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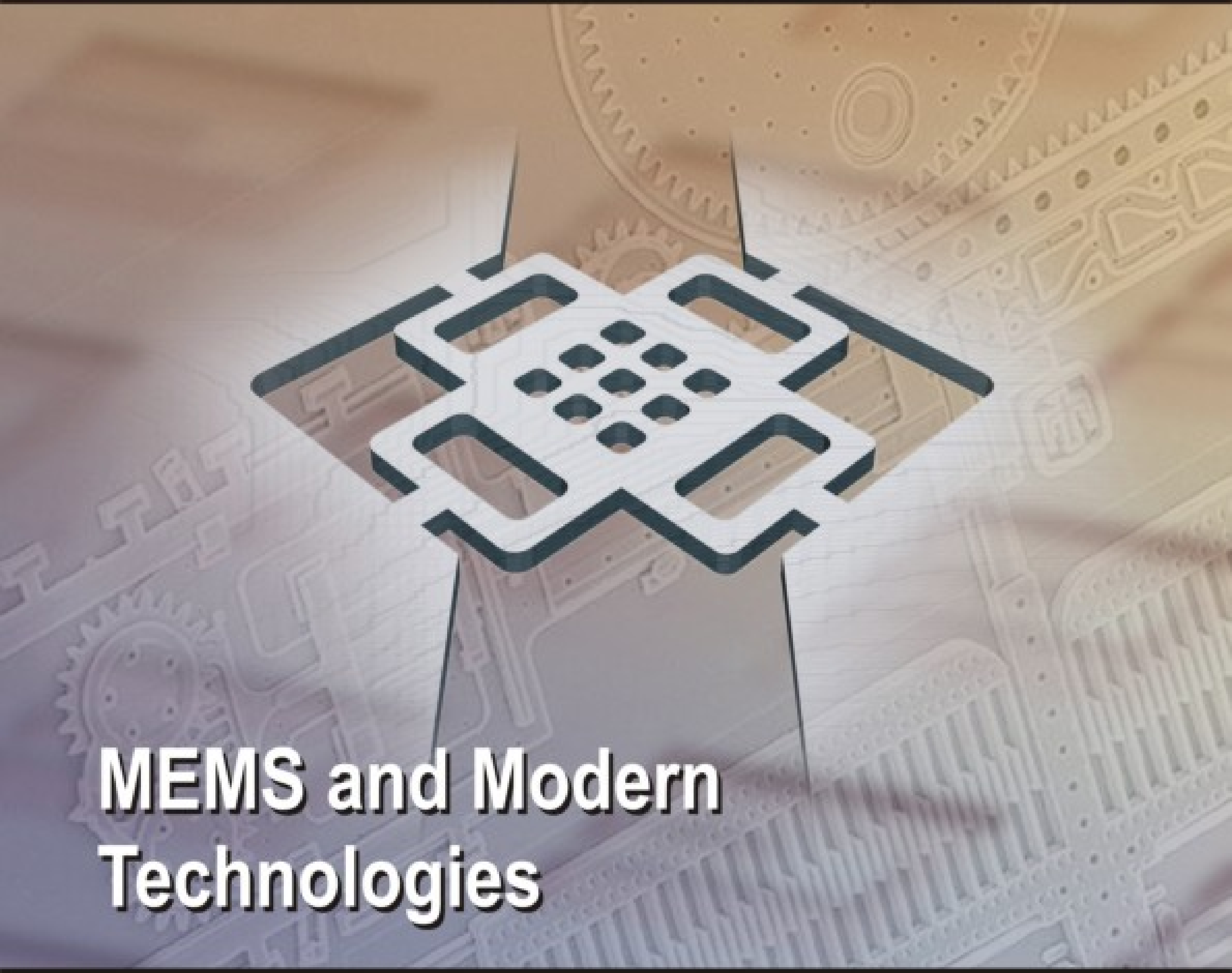

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Contents

Volume 78
Issue 4
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Editorial

Research Articles

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Investigation of the Pull-in Phenomenon in Drug Delivery Micropump Using Galerkin Method

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Abstract: In this paper, the pull-in phenomenon of an electrostatically actuated circular micropump which is used in drug delivery systems was investigated. The “pull-in” or divergence instability in electrostatically actuated micro electromechanical systems presents a ubiquitous challenge in MEMS technology of great importance. The Circular plate of the micropump was modeled as a thin plate base on Kirchhoff plate theory. The nonlinear coupled electromechanical governing differential equation was discretized using Galerkin method. The effect of number of shape functions on the accuracy of obtained results was investigated. The effects of radius, thickness and initial gap of the micropump on the pull-in voltage also were studied and the obtained results were compared with existing results. *Copyright © 2007 IFSA.*

Keywords: MEMS; Pull-in phenomenon; Micro pump; Galerkin Method

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1. Introduction

Micro-Electromechanical Systems (MEMS, also known as Microsystems - MST) combine electronics with microscale mechanical devices, resulting in microscopic machinery. In recent years, MEMS technology has grown rapidly and entered into many communication defense, automobile industries, biochemical [1] and medical [2] applications and etc. For instance accelerometers for airbag deployment in automobiles, ink jet printer heads, color projection displays, chemical sensors and scanning probe microscopy. Furthermore, the silicon semiconductor industry has provided numerous methods for fabricating microscale electronic devices. With a little ingenuity, these same fabrication methods can be used to fabricate devices that convert electrical energy into mechanical energy, and vice-versa [2].

Micropumps technology is one of the most advanced technologies in MEMS now. They are desired to handle small and precise volumes in various medical, biochemical, and chemical applications. The function of a micropump is to raise the pressure of a specific volume of a gas or liquid. Usually, micropumps are integrated with other microflow devices and sensors to allow precise control and sensing of flow of the order of microliters [3].

One of the medical applications of the MEMS technology is the drug delivery. Nowadays, implementation of high precision and performance, high reliability and small size of the drug delivery systems are the main challenges in the medical engineering. Micropumps as the main part of the drug delivery systems transfer the fluid (e.g. drug) from the drug reservoir to the body with high precision and reliability. Small size, high performance, reliability and precision of micropumps have made their useful for chemotherapy, insulin dosing in the diabetic, drug dosing in the Alazaimer, and Parkinson patients [4]. For other treatments, such as chemotherapy for cancer patients or for pain relief of terminally ill patients, a small pump that can administer small amounts of drugs at a well described and preferably programmable rate is a very necessary part of the therapy [2].

So far, various researches on the micro actuators for the application to micropumps have been performed. Generally, silicon, metal, and silicone rubber, parylene and etc. have been used as the actuator diaphragms of micropumps [5].

Generally, micropumps are classified into two categories: mechanical and non-mechanical. Mechanical micropumps include reciprocating and peristaltic pumps, and non-mechanical micropumps include electrohydrodynamic pumps. Reciprocating micropumps with different actuating techniques are widely used. The actuation principles include piezoelectric, pneumatic, electrostatic and thermopneumatic. Electrostatic actuation is gaining popularity because of its simplicity and high-flow output pressure and fast response time very small force and strokes and low power consumption [6]. During the years different micropumps have been developed with different advantages and drawbacks and different shapes of electrostatic actuating element are reported in the literature, includes rectangular, square, circular, and annular. Annular actuating elements have also been proposed for used in micropumps by some companies. They are also used in other MEMS devices, such as sensors, valves, deformable micropumps, and micro-turbo generators [3].

The research on micropumps initially emerged at Stanford University in 1980. Since then micropumps have received a lot of attention and have played an important role in the development of microfluidics systems.

During the last several decades, various designs of micropumps made of different materials and based on different pumping mechanisms have been presented. As often cited, J. G. Smits developed one of the earliest micropumps and published it in 1990[7].

The first practically successful device has been developed by Zengerle et al, who also realized one of the first vertically-stacked chip designs in silicon. The actuator is made from two silicon chips that embody the flexible pump diaphragm and a rigid counter electrode in a capacitor-like configuration. Applying a sufficiently high voltage to the actuation capacitor electrodes cause electrostatic attraction of the pump diaphragm, which in extreme gets fully attached to the counter electrode [8].

In electrostatic actuator micropumps, one of the important phenomena is pull-in phenomenon. Pull-in phenomenon is a discontinuity related to the interplay of the elastic and electrostatic forces. When a potential difference is applied between a conducting structure and a ground level the structure deforms due to electrostatic forces. The elastic forces grow about linearly with displacement whereas the electrostatic forces grow inversely proportional to the square of the distance. When the voltage is increased the displacement grows until at some point the growth rate of the electrostatic force exceeds that of the elastic force and the system cannot reach a force balance without a physical contact, thus pull-in occurs. The critical voltage is known as the pull-in voltage. The pull-in phenomenon is of great practical importance in the design of micro-electro-mechanical (MEMS) sensors, switches, and micropumps for example [9-13].

The most simple pull-in geometry is the lumped one-dimensional modeling of micropumps for which analytical expression for the pull-in position may be found. However, it turns out that it is possible to cerate a lumped model for the original distributed system.

In this paper the mechanical behavior of an electrostatic micropump employing a circular micro plate was studied. A complete theoretical framework to enable an accurate investigation and a proper understanding of the mechanical behavior of this type of micropump were presented. To this end, the circular moveable membrane is modeled as a thin Kirchhoff plate subjected to nonlinear electrostatic pressure. The deflection and the pull-in phenomenon of the circular plate under nonlinear electrostatic loading were studied using Galerkin Method. The obtained results were compared with existing results and acceptable agreements were gained.

2. Model Description

Micropumps usually consist of an actuation unit and two passive check valves. Microchannels are used for connecting the inlet port and outlet port. The pump provides the pressure gradient for moving liquid in channels, reservoirs and chambers. The principle of electrostatic actuator is very simple as shown in Figure 1. The model has two plates. One of them is fixed plate or ground plate and the other one is named moveable plate. When the voltage is applied to these plates, actuator is deflected toward ground plate. The distance between these plates (gap) is filled with dielectric like air. The general force of the actuator is inversely proportional to the second power of the gap. Electrostatic actuation has an extremely low power consumption and full MEMS compatibility. They have relatively higher frequency response [4].

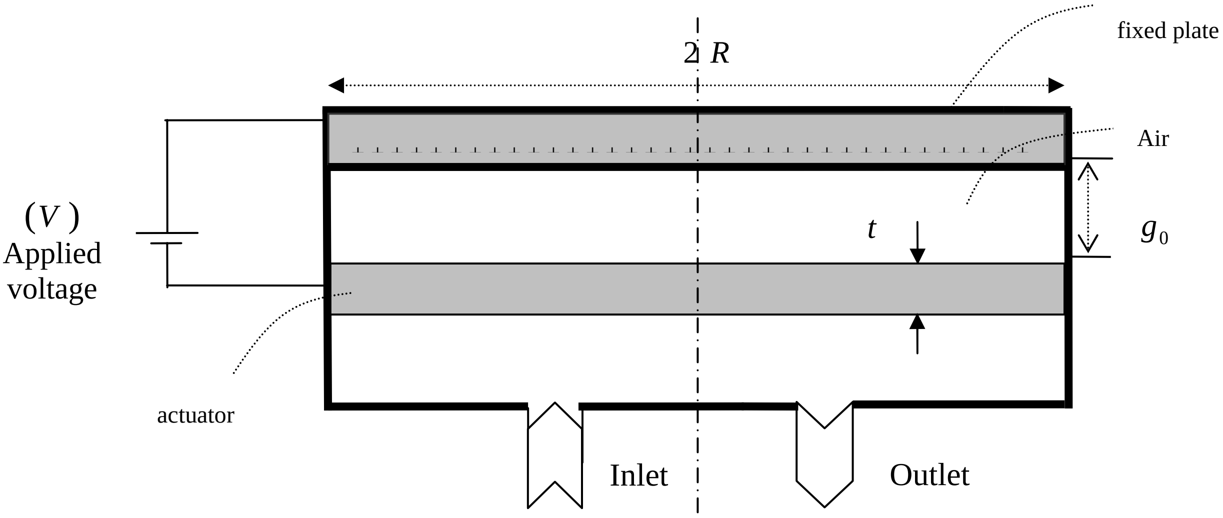


Fig. 1. View of the electrostatic micropump.

3. Mathematical Modeling

Two major group of plate theory are used in scientific and practical applications: 1) Kirchhoff plate theory; 2) Mindlin plate theory. The Kirchhoff thin plate theory is used when the thickness-to-length ratio ($t/2R$) of the plate is relatively low and so the effect of transverse shear strains is neglected in the classical theory. The Mindlin plate theory (1951) is used when the thickness-to-length ratio ($t/2R$) of the plate is relatively large and so the effect of transverse shear strains is considered [14].

Since diaphragm plate thickness is very lesser than diameter, transverse shear deformation is insignificant and can be neglected. With this important statement, all stress-strain relations that involved transverse shear deformation are vanished and the micropump plate can be modeled based on Kirchhoff classic thin plate theory.

The load acting on the plate is axisymmetrically distributed therefore the deflection surface to which the middle plane of the plate is bent will also be axisymmetrical. In all points equally distant from the center of the plate the deflections will be the same, and it is sufficient to consider deflections in one diametral section throw axis of symmetry. Let us take the origin of coordinates at the center of the deflected plate and denote by r the redial distance of points in the middle plane of the plate and by $w(r)$ their deflections.

The differential equation which is implemented in micropumps using Kirchhoff plate theory and assumption of plain strain condition is [14]:

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\}=\frac{q}{D}, \tag{1}$$

where q is the electrostatic nonlinear pressure and D is the flexural rigidity. These parameters can be expressed as:

$$q=\frac{k\varepsilon\, V^{2}}{2(g_{0}-w(r))^{2}} \tag{2}$$

and

$$D=\frac{Et^{3}}{12(1-\upsilon^{2})} \tag{3}$$

in which g_0 is the gap, k is the dielectric constant, ε is the permittivity for air and V is the applied voltage between the movable/ground plates, t is the thickness of the moveable plate, E is the modulus of elasticity of the moveable plate and υ is Poisson's ratio. The equation (1) can be rewritten in the following form:

$$L(w,V)=r^{3}\frac{d^{4}w}{dr^{4}}\left((g_{0}-w)^{2}\right)+2r^{2}\frac{d^{3}w}{dr^{3}}\left((g_{0}-w)^{2}\right)-r\frac{d^{2}w}{dr^{2}}\left((g_{0}-w)^{2}\right)+\frac{dw}{dr}\left((g_{0}-w)^{2}\right)-r^{3}\frac{k\varepsilon V^{2}}{2D}=0, \tag{4}$$

where $L(w,V)$ is a differential operator. The four spatial boundary conditions needed to find a solution is determined from the requirement that the plate surround is clamped (zero displacement and zero slop). These boundary conditions are:

$$w(\pm R)=0, \qquad \frac{dw}{dr}(\pm R)=0 \tag{5}$$

4. Numerical Solution

The general discretization methods that can be applied directly to differential equation are finite difference approximations and the various weighted residual procedures (known alternately as the Galerkin procedure) or approximate techniques of determining the stationarity of properly defined functionals. It is important to remark that the well-known finite difference method of approximation is a particular case of weighted residual approximations, when the Delta functions selected as weighting functions (Collocation method).

In the present article the weighted residual procedures to discretize governing equations was used. Let $w(r)$ is the solution of the governing nonlinear electro-mechanical coupling equation which is the continuous and smooth function on $[-R,R]$, so the $w(r)$ can be expanded with respect to base functions of $\varphi_n(r)$ as:

$$w(r)=\sum_{n=1}^{\infty}a_n\varphi_n(r) \tag{6}$$

It is clearly impossible to satisfy both the differential equation and the boundary conditions in a general case. So if the selected $\varphi_n(r)$ satisfy the existing geometrical boundary conditions, then $w\ (r)$ satisfies the same boundary conditions.

An approximate solution for Eq. (4) can be expressed as the following sequence:

$$w_N(r)=\sum_{n=1}^{N}a_n\varphi_n(r) \tag{7}$$

The sequence $w_N(r)$ converges to the function $w(r)$ in the mean, in the other word there exist an $N>0$ then [15]

$$\int_{-R}^{R}\left(w(r)-w_N(r)\right)^2 dr \prec \varepsilon^2 \tag{8}$$

By substitution the forgoing approximated solution to equation (4) we have:

$$L(w_N(r),V)=L(\sum_{n=1}^{N}a_n\varphi_n(r),V)=\mathrm{E}r(r)\ , \tag{9}$$

where the $\mathrm{E}r(r)$ represents some residual obtained by substituting of the approximate solution into the differential equation. Using the $\varphi_n(r)$ as weighting functions and applying the Galerkin-Bubnov method, a set of N nonlinear algebraic equations can be obtained as follow:

$$\int_{-R}^{R}\varphi_j(r)L\left(w_N(r),V\right)dr=\int_{-R}^{R}\varphi_j(r)L\left(\sum_{n=1}^{N}a_n\varphi_N(r),V\right)dr=0 \qquad j=1,\ldots,N \tag{10}$$

or

$$\begin{aligned}&\int_{-R}^{R}\varphi_j r^3\sum_{n=1}^{N}a_n\varphi_n^{IV}\left(g_0-\sum_{n=1}^{N}a_n\varphi_n\right)^2 dr\\&+\int_{-R}^{R}\varphi_j r^2\sum_{n=1}^{N}a_n\varphi_n'''\left(g_0-\sum_{n=1}^{N}a_n\varphi_n\right)^2 dr\\&-\int_{-R}^{R}\varphi_j r\sum_{n=1}^{N}a_n\varphi_n''\left(g_0-\sum_{n=1}^{N}a_n\varphi_n\right)^2 dr\\&+\int_{-R}^{R}\varphi_j\sum_{n=1}^{N}a_n\varphi_n'\left(g_0-\sum_{n=1}^{N}a_n\varphi_n\right)^2 dr-\int_{-R}^{R}\varphi_j r^3\frac{k\varepsilon V^2}{2D}=0\end{aligned} \qquad j=1,\ldots,N \tag{11}$$

With expansion of equation (11):

$$\begin{aligned}&\sum_{n=1}^{N}a_n g_0^2\left(K_{nj}^{(1)}+K_{nj}^{(4)}-K_{nj}^{(7)}+K_{nj}^{(10)}\right)-\sum_{m=1}^{N}\sum_{n=1}^{N}a_m a_n(2g_0)\left(K_{mnj}^{(2)}+K_{mnj}^{(5)}-K_{mnj}^{(8)}+K_{mnj}^{(11)}\right)\\&+\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N}a_m a_n a_p\left(K_{mnpj}^{(3)}+K_{mnpj}^{(6)}-K_{mnpj}^{(9)}+K_{mnpj}^{(12)}\right)-\frac{k\varepsilon V^2}{2D}\int_{-R}^{R}\varphi_j r^3 dr=0\end{aligned} \qquad j=1,\ldots,N, \tag{12}$$

where $K_{nj}^{(i)}\quad i=1,4,7,10$ and $K_{mnj}^{(i)}\quad i=2,5,8,11$ and $K_{mnpj}^{(i)}\quad i=3,6,9,12$ are represented as follow:

$$K_{nj}^{(1)}=\int_{-R}^{R}r^3\varphi_j\varphi_n^{IV}dr, \qquad K_{nmj}^{(2)}=\int_{-R}^{R}r^3\varphi_j\varphi_n\varphi_m^{IV}dr, \qquad K_{nmpj}^{(3)}=\int_{-R}^{R}r^3\varphi_j\varphi_n\varphi_m\varphi_p^{IV}dr,$$

$$K_{nj}^{(4)}=\int_{-R}^{R}2r^2\varphi_j\varphi_n'''dr, \qquad K_{nmj}^{(5)}=\int_{-R}^{R}2r^2\varphi_j\varphi_n\varphi_m'''dr, \qquad K_{nmpj}^{(6)}=\int_{-R}^{R}r^2\varphi_j\varphi_n\varphi_m\varphi_p'''dr,$$

$$K_{nj}^{(7)}=\int_{-R}^{R}r\varphi_j\varphi_n''dr, \qquad K_{nmj}^{(8)}=\int_{-R}^{R}r\varphi_j\varphi_n\varphi_m''dr, \qquad K_{nmpj}^{(9)}=\int_{-R}^{R}r\varphi_j\varphi_n\varphi_m\varphi_p''dr,$$

$$K_{nj}^{(10)}=\int_{-R}^{R}\varphi_j\varphi_n'dr, \qquad K_{nmj}^{(11)}=\int_{-R}^{R}\varphi_j\varphi_n\varphi_m'dr, \qquad K_{nmpj}^{(12)}=\int_{-R}^{R}\varphi_j\varphi_n\varphi_m\varphi_p'dr.$$

5. Numerical Results

Material and geometrical parameters for the structure of the micropump are indicated in Table 1. The form of polynomial function which is used as the base or shape function in series form as follow:

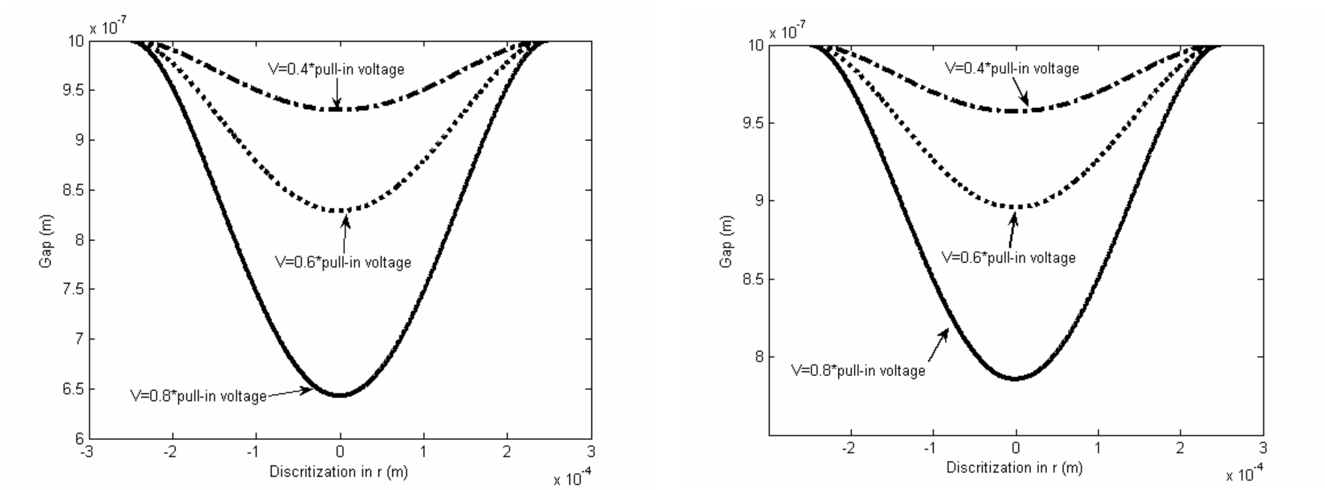
$$\left\{(r-R)^2(r+R)^2;(r-R)^2(r+R)^2(r-0.8R)(r+0.8R);\ldots\right\} \tag{13}$$

Table 1. Material and Geometrical parameters of the micropump.

Parameter	**Value**
E (Young's modulus)	169 (GPa)
υ (Poisson's ratio)	0.3
ε (Permittivity of air)	8.8541878×10^{-12} (F/m)
R (Radius of the plate)	250 µm
t (Thickness of the plate)	20 µm
g_0 (Initial gap)	1 µm

By considering the N term(s) approximation and applying Galerkin procedure, a system of N nonlinear algebraic equations is obtained. By solving obtained system of algebraic equations its roots are achieved. Some of these roots are complex and the others except one root are related to unnecessary mode shapes.

The axisymetric deflection of microplate at some given voltages is shown in figure 2.



(a) (b)

Fig. 2. Cross section view of diaphragm deflection at different given voltages, a) N=1 and b) N=2.

The result of applying different number of polynomial set of function is presented in Table 2. These results also are compared with existing results in the reference [9] and the results of Finite Difference (FD) method available in reference [10].

Table 2. Comparison of the obtained results for pull-in voltage with available results.

Type of solution way	**Pull-in voltage**	Δ_1	Δ_2
FEM[9]	314.00	-----------	-----------
FDM[10]	315.00	-----------	-----------
Galerkin Method $N=1$	435.50	38.6%	38.2%
Galerkin Method $N=2$	341.58	8.7%	8.4%

where Δ_1 is the difference percentage in comparison the results of Galerkin method with the result of reference [9] and Δ_2 in comparison with the result of reference [10].

Figure 3 shows center gap versus applied voltage for the results of Galerkin method, using different number of shape functions, and the results obtained by finite difference (FD) method available in the reference [10].

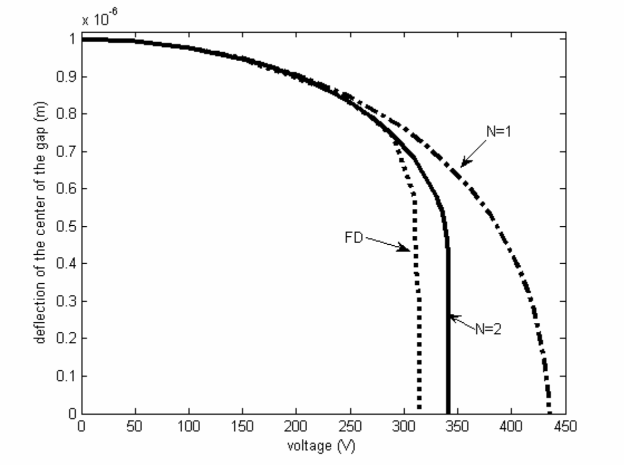
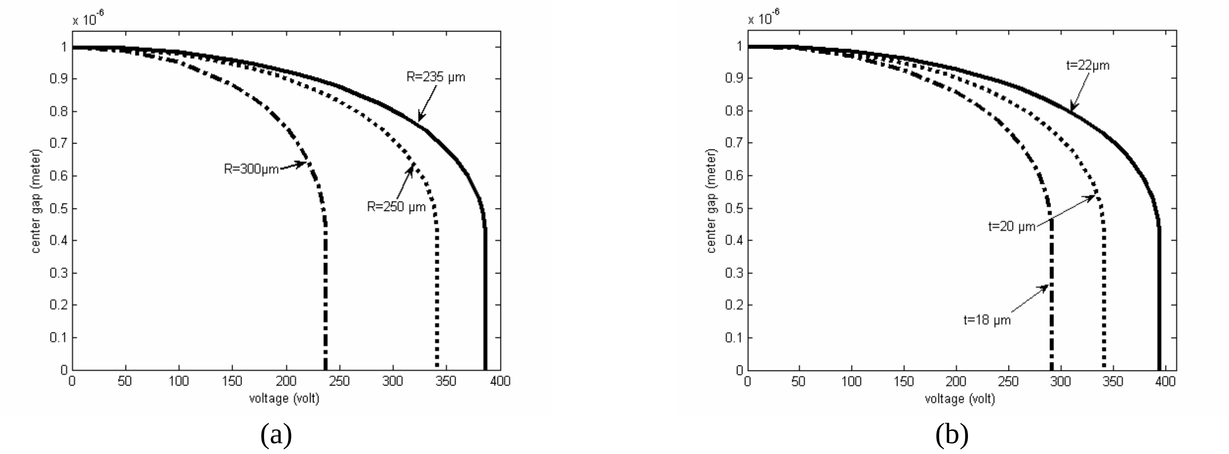
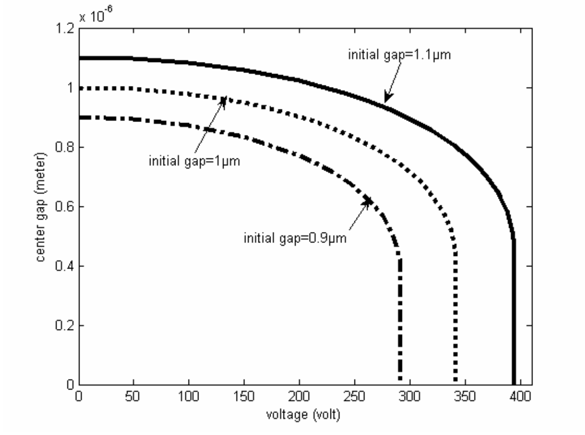


Fig. 3. Center gap versus applied voltage for N=1, 2 and f=1 and FD.



(a)

(b)



(c)

Fig. 4. Center gap versus applied voltage for different a) radius, b) thickness and c) initial gap.

The effect of radius, thickness and initial gap of the micropump on the pull-in voltage was studied for $N=2$ and the obtained results were shown in Figure 4.

6. Conclusions

In this paper, Galerkin method is successfully implemented to discretize the governed nonlinear coupled electromechanical differential equation to study the pull-in phenomenon of the micropump system. The results for polynomial set of shape or base function by changing the number of terms were obtained.

By considering the results of table 2 it was obviously clarified that if the number of shape functions increased the obtained accuracy increases.

It was clearly shown that the Pull-in voltage varies directly by increasing the radius and thickness whenever the Pull-in voltage varies inversely by increasing the initial gap.

The applications of these results is in designing the micropumps when the pull-in voltage should be computed with analytical methods, and the less-difference percent between these results and applied voltage and also the minimum-time analyzing are the important benefits of the methods.

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Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726- 5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually.

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- Microsystems;
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