknown time-varying parameter uncertainties in the filter and are assumed to satisfy

\begin{equation}
\begin{aligned}
   &\begin{bmatrix} \Delta A_i(t) \Delta B_i(t) \end{bmatrix} = M_i F_i(t) [N_1 \quad N_2 \quad N_3] \\
   &\begin{bmatrix} \Delta H_i(t) \Delta H_0 \end{bmatrix} = M_i F_i(t) [N_1 \quad N_2]
\end{aligned}
\end{equation}

where $M_i, N_i$ and $N_3$ are known real constant matrices and the unknown time-varying matrix function satisfying $F_i(t)^T F_i(t) \leq I, \forall t$.

Let $\xi(t) = [x(t)^T \tilde{x}(t)^T]^T$, $\tilde{z}(t) = z(t) - \tilde{z}(t)$. The following state-space equation for the estimation error is obtained:

\begin{equation}
\begin{aligned}
   d[\xi(t)] &= -D \xi(t) + \Delta G \xi(t-\tau(t)) + G(t) \omega(t) dt \\
   \tilde{z}(t) &= \tilde{L} \tilde{x}(t)
\end{aligned}
\end{equation}
where

\[
\Phi(t) = (\tilde{A} + \Delta \tilde{A}(t))\xi(t) + (\tilde{A}_x + \Delta \tilde{A}_x(t))G\xi(t-\tau(t)) + \\
+ (\tilde{B} + \Delta \tilde{B}(t))v(t),
\]

\[
g(t) = (\tilde{H} + \Delta \tilde{H}(t))\xi(t) + (\tilde{H}_x + \Delta \tilde{H}_x(t))G\xi(t-\tau(t)),
\]

\[
\begin{bmatrix}
\tilde{A} & 0 \\
R & \tilde{C}
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_x & 0 \\
\tilde{R}_x & \tilde{C}_x
\end{bmatrix}
\begin{bmatrix}
\Delta \tilde{A}(t) \\
\Delta \tilde{A}_x(t)
\end{bmatrix}
+ \begin{bmatrix}
\tilde{B} & 0 \\
B & \tilde{F}
\end{bmatrix}
\begin{bmatrix}
\Delta \tilde{B}(t) \\
\Delta \tilde{B}_x(t)
\end{bmatrix}.
\]

The problem under consideration is to design a dynamical non-fragile fuzzy filter in the form of (7) and (8), such that for any scalar \(0 \leq h\) and a prescribed level of noise attenuation \(\gamma > 0\), the filtering error system \((\xi)\) could be stochastic mean square stable and the error system \((\xi)\) satisfies

\[
L_2 - L_\infty\text{ performance.}
\]

**Definition 2.1.** [27] The system \((\xi)\) is said to be robust stochastic mean-square stable if there exists \(\delta(\varepsilon) > 0\) for all \(\varepsilon > 0\) such that \(E(||x(t)||^2) < \varepsilon, t > 0\), whenever \(E(||\varphi(s)||^2) < \delta(\varepsilon)\), for any uncertain variables. And in addition, \(\lim_{t \to \infty}E(||x(t)||^2) = 0\) for any initial conditions.

**Definition 2.2.** [14] The robust stochastic mean square stable system \((\xi)\) is said to satisfy the \(L_2 - L_\infty\) performance, for the given scalar \(\gamma > 0\) and any nonzero \(v(t) \in L_2[0, \infty)\), the system \((\xi)\) satisfies

\[
\|E(t)\|_2 < \gamma\|v(t)\|_2.
\]

3. Non-fragile \(L_2 - L_\infty\) Filter Design

In this section, we begin with the robust stochastic mean square stable analysis for the error system \((\xi)\). Then, a non-fragile \(L_2 - L_\infty\) fuzzy filter is addressed for the system \((\xi)\).

**Theorem 3.1.** The filtering error system \((\xi)\) is robust stochastic mean square stable with an \(L_2 - L_\infty\) attenuation level \(\gamma > 0\), if there exist matrices \(P = P^T > 0, R = R^T > 0, T, T_2, Q = Q^T > 0, i = 1, 2, 3, 4\), such that the following matrix inequalities hold:

\[
\begin{bmatrix}
\Omega & \Psi_{12} & \Psi_{12} \\
* & \Psi_{12} & 0 \\
* & * & \Psi_{12}
\end{bmatrix} < 0,
\]

\[
\Psi_{12} = \begin{bmatrix}
P & L \\
* & \gamma^2I
\end{bmatrix} > 0,
\]

where

\[
\begin{align*}
\Omega_1 & = \Omega_2 = 0 \\
\Omega_{22} & = \Omega_3 = \Omega_{23} = 0 \\
\Omega_{44} & = 0 \\
\Omega_{45} & = 0 \\
\Omega_{46} & = -I
\end{align*}
\]

\[
\Psi_{12} = [h_1G^TR, H^TP], \Psi_{25} = diag\{-h_R, -P\},
\]

\[
\begin{align*}
\Omega_1 & = R(\tilde{A} + \Delta \tilde{A}(t)) + (\tilde{A} + \Delta \tilde{A}(t))^TP + G^TQ + Q + G + S \\
\Omega_2 & = R(\tilde{A}_x + \Delta \tilde{A}_x(t)) + G^T(t) - (\tilde{A} + \Delta \tilde{A}(t))^TPD, \\
\Omega_3 & = R(Q - T_r - T_r^T - \tilde{D}^TG^T(t) - T_rG, \\
\Omega_4 & = (\tilde{A} + \Delta \tilde{A}(t))^TPD - \tilde{D}^TP(\tilde{A} + \Delta \tilde{A}(t)), \\
\Omega_{13} & = Q - T_r - T_r^T, \\
\Omega_{14} & = -T_rG\Omega_{33}, \Omega_{33} = -Q - T_r - T_r^T, \\
\Omega_{25} & = T_rG\Omega_{33},
\end{align*}
\]

**Proof.** First we choose a Lyapunov-Krasovskii candidate for system \((\xi)\) as follows:

\[
\begin{align*}
V(\xi(t)) & = (T_r^T + DG^T(t) - DG^T(t-\tau(t)))^TP(T_r^T + DG^T(t) - DG^T(t-\tau(t))) \\
& + \int_0^\tau \int_{t-h_r}^t \Phi^T(s)G^T(s)P(t-h_r)G(s)\Phi(s)ds + \\
& + \int_{t-\tau(h_r)}^{t-h_r} \xi^T(s)G^T(s)QG\xi(s)ds + \\
& + \int_{t-\tau(h_r)}^t \xi^T(s)G^T(s)QG\xi(s)ds + \\
& + \int_{t-\tau(h_r)}^t \xi^T(s)G^T(s)QG\xi(s)ds
\end{align*}
\]

When \(v(t) = 0\), from the Itô formula, the stochastic differential equation can be computed as follows:

\[
dV(\xi(t)) = (T_r^T + DG^T(t) - DG^T(t-\tau(t)))^TPv(t)ds + \\
+ \int_0^\tau (T_r^T + DG^T(t) - DG^T(t-\tau(t)))Pv(t)\sigma d\beta(t),
\]

where
\[ L^V(\xi(t) - DG \xi(t - \tau(t)), t) \]
\[ \leq 2\xi(t) - DG \xi(t - \tau(t))^T \Phi(t) + g(t)P_g(t) \]
\[ + \xi(t) GQ(t) + Q(t)G(t) \xi(t) - \xi(t - \tau(t)) GQ(t) \xi(t - \tau(t)) \]
\[ - \xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) - R_{FG}^{\tau(t)}(t) \xi(t - \tau(t)) \xi(t - \tau(t)) \]
\[ - R_{FG}^{\tau(t)}(t) GQ(t - \tau(t)) \xi(t - \tau(t)) + h(t) \Phi(t) G^T \Phi(t) \]
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds \]
\[ + \xi(t - \tau(t)) GQ(t - \tau(t)) - \xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \]
\[ + DG \xi(t - 2\tau(t)) - \xi(t - \tau(t)) \]
\[ + 2\xi(t - \tau(t)) G^T \Phi(t) G \xi(t) - \xi(t) GQ(t - \tau(t)) \xi(t - \tau(t)) \]
\[ - \xi(t - \tau(t)) GQ(t - \tau(t)) \]
\[ + \xi(t) GQ(t) G(t) \xi(t) - \xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \]
\[ + \xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \]
\[ - 2\xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \xi(t - \tau(t)) \]
\[ - \xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \xi(t - \tau(t)) \]
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds \]
\[ (13) \]

In addition, the final eight lines of (13) are equal to 0 from the Newton-Leibniz formula, where
\[ \xi(t) = \left[ \begin{array}{c} \xi^t(t) \\ \xi^t(t - \tau(t)) G^T \xi(t - \tau(t)) \end{array} \right] \]

We can obtain
\[ L^V(\xi(t) - DG \xi(t - \tau(t)), t) \]
\[ \leq \xi(t) \Omega(t) + \bar{\Psi}_{12} \psi_{22}^{-1} \bar{\Psi}_{12}^{T} + \bar{\Psi}_{13} \psi_{33}^{-1} \bar{\Psi}_{13}^{T} \]
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds \]
\[ + \xi(t - \tau(t)) G^T \Phi(t) G \xi(t) \]
\[ - 2\xi(t - \tau(t)) GQ(t - \tau(t)) \xi(t - \tau(t)) \xi(t - \tau(t)) \]
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds \]
\[ \Omega(t) = \left[ \begin{array}{cccc} \Omega_{11} & \Omega_{12} & 0 & 0 \\ \Omega_{12} & \Omega_{22} & \Omega_{23} & 0 \\ 0 & \Omega_{33} & \Omega_{34} & \Omega_{35} \\ 0 & 0 & \Omega_{44} & \Omega_{45} \\ 0 & 0 & 0 & \Omega_{55} \end{array} \right] \]
\[ \bar{\Psi}_{12} = \left[ \begin{array}{c} \bar{A} G^T \Phi \bar{B} \end{array} \right], \psi_{22}^{-1} = \left[ \begin{array}{c} \hat{T}_1 \end{array} \right] \]
\[ \bar{A} = \left[ \begin{array}{c} \bar{A}+\Delta \bar{A} \end{array} \right], \bar{B} = \Delta \bar{B} \]
\[ \hat{H} = \bar{H} + \Delta \bar{H} \]
\[ \hat{T}_1 = \left[ \begin{array}{c} 0 \ 0 \ T_1^* \ 0 \ 0 \end{array} \right], \]
\[ \hat{T}_2 = \left[ \begin{array}{c} 0 \ 0 \ T_2^* \ 0 \ 0 \end{array} \right] \]

Notice that
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds \leq 0, \]
\[ \int_{t-\tau(t)}^{t} (sG^T \xi(s)G \xi(s)) ds < 0, \]

Moreover, it has been proven that \( \xi(t) \) is measurable with \( F_t \) in system (\( \xi \)) in [16]. Hence we can derive that \( \xi(t - \tau(t)) \) is measurable with \( F_{t-h} \) when \( t > \tau(t) \), and \( \xi(t - h) \) is measurable with \( F_{t-h} \) when \( t > h \). Considering the knowledge of stochastic analysis, and taking the property of \( ho \) integral into account, we can deduce that:
\[ E[\xi(t)] = 0, \]
\[ E[\xi(t - \tau(t))] = 0, \]
\[ E[\xi(t - h)] = 0, \]

Then, from (11) and (14)-(18), it is easy to prove that there always exist constants \( c > 0 \) and \( \nu > 0 \), such that
\[ E[\xi(t)] = 0, \]
\[ E[\xi(t - \tau(t))] = 0, \]
\[ E[\xi(t - h)] = 0, \]

which means
\[ \xi(t) = 0, \]
\[ \xi(t - \tau(t)) = 0, \]
\[ \xi(t - h) = 0, \]

Diving constant \( c \) on both side of last inequality, yields
\[ E[\xi(t)] < -\frac{1}{c} E[\xi(t)] \]
\[ \xi(t) < -\frac{1}{c} \xi(t), \]
\[ \xi(t - \tau(t)) < -\frac{1}{c} \xi(t - \tau(t)), \]
\[ \xi(t - h) < -\frac{1}{c} \xi(t - h), \]

(19)
for an exist $\varepsilon > 0$. And it ensures that system $\Xi$ with $v(t) = 0$ is robustly stochastically stable according to Definition 2.1 and [16]. By considering $h_0$'s formula, there is

$$ E(\mathbb{V}(\xi(t),t)) = E\left(\int_{0}^{t} \mathbb{L}^r(\xi(s),s)ds\right) $$

Next, we establish the $L_{-\infty}$ performance of the filtering error system $\tilde{\xi}(t)$. Since $a(t)$ is a scalar zero mean Gaussian white noise process, we have $E[a(t)] = 0$, it is immediate that

$$ E\left[\mathbb{L}v(\tilde{\xi}(t) - DG \tilde{\xi}(t) + \tilde{\xi}(t-h)G^T \xi(t-h))(t),0)\right] \leq E\left[\int_{0}^{t} \mathbb{L}^r(\Omega + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T)\mathbb{G}(s)ds\right], $$

where

$$ \eta(t) = \left[\begin{array}{c} \xi(t) \\ \xi(t-(t-h)) G^T \\ \xi(t-(t-2(t-h))) G^T \\ \xi(t-(t-2(t-h)) G^T \right]. $$

Applying the Schur complement formula to (11), we can get

$$ \Omega + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T < 0 $$

for all $t > 0$. Therefore, for all $\eta(t) \neq 0$,

$$ E\left[\mathbb{L}v(\tilde{\xi}(t) - DG \tilde{\xi}(t) + \tilde{\xi}(t-h)G^T \xi(t-h))(t),0)\right] \leq E\left[\int_{0}^{t} \mathbb{L}^r(\Omega + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T)\mathbb{G}(s)ds\right], $$

and thus

$$ E\left[\mathbb{L}v(\tilde{\xi}(t) - DG \tilde{\xi}(t) + \tilde{\xi}(t-h)G^T \xi(t-h))(t),0)\right] \leq \int_{0}^{t} \mathbb{L}^r(\Omega + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T)\mathbb{G}(s)ds. $$

Then using the Schur complement to (12), we have $\tilde{L} = \mathbb{L}^T < \gamma^r P$, which guarantees

$$ E\left[\mathbb{L}v(\tilde{\xi}(t) - DG \tilde{\xi}(t) + \tilde{\xi}(t-h)G^T \xi(t-h))(t),0)\right] \leq \gamma^r \int_{0}^{t} \mathbb{L}^r(\Omega + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T + \mathbb{W}^T \mathbb{W}^{-1} \mathbb{W}^T)\mathbb{G}(s)ds. $$

Therefore, $\parallel \tilde{\xi}(t) \parallel < \gamma \parallel v(t) \parallel$, for any zero mean Gaussian white noise process $v(t)$ with unit covariance. This completes the proof.

Based on Theorem 3.1, a sufficient condition for the solvability of non-fragile $L_{-\infty}$ filtering problem for system $\Xi$ can be developed in the next theorem.

**Theorem 3.2.** Consider the uncertain neutral delay fuzzy stochastic system $\Xi$ and a constant scalar $\gamma > 0$. The robust non-fragile $L_{-\infty}$ filtering problem is solvable if there exist scalars $\varepsilon_i > 0$, $\varepsilon_j > 0$, and matrices $P_i > 0$, $P_j > 0$, $Q > 0$, $i = 1, 2, 3, 4$, $T_i, T_2, K_{ij}, K_{j2}, K_{j1}, 1 \leq j \leq r$, $\{\varepsilon_i, \varepsilon_j, i = 1, 2, 3, 4, \}$ such that the following LMIs hold:

$$ \begin{bmatrix} \varepsilon_{i1} & \varepsilon_{i2} & \cdots & \varepsilon_{i4} \\ \varepsilon_{j1} & \varepsilon_{j2} & \cdots & \varepsilon_{j4} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{r1} & \varepsilon_{r2} & \cdots & \varepsilon_{rr} \end{bmatrix} < 0, $$

$$ \begin{bmatrix} P_1 & 0 & L_T^r \\ \gamma^T & * & * \\ * & 0 & 0 \end{bmatrix} > 0, $$

$$ \begin{bmatrix} \varepsilon_i & \varepsilon_j & \varepsilon_k \\ \varepsilon_i & \varepsilon_j & \varepsilon_k \\ \varepsilon_i & \varepsilon_j & \varepsilon_k \end{bmatrix} < 0, (1 \leq i \leq r), $$

$$ \begin{bmatrix} \varepsilon_{i1} & \varepsilon_{i2} & \varepsilon_{i3} & \varepsilon_{i4} \\ \varepsilon_{j1} & \varepsilon_{j2} & \varepsilon_{j3} & \varepsilon_{j4} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{r1} & \varepsilon_{r2} & \varepsilon_{r3} & \varepsilon_{rr} \end{bmatrix} < 0, $$

$$ \begin{bmatrix} P_1 & 0 & L_T^r \\ \gamma^T & * & * \\ * & 0 & 0 \end{bmatrix} > 0, $$

$$ \begin{bmatrix} \varepsilon_i & \varepsilon_j & \varepsilon_k \\ \varepsilon_i & \varepsilon_j & \varepsilon_k \\ \varepsilon_i & \varepsilon_j & \varepsilon_k \end{bmatrix} < 0, (1 \leq i \leq r), $$

$$ \begin{bmatrix} \varepsilon_{i1} & \varepsilon_{i2} & \varepsilon_{i3} & \varepsilon_{i4} \\ \varepsilon_{j1} & \varepsilon_{j2} & \varepsilon_{j3} & \varepsilon_{j4} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{r1} & \varepsilon_{r2} & \varepsilon_{r3} & \varepsilon_{rr} \end{bmatrix} < 0, $$
\[ \begin{bmatrix} hA_A^T R & H_A^T P & 0 \\ 0 & 0 & 0 \\ hA_B^T R & H_B^T P & 0 \\ 0 & 0 & 0 \\ hB_B^T R & 0 & 0 \end{bmatrix} \]
\[ \Gamma_{12} = \begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \end{bmatrix}, \Gamma_{13} = \begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \end{bmatrix}. \]

When the LMIs (23)-(26) are feasible, the time-dependent filter desired can be chosen as
\begin{align*}
A_f &= P_2^{-1}K_{u2}, B_f = P_2^{-1}K_{x2}, L_{fi} = 1, \cdots, r.
\end{align*}

**Proof.** Define
\begin{align*}
P &= \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}.
\end{align*}

By using the same method in [14], it is easy to prove that the condition in Theorem 3.1 and the LMIs in (23)-(26) are equivalent. Thus, we can conclude that the filtering error system (\( \Sigma \)) is stochastically stable with \( L_2-L_m \) performance level \( \gamma \). In addition, the filter matrices \( A_f, B_f \) and \( L_f \) can be constructed from (27).

**Remark 1.** The desired \( L_2-L_m \) filters can be constructed by solving the LMIs in (23)-(26), which can be implemented by using standard numerical algorithms, and no tuning of parameters will be involved.

### 4. Numerical Simulation

In this section, a numerical example is given to illustrate the effectiveness and benefits of the results obtained in the previous section.

**Example:** Consider the neutral delay T-S fuzzy stochastic system (\( \Sigma \)) with model parameters given as follows:
\begin{align*}
A &= \begin{bmatrix} -2.3 & 1 \\ -1.7 & -9.2 \end{bmatrix}, A_i = \begin{bmatrix} -1.1 & -0.4 \\ -0.9 & 0.1 \end{bmatrix}, A_f = \begin{bmatrix} -2.1 & 0.6 \\ -1.7 & -1.4 \end{bmatrix}, \\
H &= \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.2 \end{bmatrix}, H_i = \begin{bmatrix} -0.01 & 0.01 \\ 0.01 & -0.05 \end{bmatrix}, H_f = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} -0.1 & -0.4 \end{bmatrix}, C_{i1} = \begin{bmatrix} -0.2 & -0.1 \end{bmatrix}, E_i = 0.4, E_f = -0.3, \\
C_2 &= \begin{bmatrix} 0.2 & -0.6 \end{bmatrix}, C_{i2} = \begin{bmatrix} -0.4 & -0.1 \end{bmatrix}, H_{i2} = \begin{bmatrix} -1.15 & 0 \end{bmatrix}, \\
L_i &= \begin{bmatrix} 1 & -0.6 \end{bmatrix}, L_f = \begin{bmatrix} -0.3 & 0.2 \end{bmatrix}, D_i = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \\
B_i &= \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix}, B_f = \begin{bmatrix} -0.2 \\ -0.5 \end{bmatrix}, A_{i2} = \begin{bmatrix} -0.18 & 0 \\ -0.22 & -0.24 \end{bmatrix}.
\end{align*}

And the parameter uncertainties are shown as:
\begin{align*}
M_i &= \begin{bmatrix} 0.1 & 0.2 \\ -0.5 & 0.1 \end{bmatrix}, M_f = \begin{bmatrix} 0.8 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}, \\
N_i &= \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, N_f = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, M_{i2} = \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & 0.5 \end{bmatrix}, \\
N_{i2} &= \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, N_{f2} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0 \end{bmatrix}, \\
M_{22} &= \begin{bmatrix} 0.1 & -0.6 \\ 0.1 & 0 \end{bmatrix}, N_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}, N_{2f} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}.
\end{align*}

The membership functions are \( h_1(x_i(t)) = \sin^2(t) \) and \( h_2(x_i(t)) = \cos^2(t) \). By using the Matlab LMI Control Toolbox, the robust non-fragile \( L_2-L_m \) filtering problem is solvable to Theorem 3.2. It can be seen that for any \( 0 < h(t) < 6 \) and the nonlinear function \( g(x(t)) = \sin(x_2(t)) \), the filtering problem can be solved with the \( L_2-L_m \) performance level \( \gamma = 0.521 \). And the fuzzy filter can be constructed as in the form of (7) and (8) with:
\begin{align*}
A_f &= \begin{bmatrix} -9.7278 & 10.3600 \\ 12.5529 & -27.7597 \end{bmatrix}, A_{i2} = \begin{bmatrix} -8.6756 & 1.0176 \\ 2.7049 & -2.0053 \end{bmatrix}, \\
B_f &= \begin{bmatrix} -1.2301 \\ 0.2148 \end{bmatrix}, B_{i2} = \begin{bmatrix} -0.4397 \\ -1.4191 \end{bmatrix}, \\
L_f &= \begin{bmatrix} -0.2341 \\ 0.0656 \end{bmatrix}, L_{i2} = \begin{bmatrix} -0.2341 \\ -0.0656 \end{bmatrix}.
\end{align*}

The simulation results of the state responses in system (\( \Sigma \)) and the filter are given in Fig. 1, where the initial condition is \( x_i(t) = [0.35, 0.3]^T \), \( \hat{x}_i(t) = [0.1, 0.1]^T \). At the same time, Fig. 2 shows the simulation results of the signal \( \hat{z}(t) \).

![Fig. 1. State responses of \( x(t) \) and \( \hat{x}(t) \).](image-url)
filter ensuring a prescribed $L_\infty$ has been developed to design the non-fragile fuzzy support.

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5. Conclusions

The problem of non-fragile robust $L_\infty-L_\infty$ filtering for neutral delay fuzzy stochastic has been

References


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