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Research in the Operational Modal Analysis of Rotating Machinery Reducer

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Abstract: In operational modal analysis process of the rotating machinery, it is always difficult to adopt usual modal analysis methods in parameter correction because of the modal parameter identification problems caused by harmonic modes. A technique to distinguish harmonic and structural modes is proposed here, that is, through frequency domain decomposition (FDD) of the vibration signals power spectral density matrix and probability density function, the two components (structural modes and harmonic modes) are successfully separated from the overall responses. The experiment indicates that structural modal parameters of the reducer can be obtained in real environment, which provides accurate references to the continuous mining machine reducer parameter correction. The method provides an efficient path for the modal analysis of rotating machinery. *Copyright © 2013 IFSA.*

Keywords: The continuous mining machine reducer, Operational mode, Frequency domain decomposition, Modal parameter identification, Probability density function.

1. Introduction

After the physical prototype of continuous coal mining machine gear reducer is manufactured, there are always problems exist in its operation because of design defects, machining errors, and so on, as might cause even more serious faults, hence, design parameters correction is needed. Inherent characteristics of the equipment can be better accessed to by modal analysis method. Modal analysis method developed in recent years can simulate real environment to analyze the modal parameter of the equipments, which provides accurate references for parameter correction and design optimization of the prototype machinery.

Operational Modal Analysis (Operational Modal Analysis short for OMA) is a technology that extracts the model parameters based on the response signal [1]. Compared with traditional method of Experimental Modal Analysis [2], OMA does not need special incentive devices, and it is especially suitable for large equipments in operation because it does not affect the normal work; as can not only satisfy the actual boundary conditions, but also obtain complete modal parameters [3]. For rotating machineries like internal combustion engines, gas turbines, reducers, and so forth, operational modal parameters obtained is more practical than those under artificial incentive work conditions [4].

The premise of Operational Modal Analysis is that the equipments are excited to white noise excitation (random excitation). However, inherent and harmonic modals (pseudo modal) [5] exist in the analysis results because of the simultaneous existing of random and cycle incentives in the operation of rotating machineries, which causes difficulties in obtaining modal parameters; hence, this method is seldom adopted in rotating machinery.

However people are still attempting to seek proper methods to apply work modal because of its inherent advantages. Rune Brincker (Denmark) extracted operational modal through stochastic subspace method on motor engines and diesel engines at work, in which, optimized parameters were obtained. [6-9]. Zheng Min (China) did operational modal tests on a model plane by using cross-correlated complex exponential methods [10], and the results obtained are in accordance with those through the optimal tuning vibration method. Dong Shu Wei (China) filtered harmonic signals and identified harmonic components in response signals by using improved algorithm of least squares complex frequency domain, some achievements were realized, yet, this method is not applicable in practical engineering because of its calculation complexity [11].

In view of the above, this paper proposes a new method to identify operational modals, that is, combining frequency domain decomposition algorithm and probability density statistics together to decomposition and extract singular value in the power spectral density matrix of vibration signal obtained by operational modal tests. Through the statistical features of the probability density curves of the signals, the inherent and harmonic modals can be identified. This can be applied in the work modal analysis of continuous mining machine reducers to extract the two different modals successfully; as can establish a better path and references for the obtaining of real inherent modal parameters of rotating machineries.

2. Frequency Domain Decomposition Theory and Solution of the Modal Parameters

Frequency Domain Decomposition (short for FDD) is one method that decomposition singular value (short for SVD) [12-14] in the response power spectral density matrix of discrete frequency.

In a modal analysis, the relationship between output response signals and input excitation signals could be expressed as equation (1):

$$G_{yy}(j\omega) = \overline{H}(j\omega) G_{xx}(j\omega) H(j\omega)^T, \tag{1}$$

where $G_xx(j\omega)$ and $G_{yy}(j\omega)$ are the power spectral density matrix of input and output respectively in the form; $H(j\omega)$ is the frequency response matrix, symbol "—" and "T" stand for complex conjugation and transposition.

When the input is white noise excitation, $G_{xx}(j\omega) = C$ is the constant, set the output response signals as equation (2):

$$\{y(t)\} = \{y_1(t), y_2(t), \cdots, y_n(t)\}^T \tag{2}$$

Then the output response power spectral density matrix could be expressed as equation (3):

$$G_{yy}(j\omega) = \begin{bmatrix} G_{11}(j\omega) & G_{12}(j\omega) & \vdots & G_{1n}(j\omega) \\ G_{21}(j\omega) & G_{22}(j\omega) & \vdots & G_{2n}(j\omega) \\ \cdots & \cdots & \vdots & \cdots \\ G_{n1}(j\omega) & G_{n2}(j\omega) & \vdots & G_{nn}(j\omega) \end{bmatrix} \tag{3}$$

Diagonal element in the matrix of $G_{yy}(j\omega)$ is auto spectrum of single measuring point signal, other elements is cross spectrum of different measuring points.

Modal parameters are obtained based on frequency response function. Modal parameters of the frequency response function are usually in the form of a residue and poles, as equation (4) below:

$$H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \frac{\overline{R}_k}{j\omega - \overline{\lambda}_k} \tag{4}$$

$\lambda_k = -\sigma_k + j\omega_{dk}$ is the pole of the frequency response function in form above; $R_k = \psi_k \gamma_k^T$ is the residue of frequency response function matrix; n is the number of system modes; σ_k is the first order modal attenuation coefficient k; ω_{dk} is the damped natural frequency; ψ_k and γ_k are the modal vibration shape vectors and modal participation factor respectively.

Bring equation (4) into equation (1), then equation (1) could be written in a minimum fraction form as equation (5) below:

$$G_{yy}(j\omega) = \sum_{k=1}^{m} \left(\frac{A_k}{j\omega - \lambda_k} + \frac{\overline{A}_k}{j\omega - \overline{\lambda}_k} + \frac{B_k}{-j\omega - \lambda_k} + \frac{\overline{B}_k}{j\omega - \overline{\lambda}_k} \right) \tag{5}$$

$A_k = \dfrac{R_k C R_k^H}{2\sigma_k}$ is the k order residue matrix response function in the form, B_k is the conjugate of transpose of A_k. When the damping is small, the residue matrix of response function is proportional to the system modal vector, which could be expressed as equation (6):

$$\lim_{damping \to light} A_k = \psi_k \gamma_k^T C \gamma_k \psi_k^T = d_k \psi_k \psi_k^T \tag{6}$$

Among which d_k is proportional constant.

In a certain affirmative frequency, only one order or two orders modal poses an important influence on the response of system usually. We write these modal as sub(ω). So the response spectrum matrix can be written as the final form, shown by equation (7):

$$G_{yy}(j\omega) = \sum_{k\in sub(\omega)} \left(\frac{d_k \psi_k \psi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\psi}_k \psi_k^T}{j\omega - \bar{\lambda}_k}\right) \qquad (7)$$

To extract the modal parameters, singular value is decomposition (SVD) in response spectrum matrix of equation (7). Then equation (8) is gotten as below:

$$G_{yy}(j\omega) = \Phi\Sigma\Phi^H \qquad (8)$$

In the form above $\Phi\Phi^H = I$, Φ is shown as equation (9), I is unit matrix.

$$\Phi = \left[\{\phi_1\}\{\phi_2\}\{\phi_3\}\cdots\{\phi_n\}\right] \qquad (9)$$

The decomposed singular value Σ stands for a series of single freedom system auto power spectrum density function. Singular value curve peak corresponds to the first order modal frequency of the K. The corresponding singular vector ϕ_1 decomposed is the modal vibration mode of estimation. The time domain signal is gotten by the function of single freedom degree of spectral density being transformed of inverse Fourier transform. The rest modal parameters of response could be obtained through its autocorrelation function.

3. The Statistic Characteristics of Random Response and Harmonic Response

For rotating machinery, under circumstances of coexistence of random excitation and harmonic excitation, the probability density function distribution form would not be changed after the signal is processed for filtering, sampling and etc, of which when the structure is inspired by Enough independent random load, the response of the structure could be regarded as a linear superposition of multiple excitations of random. Its probability density function obeys Gaussian distribution approximately, meaning a normal distribution. The normal distribution is displayed as the unimodal on the graphics. The curve of Theory Probability Density is shown as Fig. 1 (a).

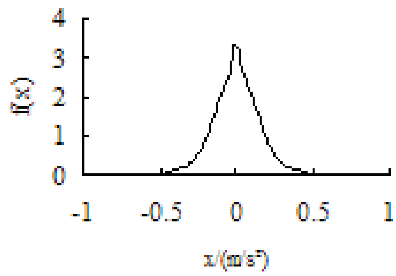
As for probability distribution of harmonic response (production after cycle motivation), assuming that two functions of probability density of random variables X and Y are $f_X(x)$, $f_Y(y)$ respectively, even $x = h(y)$, then form (10) was established.

$$f_Y(y) = \begin{cases} f_x[h(y)]h'(y) & \alpha < y < \beta \\ 0 & \end{cases} \qquad (10)$$

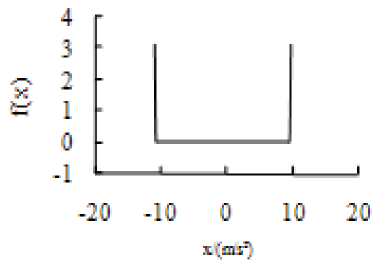
In the equation above, $h'(y)$ is the differential coefficient of $h(y)$; α, β are value ranges of y. Assuming that the simplest harmonic response is being considered firstly, $y = g(x) = a\sin(\omega x)$, take the probability density of x is constant value $f(x)$. The probability density function of harmonic response is expressed as equation (11).

$$\begin{aligned} f_Y(y) = f_X &= g^{-1}\left(\frac{1}{\omega}\arcsin\left(y/a\right)\right) \\ &= f_X \frac{1}{\omega a}\frac{1}{\sqrt{1-\left(\frac{y}{a}\right)^2}} \end{aligned} \qquad (11)$$

When $y \to \pm a$ in equation (11), $f_Y(y)$ is going to tend to infinity, there will be two different peaks in the probability density function curve of harmonic response, of which curve of the theory probability density is shown as Fig. 1 (b) [15].



(a) Random Response



(b) Harmonic Response

Fig. 1. Theoretical probability density curves.

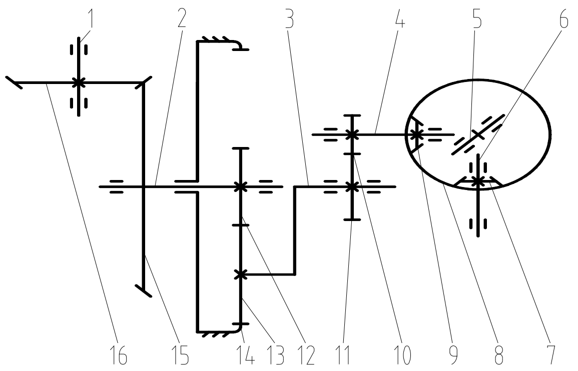
Based on the theoretical analysis above, this article is going to apply frequency decomposition technique combining with statistical property method into parameters identification of the modal analysis of some even mining machine reducer, which would distinguish each order modal of authenticity (inherent modal and harmonic modal) effectively.

4. Operational Modal Test and Parameter Identification

In the operating process of the gear reducer, Meshing point position and the number of teeth on the mesh change periodically in the process of gear transmission, cause that stress of gear change with stiffness cyclically, which makes the vibration of the reducer, contain both random vibration and vibration cycles. So at this time the modal parameters obtained include inherent modal and harmonic modal.

Some even mining machine speed reducer is a five class transmission reducer studied by this paper, of which structure is complicated, of which transmission diagram is shown as Fig. 2. After the completion of the reducer physical prototyping, there are a lot of problems, in order to get the real inherent modal parameters of the reducer entities, the operational modal test of the reducer has been done under different operational conditions in this paper.

The reducer is installed under the condition of simulation in the experiment to maximize boundary of simulation of the operational condition. A total of 72 sensors are arranged as the measuring points of response on the reducer in the experiment to meet the requirements of operation modal test. Distribution of measuring points set covers range of equipment transmission ratio, and is able to simulate the shape of equipment, shown as Fig. 3.



1. I Axis , 2. II Axis , 3. III Axis, 4. IV Axis, 5. V Axis, 6. VI Axis, 7. Gear 10, 8. Gear9, 9. Gear8, 10. Gear7, 11. Gear6, 12. Gear5, 13. Gear4, 14. Gear3, 15. Gear2, 16. Gear1.

Fig. 2. Transmission diagram of the reducer.

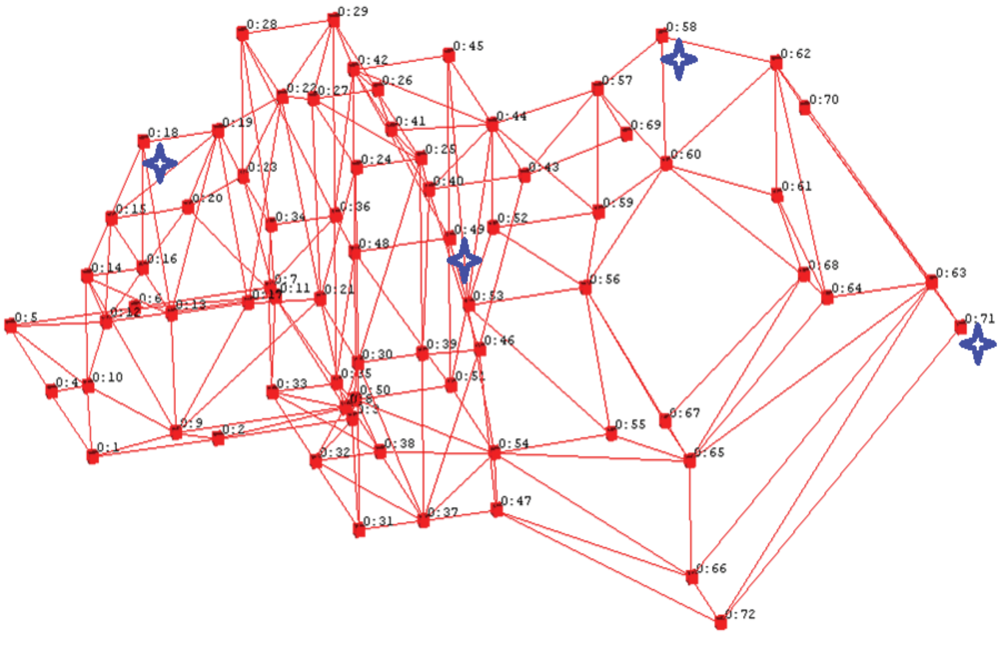


Fig. 3. The distribution of sensors.

Response signals are picked up by using American KITSLER three-way acceleration transducer. Vibration direction of the sensor must agree with the vibration direction of the system to be measured at this point when installing. Data collection is finished by using data collection analyzer. The test time of the test is set to 2 seconds. Sampling frequency is 8192 Hz. Frequency resolution is 4 Hz. The main purpose that the parameters above are set is to ensure that all the information would be picked up within the scope of the operation frequency of the equipment.

Take the vibration data of the measuring points of response into equation (3) to get the power spectral density matrix of the vibration data and then take which into equation (8) again to decomposition singular value for power spectral density matrix of measured data to get the singular value decomposition (SVD) curve. In order to make the singular value curves seen clearly, this article only chooses four curves to show (They are measuring points of 18, 49, 58, 71 of star location shown in Fig. 3, the vibration amplitude of the four points is large, and which is far away from the station, the spatial resolution of vibration mode is high.) shown as Fig. 4.

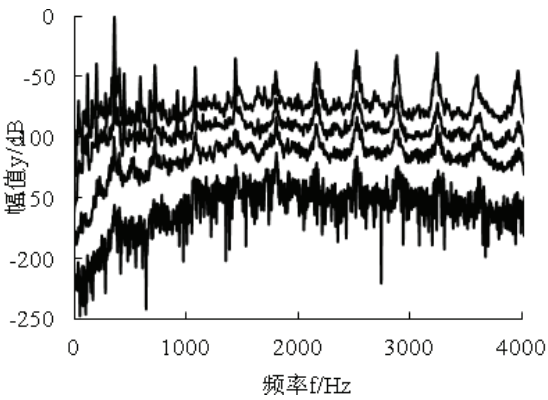


Fig. 4. Singular decomposition curves.

To distinguish between inherent modal and harmonic modal better, the Butterworth band-pass filter is designed in this paper, shown as Fig. 5. The frequency response curve which could be seen of the filter is the maximum flat within the pass-band, there are no ups and downs; the decay is slow; and which is enhancing continuously as the order of filter is increasing; of which amplitude attenuation speed of the resistance band is fast, which totally keeps the linearity and distortion of the filter. Take each peak frequency in Fig. 4 as the center frequency, half-power bandwidth as pass band width to take the response signal into Butterworth band-pass filter shown as Fig. 5 to filter. For the signal filtered are analyses by adopting the probability statistical characteristics analysis. The probability density curve of part of the modal frequency is shown in Fig. 6.

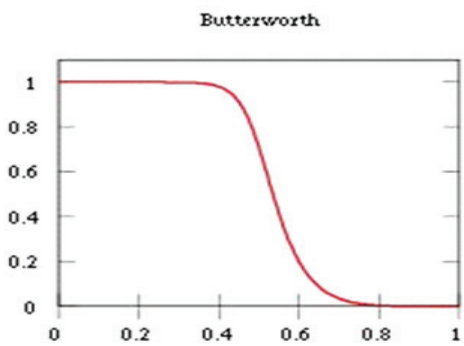
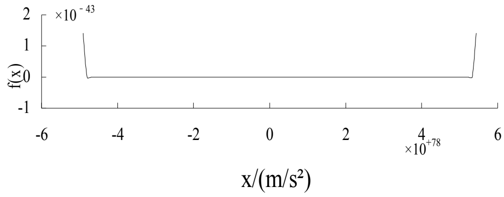
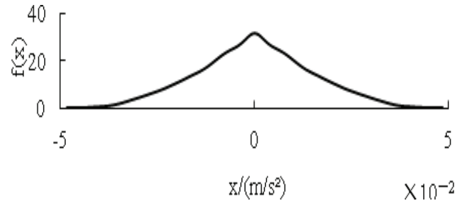


Fig. 5. Butterworth band-pass filter.

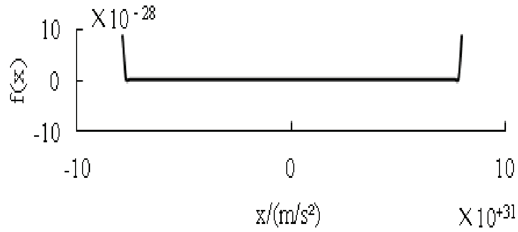
There are two peaks for modal probability distribution of 2 order modal, in the curves shown (Fig. 6 (a), Fig. 6 (c)), which could be seen from Fig. 6 and corresponds to the form of probability density curve harmonic response of inspiring the structure cyclically of the derivation above. Single peak exists in modal probability distribution of Fig. 6 (b), Fig. 6 (c), which approximates Gaussian distribution, could be confirmed as the form of probability density curve caused by the random response of structure caused by random incentive.



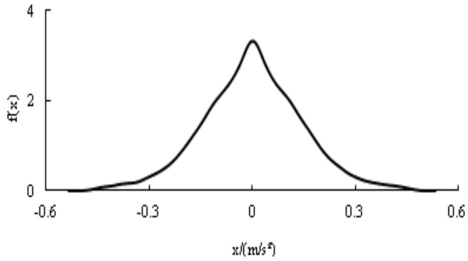
(a) 360 Hz.



(b) 448 Hz.



(c) 720 Hz.



(d) 980 Hz.

Fig. 6. Part of probability density curves of the random response and harmonic response.

On the other hand, the calculating formula for mesh frequency of the reducer gear is known as

$$f = \frac{z_1 n_1}{60} = \frac{z_2 n_2}{60}, \tag{12}$$

where in z_1, z_2 are the number of teeth of two gears respectively, n_1, n_2 are rotational speed of two gears respectively.

Equation (12) is applied to calculate transmission meshing frequency of the reducer at all levels. The first level 360 Hz, the second one is 303 Hz, the third level is 40 Hz, the fourth one is 40 Hz, and the fifth level is 40 Hz respectively. Periodical vibration is aroused when reducer gears are in the mesh per level. Therefore harmonic modal that is aroused by part of the periodical signal of system could be identified indirectly by gear-mesh frequency of this reducer per level.

It could be seen that two modal frequency 360 Hz and 720 Hz of the existence of double peak density curve obtained in Fig. 6 are just integer multiples of the mesh frequency of this reducer, which indicates that the two modal frequency is really harmonic modal produced by harmonic response that is inspired by cycle.

Singular value decomposition is done in power spectral density matrix gotten after calculating all points of measurement data and of which the probability density curve is solved. All harmonic modes could be identified that the reducer is inspired by harmonic excitation, and which is removed, and eventually real inherent modal parameters of eliminating harmonic modal of the reducer are gotten.

In order to verify the method above is effective further on, LMS Test. Lab commercial machine software is applied to analyses the operational modal of the reducer in this paper. (There is a certain authority for the commercial machine to analyses the modal, but is very expensive). Firstly a simplified model of entity of the same shape with the reducer is established in LMS software, then stationing of virtual sensors is done on the modal according to location coordinates of the same sensors in experiments above, in order to distribute vibration signal data of sensors to the corresponding virtual sensors in experiment. In this way, inherent modal parameters obtained from frequency domain decomposition technique and method studied in this paper could be compared with ones obtained from LMS Test. Lab commercial software. Situation being contrasted of inherent modal parameters obtained by two methods of identification of the first six orders shown by Table 1.

The data of comparison is shown from Table 1, inherent modal parameters obtained from commercial software LMS Test. Lab and inherent modal parameters obtained from frequency domain decomposition technique of which result is consistent. Because of Sampling frequency of FDD method of identification being restricted during the experiments (relation to the acquisition equipment), of which frequency resolution is 4 Hz. Thus, the results are integers; wherever the precision of LMS commercial software is high; the precision of resolution is also higher. The result of table 1 has proved the feasibility and effectiveness of frequency domain decomposition technique and path studied in this paper. Modal parameters obtained brings sufficient basis to correction of parameters of the reducer prototype.

Table. 1. Comparison between the results of frequency domain decomposition and LMS software.

Modal order	FDD (Hz)	LMS recognition (Hz)
1	338	336.6
2	448	447.1
3	536	542.5
4	739	741.4
5	921	911.8
6	980	979.9

5. Conclusions

1) After operational modal experiments is done on the mining machine reducer, frequency domain decomposition technique proposed in this paper is applied to calculate modal parameters to obtain significant peaks in spectrum, which provides sufficient and necessary conditions for the identification of modals in the next step.

2) Through filtering the signals at the frequency peak and analysis of the identifications of the probability density curve characteristics, the mathematical properties of each signal in inherent and harmonic modals can be identified, so as to separate inherent modals from the harmonic, which are both aroused by random and harmonic excitation coexisting in the structure. This can be applied in any equipment with the above mentioned conditions.

3) The research method provides a new path to identify modal parameter in operational modal analysis of machinery transmission systems, which establishes accurate basis for the correction of structural parameters. The same findings could be obtained without having to use expensive commercial machines in this process; hence, it is more credible and economical for any institute and individual researchers who will be engaged in the research of work modals.

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