Chapter 1

Phase Measurement and Processing from Analog to Digit and Go Deep into the Wider Digital World

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1.1. Introduction

In the processing and measurement of time and frequency quantities, the method of phase processing shows the highest precision of direct processing, which has reached 10^{-14}/s without multiplication processing, and the resolution is stable at 100 fs for a long time [1]. Therefore, we must consider the application of phase processing in the consideration of improving the measurement accuracy. Our research for many years has been devoted to the in-depth understanding of the phase [2, 3]. On the basis of the traditional belief that the phase mainly reflects the subdivision of a cycle in the period of the frequency signal, the phase is not only a signal parameter, but also a phenomenon and a method for analyzing problems. The traditional phase is only applicable to the case of the same frequency nominal value, so it is narrowband application [4]. Its scope of action is greatly affected.

The first step of our research is to change the understanding of the narrow-band application of the phase, revealing its wide-band nature, the special phase phenomenon manifested during processing, such as the phase coincidence phenomenon, and some concepts and definitions for phase processing and measurement. The traditional understanding of phase processing is only for the case of the same frequency nominal value [4, 5]. However, in order to give full play to the higher resolution of phase measurement compared with direct frequency measurement and period measurement, we not only hope to further improve the

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resolution of phase processing, but also use some special phase characteristicsbetween signals to perform broadband frequency measurement and control.

In addition, it is also the high resolution of time-frequency processing and measurement, so people try to find a way to convert non-time-frequency quantities into time-frequency quantities for processing in order to obtain higher measurement precision [6]. This is a method of measurement. However, on the other hand, through the application of phase characteristics in time-frequency processing, we can also maintain the original state of physical quantities or signals in the measurement and processing of other non-time-frequency quantities, while applying the same effect can be achieved using the analog phase processing method. The key here is to see if the object signal is periodic, linearly scaled, or can be expressed digitally. This is also one of the issues to be discussed in this chapter.

A series of previous work has enabled us to realize various flexible applications of phase processing. For example, the application of the phase between frequency signals is not limited by whether the period is the same or not, and the phase between various frequency signals is directly processed [7], analog and digital linear phase comparison [1, 8], some special phase phenomena, such as phase quadrature more novel implementations, with the help of the "border effect" of the phase coincidence detection information, the measurement accuracy is thousands of times higher than the resolution of the circuit and the device itself [9], synthesis of phase-shift controlled micro-frequency additional difference [10], etc.

At the same time, the current technological, industrial and social development is absolutely inseparable from digitalization. However, there is a contradiction between the resolution of AD conversion (related to the number of AD conversion bits) and the conversion rate in the existing understanding and application of digitization. The traditional solution to this problem is to improve the performance of the AD converter as much as possible. However, there is also a requirement that the device has more bits and a faster conversion rate at the same time. This is not something that improvements in the device can achieve.

In order to control the following concepts and processing methods of phase difference to a wider frequency range and combine and promote them with digital quantities, it is necessary for us to make some basic concepts more clear here. The most important thing is that it can extend the method that is usually only suitable for the comparison and processing between the same frequency nominal value signals to any frequency signal. The phase difference change between two arbitrary frequency signals can be described by concepts such as the least common multiple period, the greatest common factor frequency, equivalent phase detection and the quantized phase shift resolution [2, 3, 11], as shown in formula (1.1).

$$f_1 = Af_{\max c},\ f_2 = Bf_{\max c},\ f_{equ} = ABf_{\max c} = \frac{1}{\Delta P} \tag{1.1}$$

Among them, f_1 and f_2 are the corresponding frequencies of the two comparison signals respectively; A and B are two mutually prime positive integers; $f_{\max c}$ is the greatest common factor frequency; f_{equ} is the equivalent phase detection frequency, ΔP is the quantized phase shift resolution.

The phase difference variation ΔT between the two comparison signals varies with $T_{\min c}$ as the period. The formula for calculating the relative frequency difference of the measured signal is as formula (1.2)

$$\frac{\Delta f}{f_0} = \frac{\Delta T}{\tau} \tag{1.2}$$

In the formula, τ is the sampling time, and ΔT is the accumulated phase difference change of the two signals during the sampling time. This is a relatively widely used formula.

The Allan variance is calculated as [4]:

$$\sigma_y(\tau) = \frac{1}{\tau}\sqrt{\sum_{i=1}^{m}\frac{(\Delta T_{i+1} - \Delta T_i)^2}{2m}}, \tag{1.3}$$

where ΔT_i represents the amount of phase difference change, τ represents the duration corresponding to this change, m is the number of samples.

1.2. Phase Processing and Phase Comparison in Time-frequency Parameter Measurement

Our main work in phase processing and measurement for decades has been the successful phase processing and measurement between signals of the same frequency and different frequencies over a wide frequency range. In particular, it is found that the phase processing between signals of different frequencies can often reflect more obvious advantages. The work in this area includes: the application of phase synchronization between completely different frequency signals in measurement, the realization of various linear phase comparisons (including analog and digital) for sampling different nominal frequencies, the application of linear phase comparison in single sideband phase noise measurement [12, 13], and other work on phase comparability between signals of different frequencies, etc.

1.2.1. The Application of Phase Synchronization between Completely Different Frequency Signals in Measurement

Some techniques here have been proposed many years ago, but some techniques have been developed and perfected step by step in recent years. Valuable processing methods include count-based high-resolution frequency measurements that use "phase coincidence points" between signals to form measurement gates, and corresponding formulas to match it [14]. This method actually implements a "phase synchronization gate" between signals of different frequencies. Therefore, although its expression is still the periodic synchronization measurement,the multi-period phase synchronization and non-synchronization results are completely different.

The following Fig. 1.1 shows a method to make use of the special phase phenomenon between periodic signals – the phase coincidence between multiple cycles constitutes a measurement gate for measurement, which greatly improves the resolution of frequency measurement. Fig. 1.1 is the waveform diagram of the method, while Fig. 1.2 is the block diagram of the implementation method. The description here is to better articulate the performance improvements in digital measurements that are analogous to this method later on, and which are extremely widely used [15].

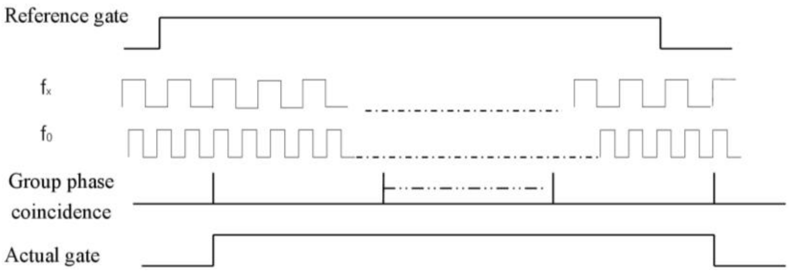


Fig. 1.1. The special phase coincidence phenomenon constitutes the measurement gate for the precise frequency measurement.

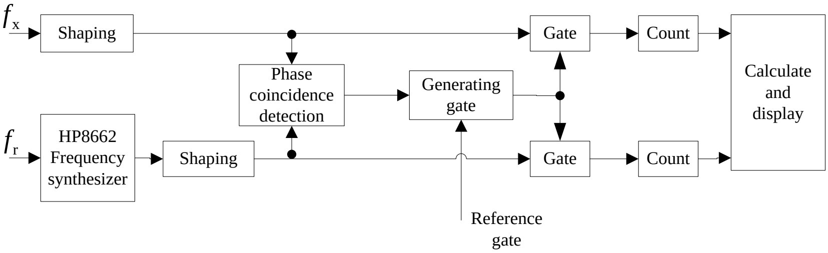


Fig. 1.2. Detecting block diagram of quantizing “phase coincidence”.

The calculation formula of the measured frequency is:

$$f_x = f_0 \frac{N_x}{N_0} \tag{1.4}$$

In the formula, f_0 is the frequency value of the standard signal, N_0 is the count value of the standard signal, and N_x is the count value of the measured signal. Although this formula is ostensibly the same as the usual multi-cycle synchronous measurement technique. But from the waveform diagram Fig. 1.1 we can see that this is a frequency measurement method for multi-cycle phase synchronization. Therefore, the measurement accuracy is greatly improved.

Compared with the frequency measurement results of the traditional direct counting method, the experimental results of this simple device have greatly improved the measurement resolution. Table 1.1 shows the results of the self-calibration with the new method.

Table 1.1. The results of the self-calibration.

Self-calibration frequencies (MHz)		ΔT	f_{equ}	**Frequency stabilities, σ(/s)**
f_0	f_x			
10.000000	5.000001	19.999996 fs	50.00001 THz	9.13×10^{-14}
10.000000	5.00001	199.9996 fs	5.00001 THz	1.30×10^{-12}
10.000000	5.0001	1.99996 ps	500.01 GHz	1.31×10^{-11}
10.000000	5.001	19.996 ps	50.01 GHz	1.82×10^{-10}
10.000000	10.000010	99.9999 fs	10.00001 THz	2.77×10^{-12}
10.000000	20.000010	49.999975 fs	20.00001 THz	7.34×10^{-13}
10.000000	100.00001	9.999999 fs	100.00001 THz	2.62×10^{-14}
10.000000	190.00001	5.263 fs	190.00001 THz	5.65×10^{-15}

In Fig. 1.3 a digital counting gate is generated by judging the zero-crossing point of the data collected by the A/D. In this gate, the reference signal and the signal under test are counted (completed by the FPGA). Finally, the frequency of the signal under test can be obtained by combining formula (1.5) [15].

$$\frac{N_x}{f_x} + \Delta\delta = \frac{N_o}{f_o} \tag{1.5}$$

Among them, f_o and f_x are the frequencies of the reference signal and the measured signal respectively, N_o and N_x are the count values of the reference signal and the measured signal during the measurement gate time, respectively, $\Delta\delta$ is the error caused by the gate signal being out of synchronization with the reference and measured signals.

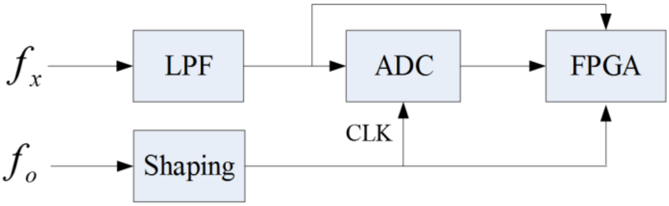


Fig. 1.3. Digital phase synchronization measurement method of periodic signals.

Fig. 1.4 is the selection waveform diagram of the digital counting gate. The linear segment of the signal under test is selected, and the value under the same specific voltage is used as the opening and closing signal of the counting gate. In practical applications, there are a large number of periodic changes, and according to the situation that the counting gate is close to 10 seconds, 1 second, 0.1 seconds, etc., the value under a specific voltage with the same sampling value is selected to open and close the gate.

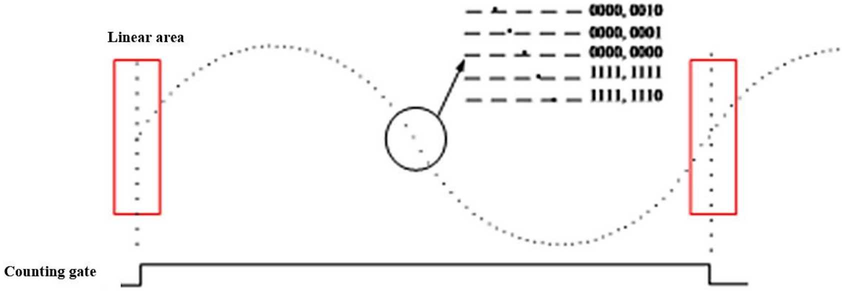


Fig. 1.4. Selection of digital counting gate.

When the AD conversion method is used, the R&S SMB100A and the Tektronix AFG3101C signal generator and the 10-bit A/D-LTC2288 of Linear Technology Corporation are used for experiments [15]. In the case of self-calibration, the 10 MHz output from OSA's constant temperature crystal oscillator 8607 is used as a common reference signal to lock SMB100A (output A/D clock signal) and AFG3101C (output measured frequency signal). In the case of mutual comparison, the two constant temperature crystal oscillators 8607 of 5 MHz and 10 MHz are used as the reference of SMB100A and AFG3101C respectively. Adjust the frequency of the measured signal, take the periodic zero-crossing point as the count mark, and the count value reflects the measured signal frequency value. The experimental results are shown in Table 1.2, and the stability is obtained by the Allan variance.

From the comparison between Table 1.1 and Table 1.2, it can be seen that the frequency measurement of phase synchronization gate has obvious precision advantages over traditional methods. However, the digital measurement of this counting method does not show obvious advantages compared with the synchronous gate measurement of analog method. Therefore, the advantages of digitization should also consider specific methods and objects.

Table 1.2. Self-calibration and mutual comparison results.

f_o / MHz	f_x / MHz	ΔT / fs	f_{equ} / THz	A/D resolution (ps)	Frequencystabilities (self-calibration)	frequency stabilities (measurement)
10.000000	10.0000001	0.99999999	1000.00001	97.656249	6.66e-14 (10s)	7.22e-13 (10s)
10.000000	10.000001	9.99999900	100.00001	97.656240	1.25e-12 (s)	6.58e-12 (s)
10.000000	10.000010	99.9999000	10.00001	97.656152	1.23e-11 (s)	1.00e-10 (s)
10.000000	10.000100	999.990000	1.00001	97.655273	3.92e-10 (s)	1.18e-9 (s)
10.000000	20.000001	4.99999975	200.00001	48.828122	9.34e-13 (s)	3.36e-12 (s)
10.000000	16.384000	24414.0625	0.04096	59.604645	6.48e-7 (s)	9.81e-7 (s)
10.000000	10.210000	97943.1929	0.01021	95.647649	2.16e-6 (s)	5.42e-6 (s)
10.000000	10.210010	97.9430970	10.21001	95.647556	3.11e-10 (s)	6.27e-10 (s)

1.2.2. Implementation of Multiple Linear Phase Comparisons at Different Nominal Frequencies

The law of phase linear change between periodic signals can easily realize the measurement of phase change, frequency and frequency stability. Over the years, the international attention has not paid enough to the measurement and processing of the linear phase comparison method, because the traditional analog linear phase comparison technology has a high measurement resolution, but its measurement response time is long, and there is still a certain gap between the resolution and the DMTD method with the highest accuracy in the world [16, 5]. Furthermore, there are dead zones and many inconveniences in use. The basic principle of phase comparison method is shown in Fig. 1.5.

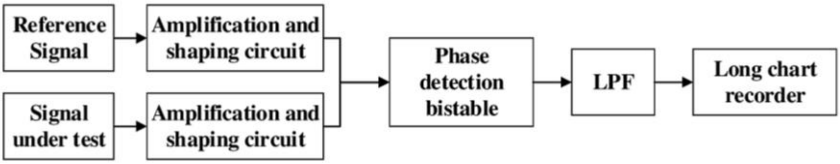


Fig. 1.5. Principle block diagram of phase comparison method.

In the phase comparison method, the measured signal f_x and the reference signal f_0 are respectively converted into square waves F_x and F_0 through the amplification and shaping circuit, and these two square waves are sent to the phase detector. The rising edge of F_0 is used as the gate opening signal of the phase detector, and the rising edge of F_x is used as the closing signal to form a pulse width signal proportional to the change of the phase difference. The duration of the high level of the pulse signal changes, which can reflect changes in the phase difference. The phase comparison method is mainly aimed at the phase comparison between the nominal value signals of the same frequency, and the comparison process is shown in Fig. 1.6.

In the phase comparison method that uses the phase detector to measure the phase difference, the level is used to represent the size of the phase. There will be a very short period of high level and low level near the 0 phase and the 2π phase. Due to the trigger of the phase detector the rate is limited, and the steep rising and falling edges cannot be obtained in a short time, so that the phase difference represented by the level width cannot be accurately measured, and there will be an ambiguous area, that is, the measurement dead zone. Therefore, based on the phase comparison method, it has been improved by controlling the phase detection area or by combining digital technology.

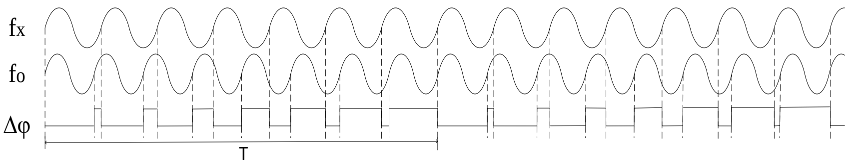


Fig. 1.6. Diagram of phase comparison at the same nominal frequency.

The phase difference is expressed in time scale as Fig. 1.7.

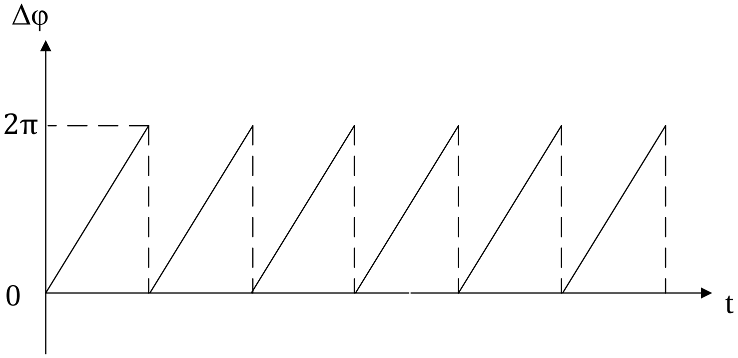


Fig. 1.7. Phase change law.

1.2.2.1. Analog Method to Control the Phase Detection Region

In the method of using analog processing to control the phase detection area, for the linear phase comparison under the same nominal frequency, one of the signals is divided into frequency, so that the signals have a multiple relationship. The phase detection region is controlled by controlling the multiple ratio of the two comparison signals. When the multiple meets certain conditions, it can be controlled in a region with better linearity. At this time, the comparison period is also extended to the period of the least common multiple of the two signals. At this time, the equivalent frequency of the phase detection is equal to the frequency value of the high-frequency signal, but a segment with good linearity in the full cycle of the low-frequency signal is selected for phase detection, which meets the phase comparison requirements for high linearity at high frequencies. As shown in Fig. 1.8 [17, 4], this is the single-channel frequency division control phase detection method. It uses the good linearity of the common switch phase detector to complete the linear phase comparison in all ranges. Once the range of the phase comparison exceeds this segment, the phase of one signal can be shifted into this segment regularly according to a fixed value that can be processed.

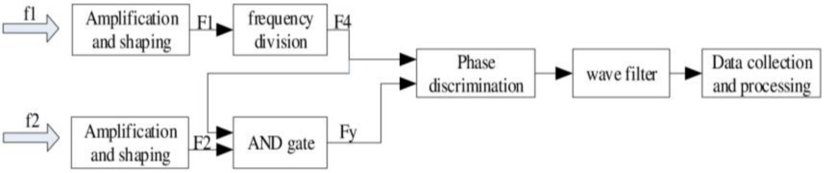


Fig. 1.8. Principle block diagram of single-channel frequency division control phase detection.

Two channels of signals with the same nominal frequency, one channel f_1 is amplified and shaped, and after frequency division, the signal F_4 is output. Its falling edge is used as a trigger signal as a phase detection signal. At the same time, F_4 is also sent to the AND gate to control the gate circuit. The other phase comparison signal F_2 is amplified and shaped and sent to the phase detection circuit as another phase detection signal. The AND gate is used here to purposefully select the pulse of F_2. Its function is that when the falling edge of F_4 makes the phase detection bistable flip, its low level also makes AND gate close.

Since the two comparison signals at the input of the phase detection circuit have a certain interval in time, the switching speed of the phase detection bistable is not required to be high, and the "dead zone" is improved. In addition, there is no narrow pulse in the square wave output by the phase detection circuit, so it is convenient to shape and obtain a regular square wave waveform. Fig. 1.9 is the corresponding working waveform.

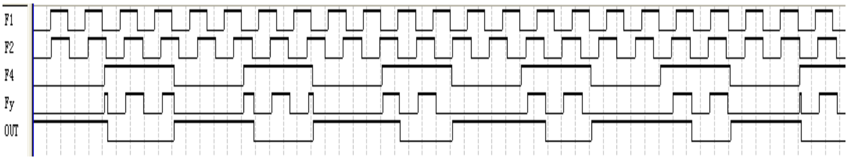


Fig. 1.9. Working Waveform.

Although the front and rear edges of the phase detection output square wave are still not very steep, since there are no narrow pulses in the waveform, this non-steepness will not cause the pulse amplitude to change with the phase change of the phase comparison signal, so it will not affect the phase comparison linearity output, to achieve phase comparison with direct linearization at high frequencies. Due to the frequency division of one signal, the amplitude of the output level change of the phase detector is smaller than that of the traditional phase detection method when the above phase detection method is used, but this method effectively improves the dead zone and nonlinear problems of the traditional phase detection circuit. The phase change trend is similar, and it is still a periodic linear change. For the measurement of the phase change rate and frequency, it is not required to know the phase value of the measured signal, but the phase change is directly required. So consciously fixing the phase shift does not affect the result of the measurement.

1.2.2.2. Digital Method to Control Phase Detection Area

The phase comparison is realized by digitization, and the phase detection area is controlled by digital processing, which can also avoid the measurement dead zone and nonlinear phenomenon caused by the phase detector. At the same time, the digital method has a simpler structure and is easier to implement than the analog method. Many other advantages are also obtained.

In the digital linear phase comparison method, ADC is the main device for phase detection.

The method of using ADC for phase detection processing does not have the measurement dead zone when the time interval in the phase detector is extremely short. However, due to the quantization error that ADC cannot avoid, when the voltage change is within the quantization resolution, the ADC cannot measure the voltage change. Therefore, when the phase information is collected, the phase detection area, that is, the linear area of the measured signal, is controlled as the phase comparison area.

The derivative of a continuous function at a certain point is the slope of its tangent at this point. The derivative of the sin x function is cos x, and its waveform is shown in Fig. 1.10.

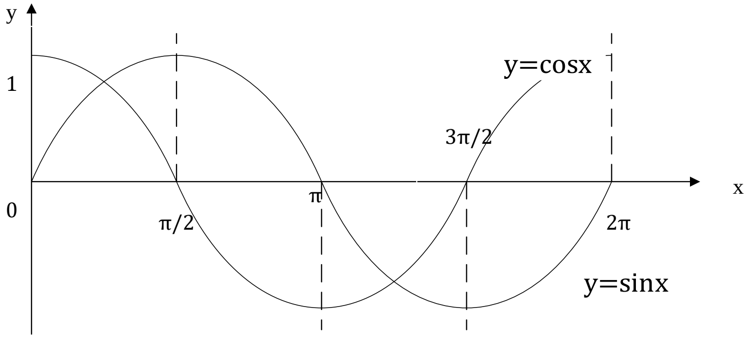


Fig. 1.10. The sine function and its derivatives.

When x is equal to 0, its derivative value is 1, which is also the maximum value of the derivative value, indicating that the rate of change is the largest at this area. When x changes from 0 to π/2, the derivative value continues to decrease to 0. The rate of change is minimized. This property means in the signal that when the phase changes at equal intervals, the voltage has the fastest rate of change near the 0 value and the slowest rate of change near the maximum value, so when collecting with a high-speed ADC, keep the same time interval, that is, for the same phase change, the voltage has a large change near the 0 value, and the accuracy of using the voltage to calculate the change of the phase difference will also be higher. When the voltage is near the maximum value, the phase difference cannot be accurately calculated because the variation is too small, causes a phase of ambiguity, especially for ADC converters with low number of bits and low resolution, the ambiguous area will be serious affects the accuracy of phase difference acquisition.

For example, for a sinusoidal signal $y=\sin(\omega t)$, ωt is the phase.

Let the time change at 0 phase (i.e. $t=0$) be ΔT_1, and the time change at $\pi/2$ phase (i.e. $t=\pi/2\omega$) as ΔT_2. ΔT_1 and ΔT_2 are equal and both are less than $\pi/4\omega$.

Then the voltage change caused by ΔT_1 is Δy_1:

$$\Delta y_1=\sin[\omega(t+\Delta T_1)]-\sin(\omega t), \tag{1.6}$$

$$\Delta y_1=\sin(\omega\Delta T_1) \tag{1.7}$$

Then the voltage change caused by ΔT_2 is Δy_2:

$$\Delta y_2=\sin[\omega(t+\Delta T_2)]-\sin(\omega t), \tag{1.8}$$

$$\Delta y_2=\cos(\omega\Delta T_2)-1 \tag{1.9}$$

Because of $\omega\Delta T_1=\omega\Delta T_2<\pi/4$, there must be $|\Delta y_1|>|\Delta y_2|$.

In order to ensure that during the phase comparison process, there is at least one sampling point in the measurement linear region of one measured period, the number of samplings needs to be increased, so that there is a multiple relationship between the reference and the measured signal.

The relationship between multiples between reference and measured signal frequencies and extent of linear region is: when the reference is A times the measured signal, the range of the linear region is not less than -180°/A~180°/A。.

When the frequency of the reference signal used as the clock is 10 times that of the measured signal, select -18°~18° as the linear range,and the 10 sampling points in one cycle of the measured signal fall into the 10 divided areas respectively[1, 18]. As shown in Fig.1.11, in the sinusoidal signal, the phase -18°~18° is used as the linear segment, and 36° is divided into 1 area at equal intervals.

When the frequency of the reference and the measured signal is 10 times integer and there is no deviation, starting from the first sampling point in the linear region, every 9 points is collected, and the 10th point will still fall in the linear region. When the frequency of the reference and the measured signal is 10 times integral but there is a slight deviation, from the first sampling point in the linear region to the 10th point in the next cycle, the phase difference will increase or decrease ΔT, after the accumulation of changes in multiple cycles, the phase will be shifted out of the linear region in a certain cycle, and can no longer be used as a linear phase acquisition point. However, in the same cycle of the measured signal, another sampling point will move into the linear region, and this point can be used as the acquisition point of the phase. From this point, count the acquisition with 10 sampling points. With this multiple relationship, the continuous sampling of the phase in the linear region is realized, and the sampling result is shown in Fig. 1.12.

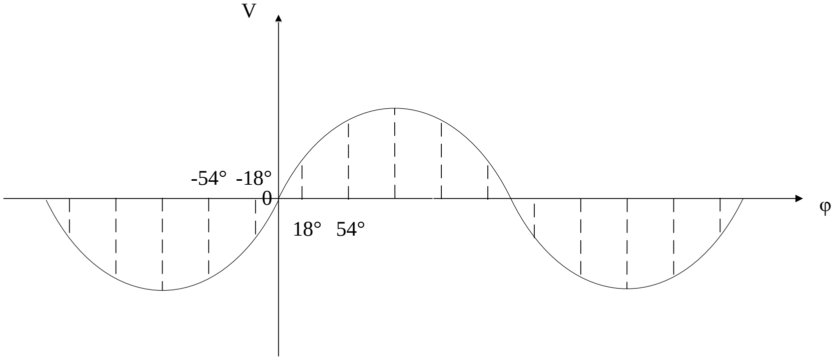


Fig. 1.11. Sampling distribution under ten octave clock.

Since there is a certain frequency difference between the comparison signals, the phase difference obtained by sampling will increase continuously by ΔT. The change trend of the phase difference obtained by the basic phase comparison method is the same, and it still presents an arithmetic sequence with linear tolerance, but the phase change range is reduced, and the phase repeats periodically in the linear region. Compared with the analog method, the reference and the measured signal frequency have a multiple relationship, which improves the response time.

The resolution of the phase comparison using the digital linear phase comparison method can reach the order of 1.5E-13/s when the 16-bit ADC is used as the phase detector.

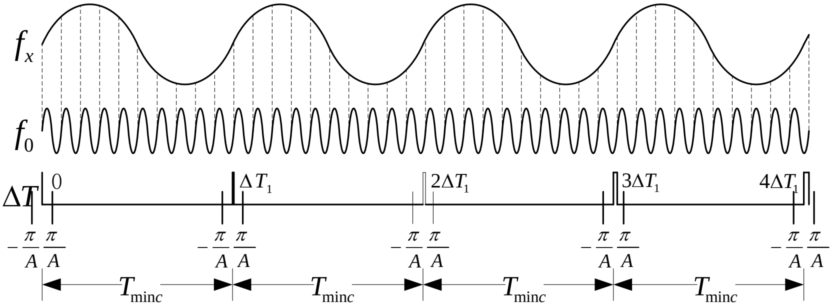


Fig. 1.12. Digital linear phase comparison under the frequency multiplication relationship.

The block diagram of the digital linear phase comparison system with multiple relation between such signals is shown in Fig. 1.13.

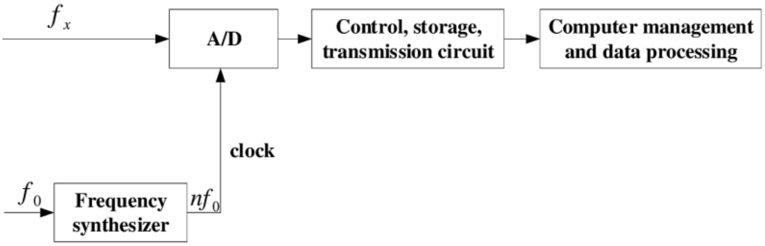


Fig. 1.13. System composition diagram.

1.2.3. Application of Linear Phase Comparison in Single Sideband Phase Noise Measurement

A novel digital linear phase comparison method–DLPC can automatically keep the orthogonal phenomenon of the signal under test without lock processing, especially in the case of noise interference and poor stability of the measured signal, there will be no loss of lock phenomenon. Therefore, the single sideband phase noise measurement can also be well applied on the basis of linear phase comparison. The mainstream phase noise measurement system needs to achieve phase quadrature control after the reference signal and the measured signal are locked with each other. This method is more convenient to use and more stable than the mainstream method. It also includes the simultaneous corresponding measurement in frequency domain and time domain. That is, the frequency stability sampling time corresponding to the deviation of the carrier frequency from 1 Hz to 1 MHz or 10 MHz in the usual phase noise measurement is also the stability index from 1 second to 1 μs or 0.1 μs, respectively. The frequency stability measurement of DLPC can cover the period from the signal carrier frequency to the unlimited time.

1.2.3.1. Overall Design of the System

The phase noise measurement system based on digital direct phase processing first realizes the linear representation of the phase change of the measured signal from 0° to 360°; by studying the single-sideband phase noise measurement algorithm, the phase noise measurement is realized. Due to the ability to quickly measure transient and short-term frequency stability, corresponding measurements of time-domain frequency stability and frequency-domain phase noise can also be performed [12, 13]. At the same time, it has the function of full stability measurement. The system is mainly composed of phase information acquisition and power spectrum estimation. The overall flow of the system is shown in Fig. 1.14.

The system adopts the digital linear comparison method to obtain the phase jitter information. The high-speed ADC is used as a linear phase detector, the output signal of the reference source is used as the clock signal of the ADC after frequency doubling. Then, the linear region of the sampled signal is judged and the phase difference information is extracted. After several sets of comparison experiments, the window length and coincidence degree are set reasonably to improve the resolution. At the same time, by designing the segmentation and splicing of the spectrum, the problem of fixed resolution during FFT calculation is solved, and the phase noise measurement with high resolution and low power consumption is realized.

1.2.3.2. Phase Information Collection

By studying the principle of linear phase comparison and analyzing the phase relationship between the signals, it is found that for two signals with a multiple of the nominal frequency, if the phase difference change data of the sampling points of the linear phase change segment is reasonably extracted, it can be found that The phase difference has a monotonic and periodic variation law. For a section at the zero-crossing point of the sine signal, the quantization fuzzy area between two adjacent sampling points is close to uniform distribution and the phase step is relatively obvious. By extracting the sampled data in the linear region and combining the border effect theory, the quantization error generated by the analog-to-digital converter during the quantization process can be effectively reduced, and the measurement accuracy can be improved.

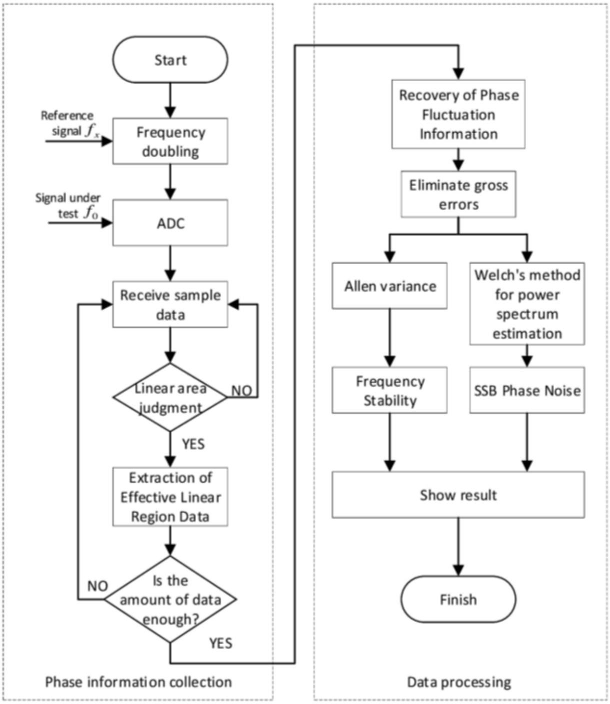


Fig. 1.14. The overall flow chart of the system.

The size of the linear interval is determined by the multiple relationship K between the two signals, so as to ensure that every K consecutive clock signals always have a signal in the linear area of the measured signal, so that the extracted phase change information is continuous, and the measurement of frequency stability in the whole domain from far carrier frequency to near carrier frequency can be realized.

1.2.3.3. Data Processing Module

In this system, the data is transmitted to the computer by serial communication and saved, and then the data is processed in MATLAB software to obtain its power spectrum distribution curve. Data processing includes processing the extracted data of the effective linear region, such as binary conversion, amplitude phase conversion, phase difference calculation and gross error removal, to obtain the phase fluctuation information, and then estimate the power spectrum of the phase information to obtain the single sideband phase noise curve. The multi-resolution power spectrum design process is shown in Fig. 1.15.

Because the focus of phase noise measurement is the change trend of power spectrum curve, there is no strict requirement for resolution. Therefore, for a group of measurement data, based on Welch method, the piecewise average of single data can be used to reduce the variance, and the appropriate data segment length and data overlap can be selected for power spectrum estimation to improve the resolution performance. In the process of power spectrum estimation, the FFT operation resolution is fixed, which will result in waste of resources or low measurement accuracy. In order to weigh the advantages and disadvantages of FFT points and resolution, we adopt the method of segmenting the frequency range of the measured signal, and select different sampling frequencies for different frequency bands to realize its power spectrum estimation.

By setting the phase contrast time, the system realizes the segmented processing of the spectrum, solves the problem of spectrum estimation resolution fixed and meets the requirements of the whole measurement range from far carrier frequency to near carrier frequency. If the number of data points involved in FFT operation is 2048, the spectrum segmentation is shown in reference [12, 13].

The resolution of different frequency bands is different. This method not only meets the requirements of resolution, but also saves resources and improves the operation speed to a certain extent.

According to the multi-resolution power spectrum design method proposed above, the phase difference information between two signals whose nominal frequency is multiple is extracted by using the measurement system. The second stability of OCXO 8607 crystal oscillatorself calibration can reach 4.106×10^{-13}, which verifies the feasibility and high precision of the phase difference information acquisition method in this measurement system.

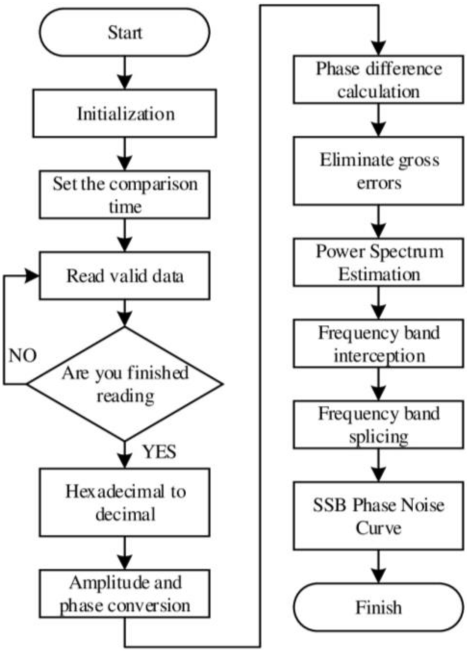


Fig. 1.15. Multiresolution power spectrum design flow chart.

According to the multi-resolution power spectrum design method proposed above, for the data of phase difference in the linear region extracted based on digital phase processing in this system, the Welch method and Hamming window are used for power spectrum estimation. The measured OCXO 8607 crystal oscillatorself calibration phase noise curve is shown in Fig. 1.16. From the figure, it can be seen that the resolution of different frequency bands is different, and this method meets the requirements of resolution. To a certain extent, it saves resources and improves the operation speed.

In the research of single sideband phase noise measurement, compared with the current phase noise measurement system with the highest resolution, the accuracy of this method still has a certain gap. However, from the perspective of meeting the vast majority of users, the above method is simple and easier to combine with traditional technology. The digital linear phase comparison method solves the problem that the phase orthogonality and the extraction of phase change information can only be realized by phase interlock in the traditional system. The function of this link simplifies the system, avoids the trouble caused by lock loss, improves work efficiency, and differences in indicator performance between different frequency sources are explained. The chapter also hopes that the counterparts who are engaged in traditional single-sideband phase noise measurement can learn from this method that can complete phase information extraction without phase quadrature control.

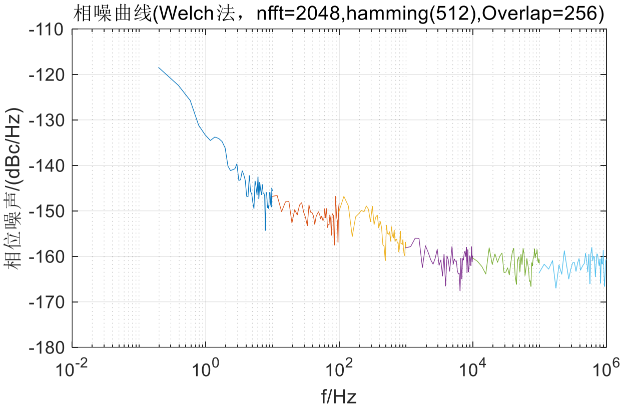


Fig. 1.16. OCXO 8607 crystal oscillatorself calibration phase noise measured by the system.

The phase noise measurement of the linear phase comparison method described earlier is based on a multiple clock method. Today's phase noise measurements are often required to work well over a wide frequency range. In this way, demands are placed on the clock. Therefore, depending on the regular change characteristics of the specific phase condition between the measured signal and the clock signal, under a simple frequency relationship, within a period of least common multiple, the phase relationship between one or more signals will be in the "linear region" of phase change.

Fig. 1.17 is the acquisition experiment of digitized linear phase difference between 10 MHz signal and 11 MHz signal. One can see within the least common multiple period 1 μs between them, there will be a "linear region" of phase change between the two waveform groups.

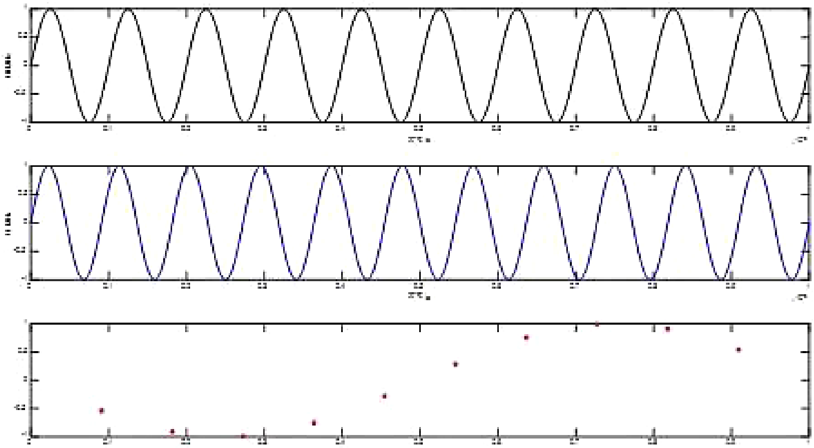


Fig. 1.17. Comparison of digital linear phase difference between 10 MHz signaland 11 MHz signal.

1.3. Exploration and Application of a Wider Range Non-time-frequency Signal Parameters and Phase Phenomena in Digital Processing

The phase phenomenon analyzed in the previous section mainly occurs and applies in the range of time-frequency and time-frequency parameters, and has shown obvious advantages in accuracy and principle. As in a wide range of measurement fields, it is often hoped to find a way to convert non time-frequency quantity into time-frequency quantity for processing, so as to obtain higher measurement accuracy [6]. This is indeed a meaningful measurement and processing method. However, according to our research, we also found that it is not necessary to undergo such conversion. Through the application of phase characteristics similar to time-frequency processing, the original state of physical quantities or signals can be maintained in the measurement and processing of other non-time-frequency quantities, and applying the analogous phase processing method can obtain the same higher precision effect [19]. Here, we work on: how the physical concept of phase and phase processing go beyond the scope of time-frequency, the frequency-phase relationship between the clock signal and the signal under test in A/D conversion, the extension of time-frequency processing method from time-frequency parameter measurement to broader periodic signal measurement and processing, fitting phenomenon and application effect of A/D jump edge and analog quantity, etc. [20].

The precision of a large number of experiments has been improved after integrating "phase processing" into periodic signal parameters and digital processing [21].

1.3.1. The Frequency-phase Relationship between the Clock Signal and the Measured Signal in A/D Conversion

AD converter and its analog-to-digital conversion technology are the key devices for digital measurement and processing. The AD converter collects the voltage value of the measured signal under the action of the clock. The quantization error of AD will occur in the usual random acquisition. It is affected and restricted by the quantization and conversion bits of the device itself and the acquisition rate. However, once we pay attention to the relationship between the frequency and phase change between the clock of AD converter and the measured signal, we will find some excellent features in targeted applications. Especially for the measured signals with periodic changes, these features focus on the targeted processing of the errors generated in the quantization process. Some cases may be conducive to the improvement of precision, and some cases will lead to the expansion of functions and the emergence of new and directional measurement technologies [21].

Fig. 1.18 shows the measuring device centered on AD [20].

In fact, the different frequency and phase variation relationship between the AD converter clock and the signal under test has become a key decisive factor for the measurement function and precision. It is also a place that must be given sufficient attention.

In the processing of different purposes, the frequency-phase relationship between the clock and the measured signal is not only similar or the same, but also very different. The phase synchronization between the AD converter clock and the measured signal, close to synchronization, and non-synchronization will bring different effects in different applications. Below are several typical frequency-to-phase relationships between the AD converter clock and the signal under test.

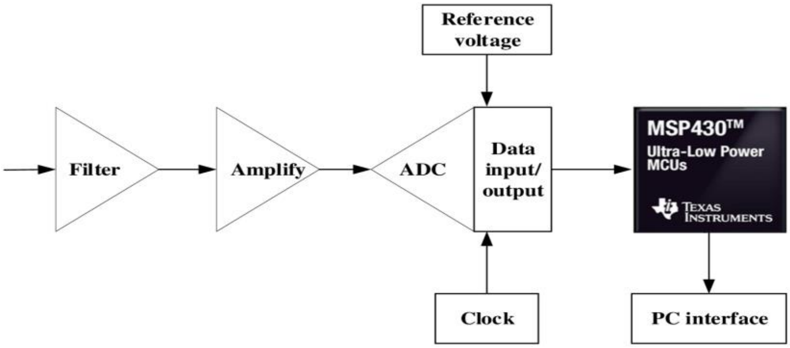


Fig. 1.18. AD-centered measurement setup.

1) When the clock and the measured signal have frequency difference or show the relationship of division and multiple in frequency, as well as the more general combination proportion relationship, and there is additional frequency difference, or even when the frequency relationship is complex, the "phase synchronization gate" of frequency measurement is formed by capturing the phase coincidence between the two signals at the closest 0 degree. Precise frequency measurement can be realized [4].

2) Similarly, when the clock and the measured signal have frequency difference or show the relationship of division and multiple in frequency, and there is additional frequency difference, the waveform of the measured signal can be recovered by connecting the collected voltage values together in time order. When the sampling rate is not high and the sampling points are limited, the original signal can be recovered through multi period measurement data, so as to obtain the complete information of the signal [21]. More attention is paid here to the integrity of the two-dimensional continuous relationship of the collected signal.

3) When the clock is a multiple of the frequency of the signal under test, and the sampling selects the linear region of the signal under test in the vicinity of 0 degrees, Fig. 1.18 is a digital linear phase comparison device

[1], and the measurement can obtain the linear segment voltage–phase difference value.

4) For periodic signals, in order to solve the contradiction between the acquisition resolution (related to the number of bits of AD) and the acquisition rate (related to the clock frequency of AD) during the acquisition of key phase information. For example, when measuring the 0 phase difference between the clock signal and the measured signal at high frequency, the periodic occurrence, and the very close phase information, we only need to pay attention to a limited part of the signal value in the measured signal waveform of sampling. Therefore, a high-speed, low-bit AD converter can be used to perform analog-to-digital conversion close to the full scale for the local area at the 0 phase difference of the measured signal waveform. If the acquisition area of our AD converter is only 1/20 to 1/50 of the full-amplitude variation of the measured signal, then the number of acquisition bits is increased by 20 to 50 times.

That is, it is possible to obtain the acquisition resolution of full waveform application with more than 20 bits by using 16 bit AD converter.

1.3.2. Application of Time-frequency Method in Extended Reconstruction of Digital AC Voltage Waveform

The current digital voltmeter is mainly used in the measurement of DC and low frequency voltage. For high-frequency voltage measurement, the method of measuring DC after rectification and filtering, and the method of measuring DC and AC with the aid of thermocouples are often used [22].

A very critical problem in digitization is the "quantization error" that may be brought about by quantization, which seriously affects the development and application of digitization. AC voltage RMS measurement and its waveform scanning display, it is often necessary to collect enough voltage signals of different phases within the measured signal cycle to be able to extract and calculate the waveform and RMS of the signal. Therefore, the analog-to-digital conversion of AD converters can only be used for DC or low frequency voltage measurement problems. In the digital storage oscilloscope, ultra-high-frequency clock signals must also be used for the display of high-frequency signals.

In order to obtain the waveform parameters and the effective value of the voltage, there must be enough sampling values in one cycle of the AC voltage. In this way, the AD converter must have higher acquisition rate and more bits at high frequency. This is also difficult to guarantee and impossible to achieve by the current traditional technology [20]. However, according to the characteristics of periodic variation of AC voltage signal, by adopting the beat cycle phase analysis method similar to that used in the frequency standard comparison [23], the frequency response and measurement precision of the high frequency measurement, which is difficult to achieve by direct digital AC voltage measurement, are obtained.

Here, the measurement method of time-frequency parameters is used to realize the principle experiment of scanning, measuring and restoring waveforms of AC voltage signals with high precision for a large number of normative frequencies. Several clock signals with fixed frequency deviation from the fractional value of the common normative frequency are used to sample and measure the voltage under the condition of multi cycle regular step-by-step phase shift of the measured signal. The waveform of the measured signal can be recovered with high precision by using the periodic phase change characteristics between signals, and the effective value of voltage can be obtained. Because the acquisition sequence has the function of phase stepping, under the condition of greatly reducing the rate of AD converter and clock signal frequency, the accuracy is improved, and a variety of measurement results such as frequency response are greatly improved.

Here, the processing of the frequency relationship between the clock and the measured signal is very important. For high-precision measurement of measured voltage at various typical frequencies, we use the regular characteristics of the phase change between periodic signals [4], use as few clock frequency signals as possible, and obtain the high-precision measurement results of the voltage at more frequencies at high frequencies. The precision of digital high-frequency voltage measurement has been significantly improved.

The current generation of high-precision, wide-frequency AC voltage signals is often obtained by using a high-stability crystal oscillator through DDS synthesis and transformation. Therefore, it has a good frequency stability index while ensuring the high precision of the voltage [21].

1.3.2.1. Law of Periodic Variation of AC Voltage and Periodic Phase Variation

Periodic signal measurement and processing often have higher resolution than other physical quantities [4]. Using the characteristic of periodic change will greatly improve the measurement accuracy of AC voltage. It's just that the current digital AC voltage measurement is directly collected by AD converter, and it does not take into account that the clock signal and the measured signal form a phase step rule under a certain frequency relationship. In the case of greatly reducing the frequency of the clock signal, the waveform parameters and voltage RMS of the signal are obtained by sampling.

The method of phase processing can reflect the details of changes in key characteristics of periodic signals. When the frequency between two frequency signals, such as the measured signal and the digitized clock signal, presents the relationship of division and multiplication, and there is a small frequency difference, the phase stepping sampling or multi period phase stepping of the measured signal can be formed. Or there will be multiple different sampling points in each cycle, and the phase stepping will be carried out respectively according to the clock sequence. We use the clock signal as the fraction of the measured signal and there is a small frequency difference, as shown in Fig. 1.19.

If the clock signal is strictly divided, multiple or proportional to the measured signal, then the relationship between the clock signal 1 and the measured signal can only be collected synchronously at a fixed phase difference point. If a certain small frequency difference is added, each acquisition point will have a fixed phase step relative to the previous one. In this way, all voltage values on the measured signal waveform can be measured at intervals but phase continuously. Not only is it convenient to restore the waveform of the signal, but also various parameters such as the voltage of the measured signal can be calculated on the basis of these data. When there is a fractional relationship between the clock signal and the measured signal and there is a certain frequency difference, sampling a triangular wave and a sine wave that is not strictly symmetrical, as shown in Fig. 1.19, each clock signal is intermittently. The phase is scanned on the waveform of the measured signal, and the multi-cycle voltage of the measured signal is sampled. Obviously, there is a step-by-step movement of the phase between the sampled voltages. Such sampling can ensure that after sampling all the phase states of the signal under test, the waveform of the signal under test can be recovered, and parameters such as the RMS voltage can be obtained.

$$F_c = f_x/m + \Delta f, \tag{1.10}$$

$$\Delta T = \tau \bullet \Delta f/f \tag{1.11}$$

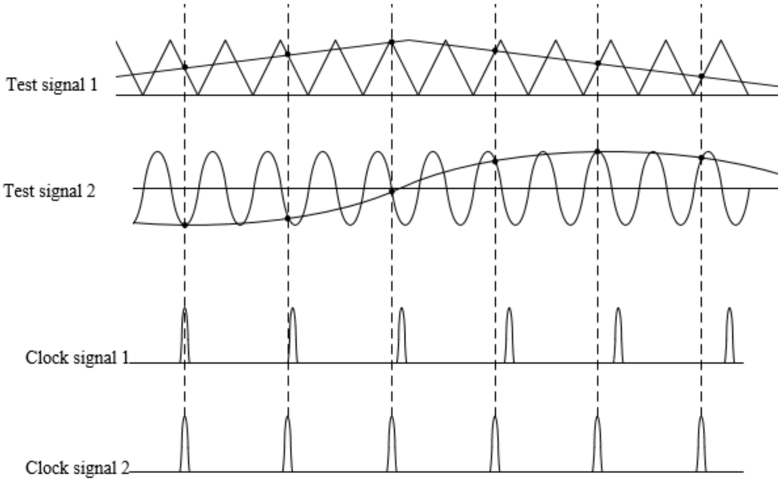


Fig. 1.19. Relative to the fractional clock of the measured signal, the phase stepping movement characteristics of the sampling point with small frequency offset.

Among them, τ is the sampling period, f_x is the measured frequency, Δf is the frequency difference between the reference and the measured signal, and ΔT here represents the step-by-step phase change under the action of the clock. When τ is equal to the period of the clock signal (that is, n times the period of the measured signal and has a fixed difference), the sampling process of the clock is also the process of τ continuously multiplying the sampling and measuring the voltage, which is a linear step process on the time axis. For example, the clock is 10 MHz + 1 kHz, and the 20 MHz signal under test is sampled by the ADC. τ takes 1 full cycle of the clock. ΔT is approximately equal to 99.99 ns × 1 kHz / 10 MHz = 0.01 ns = 10 ps.

It takes 100 ns/10 ps = 10000 points to acquire a full waveform voltage measurement. This illustrates that with a fixed frequency offset (and its fractional frequency) clock signal, voltage measurements can be made at a wide, common frequency point signal. Only the fractional frequency relationship between the clock and the measured signal is different, and the sampling time and measurement resolution will be different.

1.3.2.2. The Phase Phenomenon of a Specific lock/Measured Signal Frequency Relationship in Digital Quantization and the Corresponding Relationship with the Voltage Value

In digital signal processing, the periodic change of the phase difference of the periodic signal makes the process of quantizing the measured signal a process related to the phase processing of the clock frequency. From the perspective of the two-dimensional relationship, the instantaneous voltage value of the measured signal under different phase conditions is periodically obtained under the clock acquisition related to this phase processing. Whether the voltage value under all phase conditions of the signal under test can be scanned linearly, how much the measurement resolution of the voltage value can be obtained, and how much the noise affects the measurement are the key points of our attention. From a relatively simple and clear, easy to implement and popularize, that is, the A/D conversion clock/measured signal frequency relationship presents a fractional relationship and has a suitable frequency difference. Therefore, we use a clock signal of a specific frequency, and direct application or simple frequency division can ensure a reasonable minimum common multiple period T_{minc}[2, 3] between the signals in most cases. The required small frequency difference Δf also relatively normative. For voltages at common normative frequencies, such a sampling method can obtain parameters such as the effective value of the AC voltage with high accuracy.

Based on the above analysis and the combination of the periodicity of phase change between periodic signals and the digital quantization process, we selected the constant temperature and high stability crystal oscillator group of 10 MHz + 1 kHz and 1 MHz + 0.1 kHz signals as the clock of the high-frequency AC voltage signal measurement system signal.

Among them, 10 MHz + 1 kHz signal is for a large number of multiple frequency signals of 10MHz signal (such as 20 MHz, 30 MHz, 40 MHz, 50 MHz, 60 MHz, 70 MHz, 80 MHz, 90 MHz, 100 MHz, etc.). These nine multiple frequency points of the clock will form a sampling relationship for the measured voltage similar to Fig. 1.19. The T_{minC} guaranteed by 9 frequency points is 1 / 1 kHz = 1 ms. The cycle of clock sampling is 99.99 ns. The number of voltage points collected for an effective waveform is 1ms / 100 ns = 10000.

And 1 MHz + 0.1 kHz signal for a large number of multiple frequency signals of 1 MHz (such as multiple frequency, 2 MHz, 3 MHz, ~9 MHz, 11 MHz, 12 MHz ~ 19 MHz, 21 MHz ~ 29 MHz, with 1 MHz interval, until more than 100 MHz, or even higher.). The sampling relationship for the measured voltage similar to Fig. 1.19 will be formed. The multiple frequency point of a large number of clocks under multiple frequencies can ensure that $T_{min}$is 1 / 0.1kHz = 10ms. The cycle of clock sampling is 999.9 ns, and the number of voltage points collected for a valid waveform is 10 ms / 1000 ns = 10000.

As far as the frequency relationship between the clock and the measured signal in our example is concerned, we are concerned with the acquisition of measurement and processing information related to the voltage waveform and AC voltage parameters of the measured signal. We hope to obtain with high precision: the collected voltages have a relatively fixed phase shift with each other, that is, in the extension of the measured periodic analog quantity, the voltage and accurate change rate of the measured signal can be obtained synchronously under the action of the clock; The voltage signal corresponding to all phase differences of the measured signal from 0 to 360 degrees can be detected in sequence, and its waveform can be recovered with high precision.

Therefore, the acquisition period of the signal can be determined by taking the minimum common multiple period T_{minc} between the clock and the main part of the frequency between the measured signal as the period; The period required to obtain the complete measured signal waveform is determined by taking the minimum common multiple period T_{minc} of the frequency between the clock and the measured signal as the cycle. For example, with 1 MHz + 0.1 kHz clock signal, for the measured signal of 21 MHz, the clock acquisition cycle is close to 1 μs. The complete waveform acquisition cycle is 1 0ms. The number of voltage / phase data collected is 10 ms / 1 μS = 10000. In this way, the measured signal period of 1 / 21 MHz = 47.62 ns can actually be divided into 10000 parts. The theoretical phase resolution can reach 4.762 ps. For the voltage sampling of voltage waveform, the accuracy of sampling in time determines the accuracy of the collected AC voltage.

The measurement of the effective value and peak value of the digital high and low frequency AC voltage signal is aimed at the signal with periodic variation characteristics. Not only do we take into account the digitization of these signals, but we can also pay attention to periodic changes in their quantified characteristics. This means that the original waveform of the signal is recovered through the multi-cycle phase step voltage measurement by using the strict periodic change characteristics of the signal, and the purpose of the measurement is completed.

Fig. 1.20 is a block diagram of the experiment for the new method to measure high frequency AC voltage.

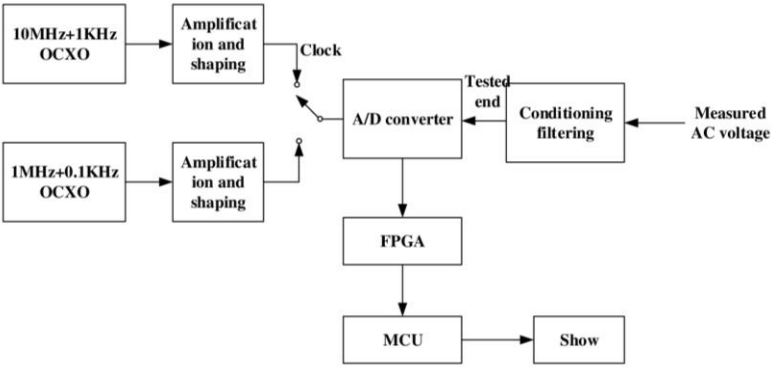


Fig. 1.20. Experimental block diagram for measuring high frequency AC voltage.

Here, the clock-to-signal frequency relationship between signals is used as a fraction and there is a difference, the beat value or combined beat value between the signals can be equal to the least common multiple period between them [3, 4]. The least common multiple period is introduced here to use the clock signal more flexibly on the one hand; on the other hand, it is still applicable when the signals do not necessarily have a simple beat relationship but have a frequency combination relationship and a certain frequency difference.

In the case of cycle time expansion, the waveform is similar to the original condition of the signal, but the cycle time is expanded and multiplied. In this reconstructed signal waveform, all voltage measurement data are from the sampling of the original input voltage.

We can complete the measurement of signal voltage RMS and other parameters without distortion at a lower frequency equivalent to the measured high frequency.

1.3.2.3. Experiment, Analysis and Comparison

Under the measurement scheme in Fig. 1.20, the waveform diagram of the specific AC voltage sampling measurement in Fig. 1.19 is realized. This method greatly improves the measurement accuracy of high frequency AC voltage. In the specific implementation, when the hardware block diagram is almost unchanged, the change of the frequency relationship between the clock signal and the measured signal and the change of the software processing method will ensure the measurement function and measurement precision of the system in a wide frequency range. The generation of a large number of wide-range precision AC voltage signals often comes from a synthetic signal generator or DDS. The traditional method only considers that the clock signal is simply a large multiple of the frequency of the measured signal, so that more voltage-digital values can be collected in one cycle time of the measured signal, and the relevant information of the AC voltage can be accurately obtained from it. In order to obtain more collection points, the upper frequency limit of the high frequency that the clock signal can multiply, restricts the application of this method.

The new method actually acts as an equivalent multiplication clock sampling effect, using a lower frequency, compared with the frequency fraction of the measured signal, the difference-related clock realizes the waveform-voltage (phase) sweep of the measured AC signal greatly improving the frequency response of the rms measurement of high-frequency voltage.

The frequency difference selection based on the fractional relationship here is to consider the number of phase data collections of the measured signal that can be accommodated in the obtained "beat period" or the least common multiple period. For example, in the case of using 8-bit AD, if you want to be able to collect 256 different phase difference data for the 10MHz signal under test, it is very difficult for the traditional method to require the clock frequency to reach 2560 MHz. Especially for high-precision ADs such as 20-bit, the gap is too large. According to the sampling and processing methods described in this paper, a sufficient number of samples can be obtained at a lower frequency clock. For example, when the clock frequency has a multiple relationship with the measured signal frequency, there are:

$$\text{Number of samples} = T_{\text{minc}}/t, \qquad (1.12)$$

wheret is the effective period of clock sampling. In order to facilitate signal generation and have more sampling data, and when the number of bits of AD converter is small, the jump edge information can be selected from each quantization step, the proportion of formula (1.12) can be appropriately selected to be larger.

According to the definition of the effective value of sinusoidal AC signal:

$$\text{AC instantaneous voltage:}\, u(t) = A\sin(\omega t + \varphi), \qquad (1.13)$$

$$\text{Voltage RMS } V_{RMS} = \frac{1}{T}\int_0^T u(t)^2\, dt = \frac{A}{\sqrt{2}} \qquad (1.14)$$

Formula (1.14) shows that for the calculation of the rms value of high-frequency AC voltage, it can be calculated by integrating the collected digital/voltage over time data, or it can be derived from the measurement or calculation of the voltage peak [20]. The processing cycle here has become T_{minc}, not the original signal cycle.

This experiment verifies in principle that digital voltage measurement at high frequency can be achieved and can obtain high measurement accuracy. In-depth research in this direction can comprehensively solve the problem of precision measurement of digital high-frequency voltage.

As shown in Table 1.3, it can be seen from the comparison of the measurement results of the 16-bit AD converter and the measurement results of other methods. That the high frequency range of the digitally extended signal under test is compared with the traditional method, based on the sampling of the clock signal with the frequency difference, combined with the digital quantization hopping edge effect, the obtained high frequency response characteristic is the best. Although the AD converter with more than 20 bits has relatively high precision, it can only work under the condition of DC or low frequency. Direct use of high-speed, low-digit AD converters still cannot meet high-frequency, high-precision AC voltage measurement [20, 6]. The table also shows the measurement results of the 8-bit AD converter at 100 MHz clock.

Therefore, by using the voltage acquisition method of the beat or the least common multiple period, the phase step method, and the extended signal waveform, the high-precision measurement result of the AC voltage can be obtained at the highest possible frequency. This is a basic experiment of this principle. However, with a fixed frequency clock signal, some discontinuities will appear in the measurement. Such as those whose frequency is the same as that of the clock signal, or the signal that presents the relationship between fractional or multiple frequencies. Therefore, in practical applications, a DDS can be added, and according to the frequency of the signal under test, the output clock signal that satisfies the relationship of division and multiple with the signal under test and has a slight frequency difference.

Table 1.3. Precision comparison of digitized voltage rms measurements forspecialized and general methods.

Input signal frequency/ Hz	6.5 digit multimeterGDM 8261A (21bits)/mV	Edge Spread Measurements for ADC Clock Tracking	8-bit ADC direct measurement/mV
1 M	321.9354	400.4	487
2 M	289.3489	400.9	489
3 M	268.7763	400.4	492.5
4 M	301.2725	400.5	494.5
5 M	304.976	400.7	491.5
6 M	269.7022	400.4	495.5
7 M	15.26574	400.2	496
8 M	9.236222	400.1	496
9 M	2.506652	400.8	496
10 M	0.316154	400.4	491.5
20 M	0.022582	400.9	482
50 M	0.022582	401.1	441.5

The measured AC voltage signal used in our experiment mainly comes from a high-stability crystal oscillator and is generated by a synthetic signal generator. This is in line with the high-frequency signals with good stability indicators used in most laboratories and the signals formed by DDS [6]. AC voltages that can achieve and need higher RMS measurement precision indicators often belong to this category.

However, for those high-frequency voltage signals with poor stability, the multiplication of noise will affect the reconstruction precision of the signal. This is to further expand the application scope of the method, which must be considered in the noise processing.

1.3.2.4. Summary

The large expansion to the high frequency direction is one of the directions that digitalization must develop. The precise measurement of high-frequency voltage signals is often limited by the quantization error and sampling rate, and the periodic phase change characteristics between signals are used, and the waveform of the signal itself is recovered by regular phase step sampling of multiple cycles, which is to improve the digital AC voltage efficient way to measure range.The idea of signal reconstruction here is to recover the original signal through multi period measurement data through specific frequency relationship, so as to obtain the complete information of the signal when the sampling rate is not high, the sampling points of each signal period are limited, or even there is only one or less than one. This includes methodological updates and changes in software processing. The key to this method is to ensure the regularity of the phase sampling of the clock signal relative to the measured signal [24]. The simplest is to ensure the phase step when acquiring the waveform voltage [2, 3].Make full use of the interval between periodic signals, taking the least common multiple period as the periodic characteristic of the phase change between signals, only a high stability crystal oscillator with a certain difference from the standard nominal frequency can realize high-precision measurement of a large number of high-frequency voltage signals with standard frequency values in a wide frequency range. It should be noted that this method also partially refers to the beat period of time-frequency measurement or its phase analysis method in principle. This method with multiplication effect is to expand the influence of noise in time-frequency parameter measurement to facilitate processing in stability measurement, but in precision voltage measurement, for signals with poor noise indicators, real-time sampling of noise should be added [22] and reasonable corrections are made in the final processing results.

1.3.3. Phase Phenomenon in the Measurement of the Magnitude of Linear Scale Represented by AD Converters

In the previous discussion, we mainly take the advantages of the phase processing method to extend the time-frequency processing method to the precise measurement of other periodic signals and achieved obvious results. In the further study, the focus will be put on the application of a wider range of aperiodic signals. The most typical example here is the application of the time-frequency processing method on the magnitude of the linear scale, which can be compared to the special phase phenomenon of ‘phase coincidence phenomenon’ in frequency measurement. By comparison, the periodic signal is a repetitive and cyclic change phenomenon in the perspective of the phase relationship, while the signal of the linear scale is uniformly stepped in the extension of a certain direction.

The vernier method in length measurement can be further considered here, but the vernier method takes two different length scales to measure a measured scale [25, 26]. From the perspective of phase, the relevant explanation of the vernier method is: for the typical value of the linear change scale, relying on two rulers of different scales, we could get a resolution in the length measurement that is significantly higher than the smallest interval of the length scale can be achieved through the coincidence between the scale lines - "phase coincidence phenomenon". The difference level of the coincidence condition of the tick marks itself shows a higher precision.

In the measurement shown in Fig. 1.21, if the measured length corresponds exactly to the position from 0 to 10 of the right sub-ruler, then the error of the obtained 99 mm measurement result should not be 1 mm, but 1/100 mm.In the case of measuring speed, if we can know the high-precision length and position information of the coincident points, and can know the moment values when these corresponding positions occur. Then by formula

$$V = \frac{X2-X1}{t2-t1} \tag{1.15}$$

In the result of calculating the velocity V, the quantization error of the scale in the measuring device can be largely suppressed.

On the other hand, it should also be possible to use a standard length ruler and a very stable length reference to determine the precision and stability of another length ruler. As is shown in Fig. 1.21 [25], if one of the two rulers is not particularly stable, it will affect the position of the coincidence point of the latter two rulers, either forward or backward, so that we can get our desired result, that is the position of the coincident point close to the right in the figure, which will change as one of that not particularly stable length scales changes.

Fig. 1.21. Schematic diagram of vernier caliper with large difference.

The combination of the principle and method with the phase concept can be verified in a wider range of digital applications.

For an actual digital quantization system, the quantization bit is N. When the reference voltage is V_{REF}, the quantization resolution (size of dynamic blur area) is expressed as:

$$LSB = \frac{1}{2^N} V_{REF} \tag{1.16}$$

Maximum error of measurement V^* is:

$$V^* = \frac{1}{2} LSB = \frac{1}{2}\left(\frac{1}{2^N} V_{REF} \right) \tag{1.17}$$

As shown in Fig. 1.21, fine tuning the reference voltage of the A/D converter can adjust its quantization value, and the fine tuning of the reference voltage can make the two A/D converters show higher precision in the solution of further measurement derived quantity just like the two scales in the length vernier method.

In the experimental work of frequency measurement and its improvement, phase coincidence provides a good research foundation for the development of digital technology. The phase coincidence between frequency signals provides multi period strict synchronization with each other; moreover, in the process of digital processing, the quantization space between the coincidence edges of two A/D conversions provides the corresponding background for further high-precision measurement.

The voltage change between the quantization jump edges of AD conversion with two different quantization values is strictly equal to the quantization value of digital conversion, and the determination of coincidence fitting time forms an accurate description of the voltage change rate during this period - it can derive the waveform parameters of the signal, the phase change rate related to the change of linear phase difference, frequency and frequency stability, etc.

As for the application of the measurement method associated with the phase concept of digital A/D converter, firstly, the two A/D converters make a certain difference in their quantization values by adjusting the reference voltage of one of them, which constitutes the possible cursor conditions between them. At present, in the aperiodic signal measurement of linear scale physical quantities such as A/D converter, our first application example is to test the health and instability of the tested A/D converter through vernier experiment. For convenience, we choose stable low-noise and low-frequency sinusoidal or triangular wave signal as the comparison measurement source of two A/D converters (Fig. 1.22). In this way, in a sampling area where the measured signal rises or falls in one cycle, one or several complete cursor measurement intervals as shown in Fig. 1.21 can be selected from the data collected by the two A/D converters, that is, to capture the coincidence signal of the digital cursor, including the value of the same point of voltage collected by the two A/D converters and the time of occurrence. In practice, there can be multiple combinations of such cursor intervals. With the periodic change of the measured signal, we have the opportunity to measure such a specific and complete cursor interval. What we are concerned about is that after collecting a large amount of interval data, we can obtain the health status and instability of the tested A/D converter.

Since the analog quantity is quantized in the characteristics of signal conversion in digital measurement, it will show the same quantized value within a certain range of analog value. Since there are different analog change values within the scope covered by the quantitative value, even if it is digitized, the errors displayed at the time of occurrence are completely different, such as measurement under dynamic conditions (indirect measurement), expressing the characteristics, conditions and objects of indirect measurement, etc.

Fig. 1.22 is a block diagram of the health status and instability measurement of the A/D converter we use. The quantization values of the two A/D converters are slightly different. A series of cursor coincidence points can be obtained during the use of dual A/D converter with small difference in quantization value. The voltage information of these coincidence points can give the resolution of sampling precision much higher than the quantization value of A/D converter. Therefore, we can get points with higher relative precision from the collected information. FPGA receives the output of dual A/D converter and records the time of data occurrence. MCU compares the output data of two A/D converters, captures the coincidence signal of the cursor, including the value of the same voltage point and the time of occurrence during coincidence, and calculates the time interval between two adjacent and multiple coincidence points. By repeating this operation, a large number of interval values and the selected cursor detection area can be obtained. According to a series of repeated fluctuations of the acquisition voltage at both ends of the vernier detection area as shown in Fig. 1.23, the fluctuation change of the tested A/D converter can be obtained, and the results of other quantities can be derived from the data of the tested A/D converter.

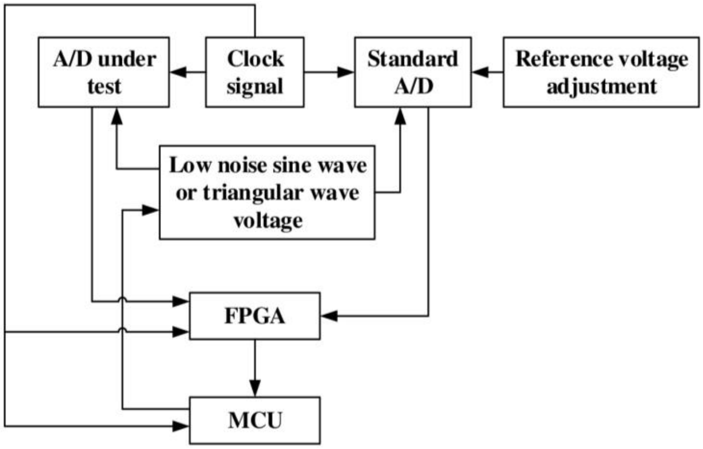


Fig. 1.22. Block diagram of health status and instability measurement of A/D converter.

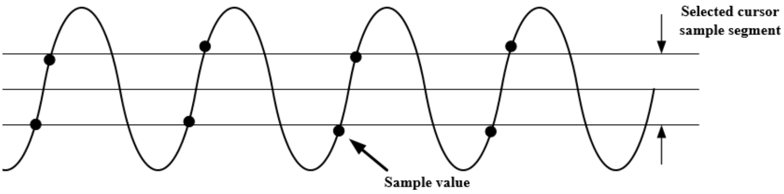


Fig. 1.23. Waveform diagram of working process.

The specific performance is shown in Fig. 1.23. At the coincidence point of the vernier device, the two A/D converters should be very close to the same voltage transitions at the same time under the action of the clock. This is also the "synchronization" of the transition edges of the two A/D converters and the analog/digital fitting edges. This method is widely used, for example, it can only fine-tune the reference voltage of the stable A/D converter, check the measured signal, and check whether the stability of the A/D converter is within 1 % of its quantized value. The vernier measurement method using dual AD converters with slight differences can obtain high measurement precision. But just like the frequency counters for phase coincidence detection we studied earlier, this approach will sacrifice the response time of the measurement.

An interesting and important experimental result that we found is that as the number of bits of the A/D converter increases, the basic stability/quantization value does not stay the same, but decreases. This happens to indicate the basic transition border stability index of an A/D converter. Even if the high-bit A/D converter chooses a better base chip, with more bits converted, the ratio of A/D converter stability to this quantization resolution may also be degraded.

The current disadvantage of this method is that as the interval between the coincident points of the vernier is larger, the multiplication factor of the vernier will be larger, the space required for high-precision measurement is correspondingly larger [26], and the required voltage variation range is larger. Furthermore, in terms of the number of digits of the A/D converter, if the method of vernier measurement of dual A/D converters whose precision is improved by 100 times is adopted, a step space of 100 digits is occupied. For a 10-bit A/D converter, it occupies almost 1/10 of its operating range. This must be considered.

Table 1.4 shows the corresponding relationship between the period and phase of time-frequency magnitude and the quantization and quantization step of AD conversion, which is more widely used in digitization. That is to say, the results obtained in the research of phase problems can be used for reference in the extensive digital measurement.

Table 1.4. Relationship between period, phase and AD conversion quantization and quantization step.

	Signal change unit	**Coarse impact**	**Phase relationship**	**Phase coincidence**
Frequency signal	Period	+/-1period	For 1 period further scale	Fitting of specific phase conditions with intervals
Digital quantization	Quantitative scale	1 quantization step	Existing Quantitative Unit Limits difficult to distinguish directly	Quantitatively converts the fit of digital to analog measurands
	Measurement Resolution Ambiguity Region	Blur area edge	Special Phase Phenomenon Edge Resolution	
Frequency signal	Resolution of the phase coincidence detection circuit	Phase Coincidence Detection of Ambiguous Region Edges	Stability of coincidence detection resolution	
Digital quantization	Close to quantitative scale	Transition edge of AD conversion	Increase by the phase shift multiplication factor	

1.4. A Personal Computer Instrument System

We have designed a personal computer instrument system based on the content introduced earlier in this chapter and a personal computer [21]. Give full play to all the hardware and software processing capacity and resources of the personal computer, and all functions of the system are completed under the combined action of the external small accessory box and copy software package. Fig. 1.24 is the block diagram of this system. Figs. 1.25 and 1.26 show some functional interfaces and overall appearance photos of the personal computer instrument system.

The instrument system has a single instrument enclosure configuration (called an instrument section),which also has an enclosure, faceplate, and connecting wires. The connection line includes the input of the measured signal, the separate power supply, the interface connection with the computer, the selection connection and socket of the external reference source, etc. For the function conversion of the system, it can be carried out from the switch of this instrument part, or it can be operated with the mouse from the interface on the computer screen. (with the necessary options, there may be separate display numbers, etc.).

Although the operation of specific parameter measurement of the block diagram of this system is not exactly the same (the difference in details), constitutes the main measurement function of this system.

This system block diagram is superficially (especially from the perspective of some waveforms) similar to the beat period or beat phase analysis method in the frequency standard comparison [4, 21]. But there is no mixing link and mixing effect. In the case of digitization, there is a difference between the clock and the measured signal or in the case of fractional or multiple, which results in the phase stepping of the signals between each other according to the period of the clock or the signal. A temporal, equivalent phase shift and processing result is thus produced. This is a guarantee of high precision in processing

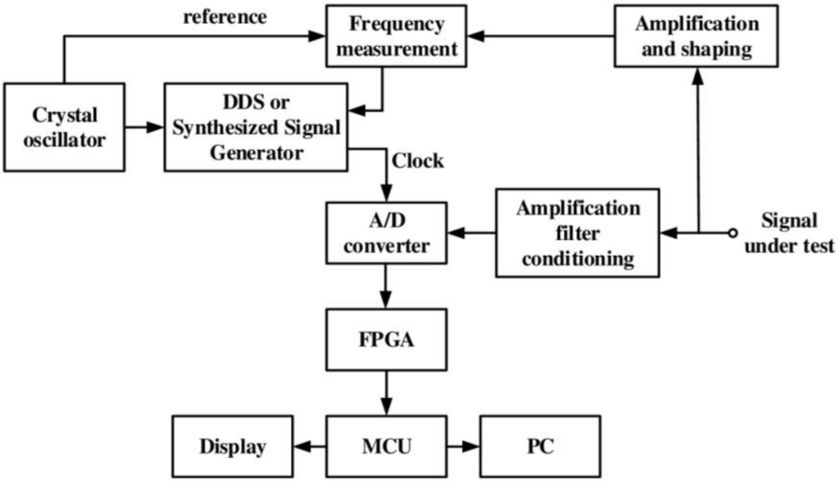
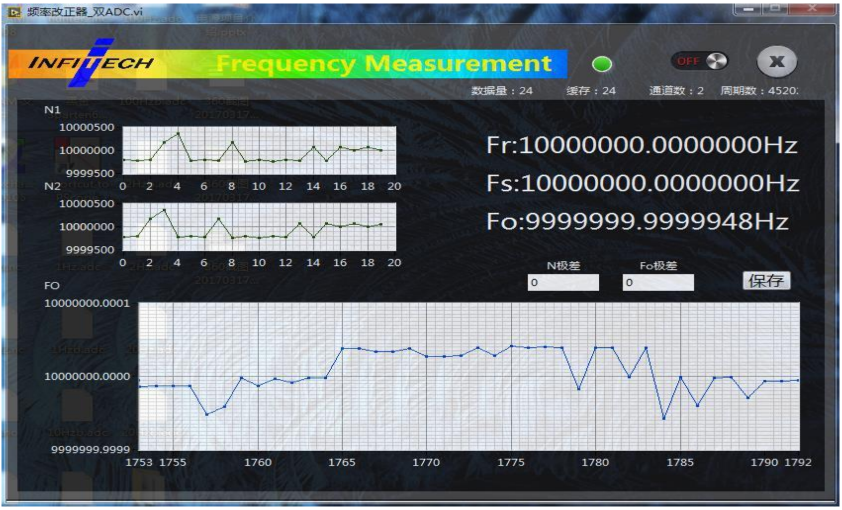


Fig. 1.24. Block diagram of instrument system.



数据量：24 缓存：24 通道数：2 周期数：4520:

Fr:10000000.0000000Hz

Fs:10000000.0000000Hz

Fo:9999999.9999948Hz

N极差 0

Fo极差 0

.

Fig. 1.25. Some functional interfaces of the instrument system.

Fig. 1.26. Overall picture of personal computer instrument system.

This personal instrument system can use one of its computer accessories to realize the measurement of more than 10 most commonly used instruments, and ensure that the precision of many functions is much higher than that of traditional instruments.

In general, the instrument system adopts a targeted and high-resolution measuring instrument system whose output signal frequency of DDS changes with the measured signal. So DDS and AD converter are the most critical components of this instrument. To further improve and ensure the system precision, there must also be such two parts of high level and high quality.

1.4.1. Functions of the Personal Computer Instrument System

(1) Digital sampling oscilloscope function: display periodic signal and aperiodic signal waveform, and mainly periodic signal. The measurement range is from DC to 500MHz range. And can visually display the noise condition of the measured signal through the setting change of the clock sampling frequency. The latter is an irreplaceable function compared to traditional oscilloscopes.

On the one hand, in this system, a low-frequency clock signal can be used to obtain the effect of AC voltage measurement at a high frequency. This accommodates the situation where the frequency of the clock DDS cannot be very high. On the other hand, the adjustment of the small frequency difference term in the clock is beneficial to the adjustment of measurement precision and measurement response time.

(2) AC and DC voltage measurement function: It is characterized by AC voltage measurement (such as peak value, average value, effective value, etc.) and high-frequency voltage measurement.

(3) Time-frequency parameter measurement function: Tomeasurefrequency, frequency stability, phase change, time interval, phase noise, medium and long-term frequency drift, etc. High-resolution time interval measurements are an option for instrument upgrades.

(4) (Instrument option) frequency stability index and single-sideband phase noise measurement of the corresponding high-precision time-frequency domain measurement. By converting from time domain to frequency domain, the equipment can display the SSB phase noise as easily as possible.

(5) Other periodic signal parameters: such as sampling measurement of the phase change rate of non-sinusoidal signals, etc.

(6) Sensor signal measurement and display: the electrical signals converted from different sensors are sent to the digital processing part of the instrument for digital processing. It can analyze the sensor output signal with high precision and give its various derived quantities.

(7) Pulse parameter sampling measurement capability: Sampling and measurement for the rising edge, falling edge, overshoot, etc. of periodic pulse signals. Endpoints for sampling can be set.

(8) Increase the measurement functions of the digital multimeter: such as current, resistance, etc.

(9) High-precision signal generator for DDS generation and output: The system is equipped with 2 DDSs, one of which is used for voltage measurement and is also a variable clock sampling signal of a digital oscilloscope; the other is dedicated to generating and outputting frequency/voltage signal.

(10) The learning function of the measurement method: that is, the cooperation between each module in the system and the processing center can have the cooperation of different processing methods. In this system, there are settings of direct measurement method and indirect measurement method. For a large number of derived quantities in the measurement, the indirect measurement method can be selected for measurement.

(11) Can form the function of the measurement system: for example, the scanning output of DDS passes through the measured device or network, and then combined with the oscilloscope and AC voltage measurement function of the system, the frequency characteristics of the measured device can be detected.

1.4.2. Key Components of the System

(1) Computer (laptop computer).

(2) Clock system: It includes frequency measurement for the signal under test, and a low-noise DDS clock generator is attached to track the signal under test.

Because this is a digital computer instrument system, the clock part is a very important core component. On the one hand, this clock is a reference or standard for a large number of measurement objects (such as frequency, voltage, etc.), so its reference source plus DDS conversion should be stable and accurate. On the other hand, the frequency value of the clock signal is adjusted or controlled according to the requirements of the measurement purpose, object, accuracy, and response time. These controls are all carried out by the computer or the microcontroller in the system.

The clock system of the system is the reference frequency standard of the system. The reference frequency standard usually adopts a high-stability crystal oscillator, and can also be converted into an externally selected rubidium atomic frequency standard.

(3) Waveform display and generation module.

(4) Time-frequency parameter processing module.

(5) Signal parameter processing module such as AC voltage.

(6) Separate DDS signal generator module.

(7) Sensor signal processing and measurement modules combined with sensors.

(8) The work of the usual measurement purpose realized from the functional aspect, such as the common measurement target that the accuracy of DC and low frequency is not high.

(9) The software package of the personal instrument system provided to the computer, which has the functions of the instrument system only after being implanted in the computer. The software package includes LabVIEW processing software (laboratory virtual instrument engineering platform) as the host computer.

(10) The frequency values of DDS in various measurement functions are obviously different. This is also a distinct feature of this personal instrumentation system. In many cases, it is used as the clock signal for the AD converter in the instrument. In the oscilloscope function and AC voltage measurement, its frequency is the division or multiple of the measured signal frequency and has a small frequency difference (such as 100 Hz, 1 kHz, etc.). In the frequency measurement of most frequency signals, the frequency of the DDS output signal often has a smaller frequency difference (such as 10 Hz) when it is equal to the frequency of the measured signal. In the linear phase comparison, the frequency of the DDS output signal should be a multiple (such as 10 times) of the frequency of the measured signal.

(11) The analog signal processing module at the front end of the instrument.

1.4.3. Features

(1) High precision (but because it is a measuring instrument, because it is limited by the corresponding system, the precision given by text or surface will be lower than the actual situation). For example, the precision of time-frequency parameter measurement can reach the order of picoseconds; the measurement accuracy of the rms value of high-frequency AC voltage can reach 0.3 %.

(2) High-frequency processing capability. It does not rely on the higher frequency of the clock signal, but the division, multiple frequency and a certain frequency difference between the clock signal and the measured voltage signal. The measurement upper limit of high-precision digital high-frequency voltage value can reach higher than 500 MHz.

(3) Fast response time.

(4) Diversified functions.

(5) It adopts its own original measurement method in each measurement function: for example, in the waveform display and AC voltage measurement, the DDS clock is used to track the measured signal to form the division and multiple of the frequency of the measured signal and there is a certain frequency difference. The method of phase step sampling and waveform reconstruction is realized; the method similar to the DDS tracking method above is also used in the measurement of periodic signal parameters such as frequency, and the synchronization gate formed by the phase coincidence detection between the signals eliminates +/- 1 counting measurement. In the linear phase comparison, the multiple relationship between the clock and the measured signal is used, so that the sampling of the linear region of the measured sinusoidal signal can be guaranteed, etc.

(6) The combination and optimization ability of measurement methods is convenient to obtain the best measurement effect on the basis of existing resources.

(7) The ability of self-organizing measurement system: use various modules within the system to form different measurement systems. For example, the signal generator module, the oscilloscope and the AC voltage measurement module can complete the measurement and

description of the frequency response and frequency characteristics of the network and the device.

1.4.4. Positioning of the Instrument

This instrument is suitable for all kinds of people with personal computers, and can meet the measurement requirements of different industries at a low price of several thousand yuan.

From the measurement precision of the instrument itself, it should belong to precision measurement instrument. However, when this multi parameter precision instrument is considered according to the measurement standard, there will be many management problems in the actual operation. Therefore, for the instruments sold in large quantities, it is still considered as non standard measuring instruments to reduce the accuracy of sales. If the batch is not a large part of the instrument, it shall be considered according to the standard measuring instrument. The measurement department shall issue the verification certificate and give the validity period of use.

Instruments can be divided into special personal instrument systems and civil systems, and there can also be instrument systems that combine learning and practical applications for students.

In addition, the personal instrument system can also be used as a research platform for measurement methods for different measurement purposes, and as a software and hardware basis for the development of different measurement technologies.

1.5. Conclusion

The phase question is usually posed and applied to frequency as well as periodic signals. Due to the high precision of phase processing in frequency signal processing and the convenience it has shown in recent years, on the one hand, we deeply discuss the method of digital phase processing, especially the research on digital linear phase comparison method [1, 8, 18]; One aspect is to try to apply the concept and processing method of phase and the border effect found in it to the quantization process using more widely digitization [27, 21]. For the former, the phase relationship and variation law between periodic signals have been further revealed and understood. For the latter, it is groundbreaking in the field of digital application.

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