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Multi-Model Adaptive Fuzzy Controller for a CSTR Process

[*] Shubham Gogoria, Tanvir Parhar, Jaganatha Pandian B.
Electronics and Instrumentation, VIT University, 632014, India
[*] Tel.: +91-8681910995
[*] E-mail: gogoriyashubham@yahoo.in

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Abstract: Continuous Stirred Tank Reactors are intensively used to control exothermic reactions in chemical industries. It is a very complex multi-variable system with non-linear characteristics. This paper deals with linearization of the mathematical model of a CSTR Process. Multi model adaptive fuzzy controller has been designed to control the reactor concentration and temperature of CSTR process. This method combines the output of multiple Fuzzy controllers, which are operated at various operating points. The proposed solution is a straightforward implementation of Fuzzy controller with gain scheduler to control the linearly inseparable parameters of a highly non-linear process. *Copyright © 2015 IFSA Publishing, S. L.*

Keywords: CSTR, Linearization, State-space, Fuzzy, Weight distributor, Adaptive.

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1. Introduction

CSTR processes have been rigorously used in chemical industries for a long time. Proportional-Integral-Derivative (PID) controllers have been used in process control most extensively. But these main-stream algorithms are incapable of controlling complex non-linear complex systems with accuracy. Earlier control systems were constricted by lack of computational power. But, with the increase of computing technology, it has become feasible to implement computationally expensive algorithms, like Fuzzy logic controllers. Attempts have been made to implement a PID controller over CSTR process, using cascaded control algorithms [1-3] Multi-loop PID controllers and Neuro-PID controllers have also been devised for an optimal concentration and temperature control [4-6]. But problems with most of the approaches is one of the two process variables (either process temperature or concentration) have been taken as a constant. Where in practical conditions, it is not the case. Both the variables depend on each other, through a differential relation [7]. Hence, this paper focuses on controlling both the output variables, using a novel Multi-model Fuzzy controller approach. The conventional mathematical model of the CSTR process is used [8]. The non-linear equations are linearized at various operating points and cast into state space models [9-10] Using the state-space model at different operational points, transfer function matrices have been formed [11]. A Fuzzy controller maps inputs to outputs, using a set of rules that might be linear or non-linear relation. The transfer function matrix is used to tune Fuzzy Logic controller parameters. A weighing algorithm has been designed, to select the appropriate controller and state-space model, corresponding to the input signal. Since the Fuzzy controller has an oscillatory manipulative variable, so an integrator has been implemented to eliminate fluctuations.

2. System Modelling

The modelling of the CSTR process has been done, keeping the following assumptions in mind:

1. Volume of the reactor remains constant, during the process.
2. The chemicals are mixed perfectly inside the vessel.
3. The reaction follows the following dynamics
$A \longrightarrow B$ + Heat (Q).

2.1. Mass Balance Equation

In as CSTR process, two state variables are controlled, namely reactor temperature (T) and reactor concentration (C). The following differential equations symbolize the process in time domain.

$$\frac{dC_A(t)}{dt} = \frac{F(t)}{V}\left(C_{af}(t) - C_a(t)\right) - K_0 C_a(t)\exp\left(\frac{-E}{RT(t)}\right) \tag{1}$$

and

$$\frac{dT(t)}{dt} = \frac{F(t)}{V}\left(T_f(t) - T(t)\right) - \frac{(-\Delta H)K_0C_a(t)}{\rho C_p}\exp\left(-\frac{E}{RT(t)}\right) + \frac{F_c(t)\rho_c C_{pc}}{\rho C_p V}(1 - \exp\left(\frac{-hA}{F_c\rho_c C_{pc}}\right)) *(T_c(t) - T(t)) \tag{2}$$

where:
F = Feed flow rate;
V = Volume of reactor;
C_{af} = Feed concentration;
C_a = Reactor concentration;
K_0 = Reaction rate constant;
E = Activation energy;
R = Ideal gas constant;
T = Reactor temperature in K;
H = Heat of reaction;
hA = Heat transfer coefficient;
T_f = Feed temperature;
T_c = Coolant temperature;
ρ; ρ_c = Liquid densities;
C_p; C_{pc} = Specific heats.

State input variables are given by

$$x(t) = [C_A : T] \tag{3}$$

$$u(t) = [F : F_C] \tag{4}$$

2.2. Linearization

The nonlinear equations are formed into state space variables as follows:

$$\dot{x} = Au + Bu \tag{5}$$
$$y = Cu \tag{6}$$

Linearizing (1) and (2), transfer function matrices are obtained for the CSTR model. The idea behind computing the matrices is to develop a state space model for the system.

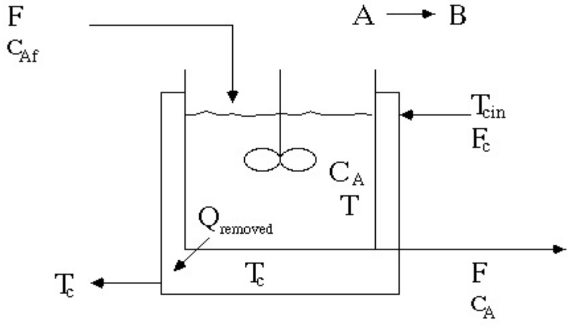

Fig. 1. The Continuously Stirred Tank Reactor.

Where matrix A and B are Jacobian matrices of state and input variables respectively and C is output matrix.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$A_{11} = \frac{F}{V}K_p \tag{7}$$

$$A_{12} = C_a K_p{}' \tag{8}$$

$$A_{21} = \frac{\Delta H K_p}{\rho C_p} \tag{9}$$

$$A_{22} = -\frac{Q}{V} + \frac{-\Delta H C_a K_p{}'}{\rho C_p} + \frac{\rho C C_{pc} F_c}{\rho C_p V} + \frac{\rho C_{pc} F_c}{C_p V} * \exp(\frac{-hA}{F_c C_p \rho}) \tag{10}$$

here

$$K_p = K_0 \exp\left(\frac{-E}{RT}\right) \tag{11}$$

The Jacobean matrix B is given by

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix},$$

where

$$B_{11} = \frac{C_{af} - C_a}{V}, \tag{12}$$

$$B_{21} = 0, \tag{13}$$

$$B_{21} = \frac{T_f - T}{V}, \tag{14}$$

$$B_{22} = \frac{-hA}{F_c \rho_c C_{pc}} [F_c \left(- \exp\left(\frac{-hA}{F_c \rho_c C_{pc}}\right) \left(\frac{hA}{F_c^2 \rho_c C_{pc}}\right)\right) - \exp\left(\frac{-hA}{F_c \rho_c C_{pc}}\right)] \quad (15)$$

The output matrix C is given by

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Table 1 shows steady state parameters for the process.

Table 1. Steady State Operating Data.

S. No.	Parameters	Values
1	C_a	0.0882 mol/l
2	T	441.2 K
3	F_c	100 l/min
4	F	100 l/min
5	C_{af}	1mol/l
6	T_f	350 K
7	T_c	350 K
8	V	100 l
9	hA	7×10^5 cal/min K
10	E/R	10000 K
11	ΔH	2×10^5 cal/min K
12	ρ_c, ρ	1000 g/l
13	C_p, C_{pc}	1cal/(gK)
14	K_0	$72\times10^{10}\ min^{-1}$

Table 2 shows the all the operating points around which system has been linearized. The system is inherently stable at all five operating points, as their Eigen values are negative [12]. Table 3 shows Eigen values.

Table 2. Operating Points.

S. No.	Feed Flow (LPM)	Coolant Flow (LPM)	Conc. (mol/l)	Temp (K)
1	102	97	0.0762	444.7
2	100	100	0.0882	441.2
3	100	103	0.0989	438.77
4	97	103	0.1055	436.8
5	98	109	0.1275	433

Table 3. Eigen Values.

Operating Point	Eigen Value
1	-22.8703, -1.0204
2	-20.6802, -1.0005
3	-19.4563, -1.0006
4	-18.2740, -0.9705
5	-16.8432, -0.9808

2.3. Transfer Function Matrix

In this section five transfer function matrices are obtained at five different operating points. They are –

At operation point 1

$$\begin{bmatrix} \frac{0.092s + 0.4123}{s^2 + 23.891s + 23.337} & \frac{0.0448}{s^2 + 23.891s + 23.337} \\ \frac{-0.947s - 35.452}{s^2 + 23.891s + 23.337} & \frac{-0.9413s - 12.576}{s^2 + 23.891s + 23.337} \end{bmatrix}$$

At operating point 2

$$\begin{bmatrix} \frac{0.0091s + 0.1371}{s^2 + 21.681s + 20.69} & \frac{0.0424}{s^2 + 21.681s + 20.69} \\ \frac{-0.912s - 21.154}{s^2 + 21.681s + 20.69} & \frac{-0.9053 - 10.252}{s^2 + 21.681s + 20.69} \end{bmatrix}$$

At operating point 3

$$\begin{bmatrix} \frac{0.009s + 0.135}{s^2 + 20.503s + 19.879} & \frac{0.0413}{s^2 + 20.503s + 19.879} \\ \frac{-0.8877s - 25.4}{s^2 + 20.503s + 19.879} & \frac{-0.8823s - 8.9347}{s^2 + 20.503s + 19.879} \end{bmatrix}$$

At operating point 4

$$\begin{bmatrix} \frac{0.0089s + 0.1298}{s^2 19.344s + 18.3697} & \frac{0.0392}{s^2 19.344s + 18.3697} \\ \frac{-0.868s - 22.717}{s^2 19.344s + 18.3697} & \frac{-0.8628s - 7.969}{s^2 19.344s + 18.3697} \end{bmatrix}$$

At operating point 5

$$\begin{bmatrix} \frac{0.080087s + 0.1266}{s^2 + 17.903s + 17.225} & \frac{0.0.377}{s^2 + 17.903s + 17.225} \\ \frac{-0.83s - 18.153}{s^2 + 17.903s + 17.225} & \frac{-0.825s - 6.3863}{s^2 + 17.903s + 17.225} \end{bmatrix}$$

3. Controller Design

3.1. Fuzzy Logic

A fuzzy logic controller is widely used in machine control approaches that require computing based on "degrees of truth" rather than the usual "true or false" Boolean logic. The mapping between the input variables and the outputs is done through a set of pre-defined functions called membership function, which are also known as "Fuzzy Set" [13]. The architecture for a fuzzy controller is as shown in Fig. 2.

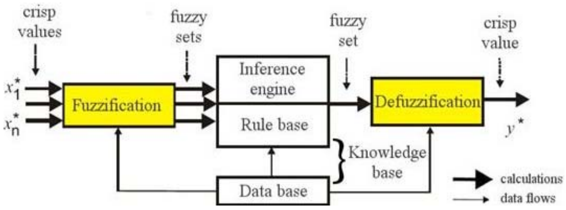

Fig. 2. Fuzzy Controller.

3.2. Fuzzy Logic Controller

The fuzzy controller is designed to control the temperature and concentration of the chemical product. It uses four input variables which are error and differential error of concentration and temperature of the reactor. The manipulated variables are feed flow rate and coolant flow rate. Separate controllers are used to control the manipulative variables.

If a single fuzzy controller is used for all operating points, it will have ten outputs (F, F_c for all five points). So the total input membership functions will be 7^4 and the total output membership functions will be 7^{10}. Hence it becomes very difficult to make all the rules for fuzzy controller manually. So, instead of one fuzzy controller, five pairs of fuzzy controllers are used, for each operating point. Each controller has a similar set of rules (Table 4 and 5 shows the rules). So, it becomes less tiresome to create the rules manually [14].

Table 4. Fuzzy Rules for Feed Flow rate.

E ***ΔE***	**HN**	**MN**	**LN**	**Z**	**LP**	**MP**	**HP**
HN	HP	HP	MP	LP	Z	LN	HN
MN	HP	HP	MP	LP	Z	LN	HN
LN	HP	HP	LP	LP	Z	MN	HN
Z	HP	MP	LP	Z	LN	MN	HN
LP	HP	MP	Z	LN	LN	HN	HN
MP	HP	LP	Z	LN	MN	HN	HN
HP	HP	LP	Z	LN	MN	HN	HN

Table 5. Fuzzy Rules for Coolant Flow rate.

E ***ΔE***	**HP**	**MP**	**LP**	**Z**	**LN**	**MN**	**HN**
HN	HP	HP	MP	LP	Z	LN	HN
MN	HP	HP	MP	LP	Z	LN	HN
LN	HP	HP	LP	LP	Z	MN	HN
Z	HP	MP	LP	Z	LN	MN	HN
LP	HP	MP	Z	LN	LN	HN	HN
MP	HP	LP	Z	LN	MN	HN	HN
HP	HP	LP	Z	LN	MN	HN	HN

The fuzzy logic controller uses triangular membership functions. Fig. 3 and Fig. 4 depict the membership function of input and output variable respectively.

The system has been linearized at five operating points. Since for each operating point a pair of fuzzy controllers needed, hence a total of ten controllers or five pairs have been designed for the process.

The transfer function matrices were used to tune each pair of fuzzy controllers simultaneously.

3.3. Weighing Algorithm

Since, each pair of fuzzy controller has been tuned around a particular operating point, so a scheduler assigns different weights to the outputs of the fuzzy controllers [15], corresponding to the present input, in accordance with the following algorithm.

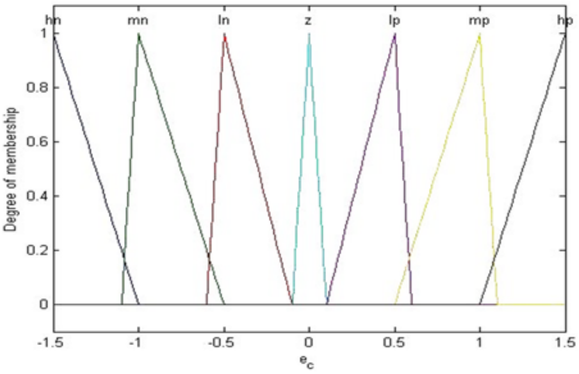

Fig. 3. Input Membership function.

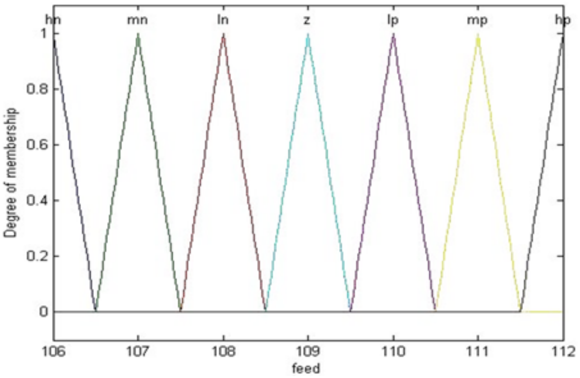

Fig. 4. Output Membership function.

If there are n operating points and y is the input at that instant,

1. Initialize a variable i to 0.
2. Iterate steps 3 and 4 till i is less than n.
3. Check if y lies between n^{th} and $(n+1)^{th}$ operating point.
    - if yes, then update the weight of i^{th} fuzzy output as

      $$w[i] = 1 - \frac{[x[i] - y]}{[x[i] + x[i+1]]}$$

      And the output of $(n+1)^{th}$ Fuzzy
      As $w[i+1] = 1 - w[i]$
      And increase i to i+1.
    - else, update the weight of i^{th} Fuzzy to 0
4. Update $i = i + 1$.

The system has a predefined set-point of temperature and concentration. The output of the controller is F and F_c which goes into actuator. The actuator output serves as the input for the process. The error and differential error, with respect to the set point are given to the controller.

The weight scheduler assigns weight to the corresponding controllers as per the set-point at that instant. So, the process is adapts the best controller needed to track the set-point. The SIMULINK model for the whole process is shown in Fig. 5.

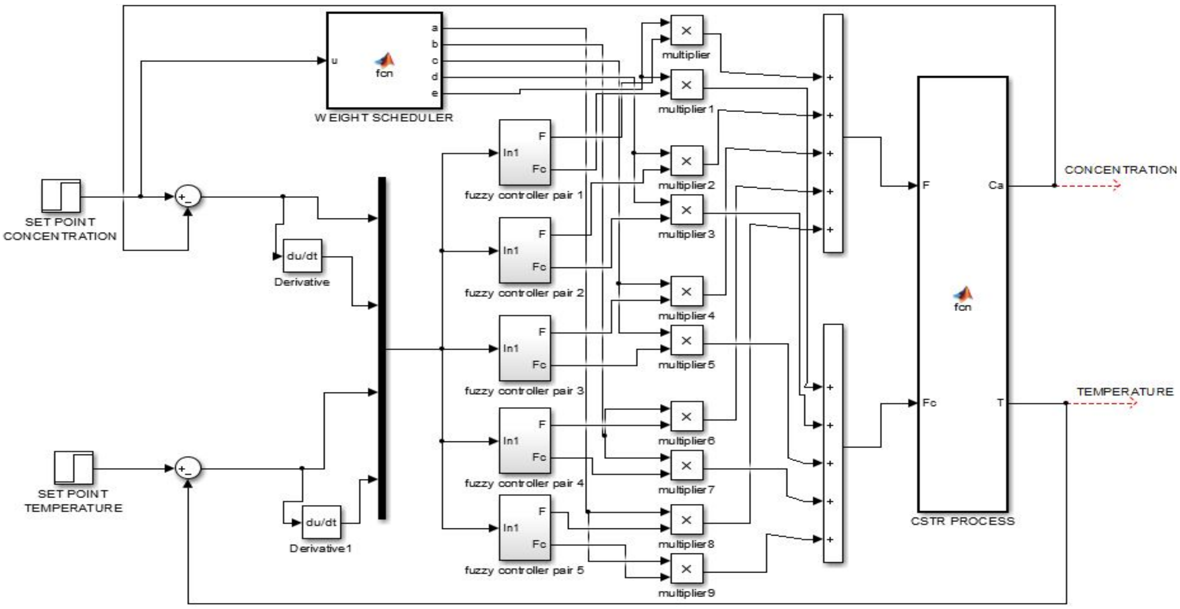

Fig. 5. CSTR Model.

4. Results

4.1. Simulation

The system has been linearized and simulated in MATLAB 2013. In order to test the set-point tracking capability of the control algorithm, various set-points have been tested at continuous intervals of time. The initial conditions of the system were:

$$C_a = 0.762\ mol/l$$
$$T = 444.7\ K$$
$$F = 102\ l/min$$
$$F_c = 97\ lit/min$$

From the responses it can be said that the controller designed for the CSTR process is able to maintain the desired set-points for dynamic change in concentration and temperature. The variation in controller output is presented in Fig. 6 and Fig. 7.

Fig. 8 and Fig. 9 represents the set point tracking of reactor concentration and temperature.

5. Conclusions

In this paper the authors have proposed an Adaptive Fuzzy controller for set-point tracking of a CSTR process. The objective was to control both the reactor concentration and temperature with minimal error.

CSTR processes have wide industrial applications. These processes need an accurate temperature and concentration to yield desirable products. The Adaptive-Fuzzy controller has an almost negligible steady state error for various operational points. Hence, it makes Adaptive-Fuzzy controller an apt controller for industrial processes.

By linearizing the system around different operating points, a better approximation of the system can be made and a weighed combination of Fuzzy controllers can be used to take a control action around the given set-point.

MATLAB provides with an comprehensive environment for algorithm development and simulation. SIMULINK is an add-on that is used to model the process in a block diagram format.

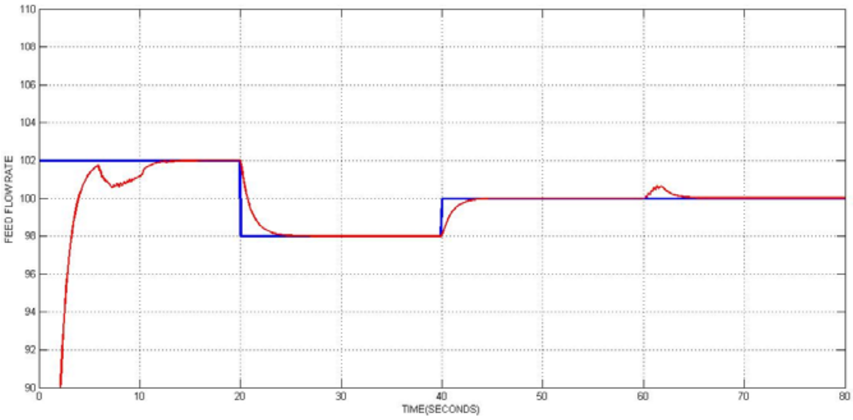

Fig. 6. Feed Flow.

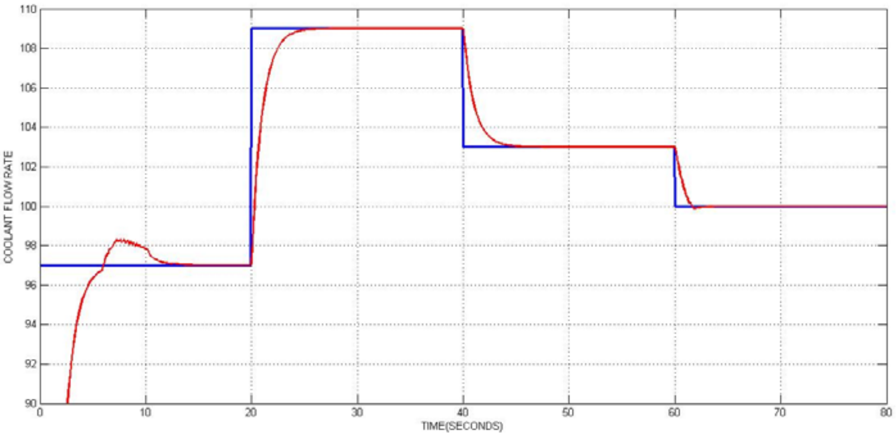

Fig. 7. Coolant Flow.

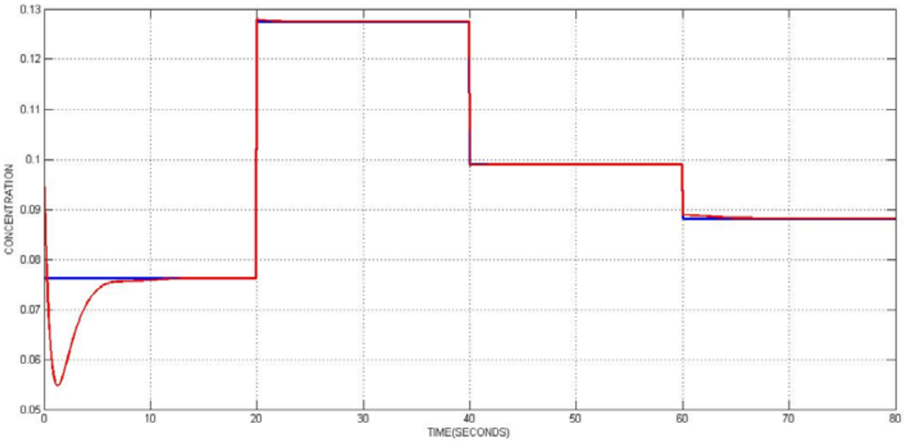

Fig. 8. Reactor Concentration.

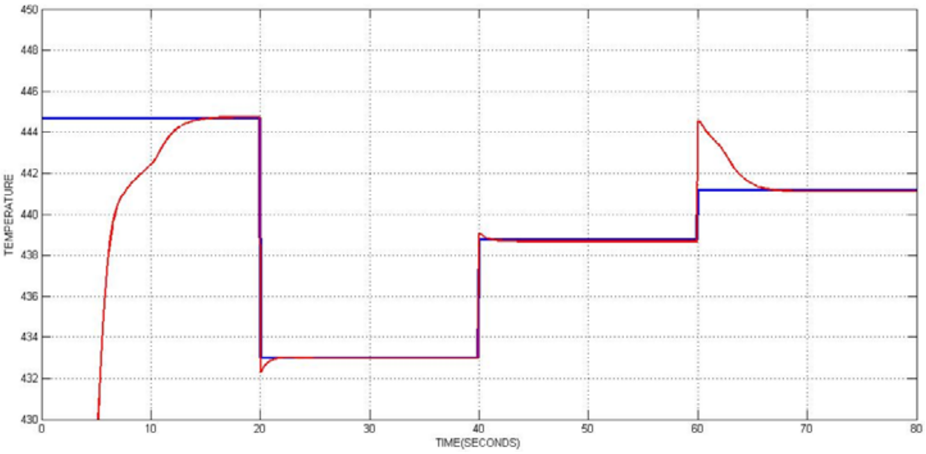

Fig. 9. Reactor Temperature.

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