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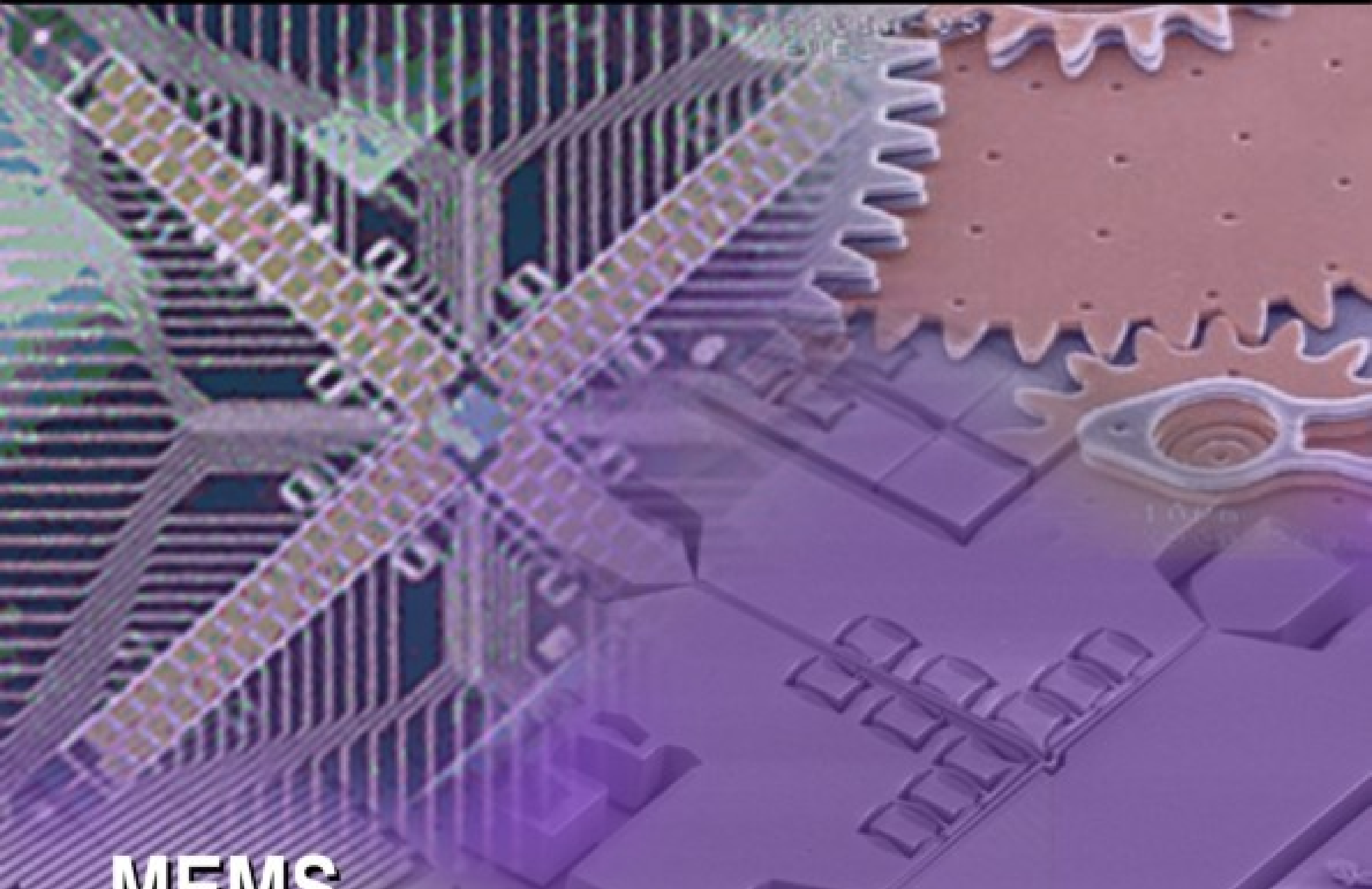

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Design of MEMS Cantilever - Hand Calculation

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Abstract: The present article describes the basic hand calculations for design of MEMS cantilever for beginners. The MATLAB software code was written to analysis the all formulae. Further the article gives insight of important parameters, its dependence and consideration for a good design. *Copyright © 2008 IFSA.*

Keywords: MEMS, Cantilever, Spring constant, Pull-in and hold down voltage, Resonant frequency

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1. Introduction

It is a common practice for scientist and engineers working in Micro-Electro-Mechanical System (MEMS) area to simulate the structures using MEMS simulation software like Coventorware [1] and IntelliSuite [2] before actual fabrication. The software helps to build the structure, mesh it and then simulate. The main intention in using this software's is to optimize the structure dimensions to get the required output which saves time and money during fabrication. Before simulation beginners should have an idea of required dimensions of the device, materials, material properties and tentative fabrication process. The MEMS simulation software's are also used to study and analyze much more critical issues of MEMS structures like modal, harmonic, contact, hysteresis, temperature sensitivity and piezoelectric analysis. This is done by computing parameters like stress, strain, reaction forces, contact forces, contact heating etc on MEMS structures. For example for designing cantilever as an actuator one should know the Spring constant for cantilever, Pull-in voltage, hold-down voltage, Resonant frequency for cantilever to be used as resonators or any high frequency applications. The most important point is parameters dependence on dimensions and material properties of structure.

However it must be mentioned that these software's are not available to all and are expensive. The present article therefore describes the basic hand calculations for design of MEMS cantilever for beginners. The MATLAB software code was written to analysis the all formulae. Further the article gives insight of important parameters, its dependence and consideration for a good design.

2. MEMS Cantilever

By definition "A cantilever is a beam supported on only one end". The beam carries the load to the support where it is resisted by moment and shear stress. Cantilever construction allows for overhanging structures without external bracing.

Cantilevered beams are the most ubiquitous structures in the field of microelectromechanical systems (MEMS). MEMS cantilevers are commonly fabricated from silicon (Si), silicon nitride (SiN), or polymers. The fabrication process typically involves undercutting the cantilever structure to release it, often with an anisotropic wet or dry etching technique. MEMS cantilever has number of applications. A large number of research groups are attempting to develop cantilever arrays as biosensors for medical diagnostic applications. MEMS cantilevers are also finding application as radio frequency filters and resonators. In current article we concentrate on MEMS cantilever in Sensor and RF MEMS application. Many references are available on both topics [4 - 21].

Two equations is a key to understanding the behavior of MEMS cantilevers. The first is *Stoney's formula*, which relates cantilever end deflection δ to applied stress σ:

$$\delta = \frac{3\sigma(1-\nu)}{E}\left(\frac{L}{t}\right)^2, \tag{1}$$

where ν is Poisson's ratio, *E* is Young's modulus, *L* is the beam length and *t* is the cantilever thickness. Very sensitive optical and capacitive methods have been developed to measure changes in the static deflection of cantilever beams used in dc-coupled sensors.

The second is the formula relating the cantilever spring constant K to the cantilever dimensions and material constants which is mentioned below and explained in the design section further.

$$K = \frac{q_o l_c}{Y_{\max}}, \tag{2}$$

where q_o is the uniformly distributed load over a length, l_c, of the beam. The principal advantage of MEMS cantilevers is their cheapness and ease of fabrication in large arrays. The challenge for their practical application lies in the square and cubic dependences of cantilever performance specifications on dimensions. These superlinear dependences mean that cantilevers are quite sensitive to variation in process parameters.

3. Design for MEMS Cantilever

It has been already mentioned that cantilever acts as a basic block in many MEMS application. Giving design for each and every application is not possible. Thus in this article we focus on use of cantilever in sensor and RF MEMS switch application. The basic parameters which define the performance of cantilever as a sensor are spring constant and resonant frequency. On other hand the basic parameters which define the performance of cantilever as a RF MEMS Switch are spring constant, resonant frequency, pull-in voltage, and hold down voltage. It should be noted that spring constant and resonant frequency parameters are in common to both, and thus will be discussed at the same point.

Please note for all the plots below the following parameters are kept constant:
- Young's modulus of Si_3N_4 is E=210 GPa;
- Width of Cantilever beam is b=150µm;
- Thickness of Cantilever beam is h=1µm;
- Relative permittivity of Si_3N_4 is $\varepsilon_0 = 6$;
- Gap between electrodes at zero actuation voltage is d = 3µm;
- Gap between electrodes at actuation voltage V_{PI} is d_0 = 0.5µm.

3.1. Spring Constant

The first step in understanding the mechanical operation of sensor/RF MEMS switch is to consider spring constant 'K' expressed in N/m unit. Spring constant defined for a beam is also called as beam stiffness. Stiffness is differently defined for fixed-fixed and fixed-free type of beams. Fixed- fixed type of beam is membrane type of beam where both ends are fixed with anchors as shown in Fig. 1A.

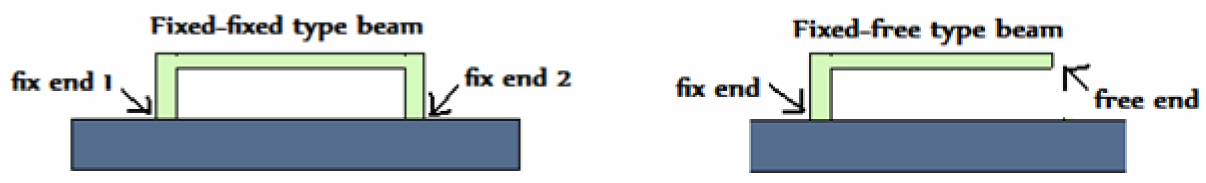

Fig. 1. A) Fixed-fixed type beam; B) Fixed-free type of beam.

Fixed-free type of beams is cantilever type of beam where one end is fixed and other is free as shown in Fig. 1B. Cantilever type fixed-free beam is used for sensor and DC-contact RF-MEMS series switch. Cantilever beam stiffness *K* is defined as the ratio of total applied load 'q_0 l_c' to maximum deflection Y_{max} [22, 23] as given in equation (2) before.

A fixed-free beam with one free movable edge is subjected to transverse load q_0 distributed about the free end of the beam. It can be easily seen from Fig. 2, for a fixed-free beam, Y_{max} is at the free end of the beam. Making use of the load-deflection properties of the beams, accounting for transverse and axial loading simultaneously, and applying superposition principle, the effective stiffness, *K* of a fixed-free beam at the position of maximum deflection, Y_{max} is given by [26]

$$K(\lambda_r) = \frac{2Ebh^3}{3l^3}\left[\frac{3}{8-6\lambda_r+\lambda_r^{\,2}}\right], \tag{3}$$

where $\lambda_r = l_c / l$
E is the young's modulus of the cantilever material;
b is the width of cantilever;
h is the thickness of cantilever;
l is the length of cantilever;
l_c is the length over which equal distributed transverse load is applied.

Form the above equation it is clear that length of cantilever is one of the important parameter as more the length of cantilever less the spring constant and in turn reduces the pull-in voltage of switch in case of RF MEMS switch and would increase deflection in case of sensor.

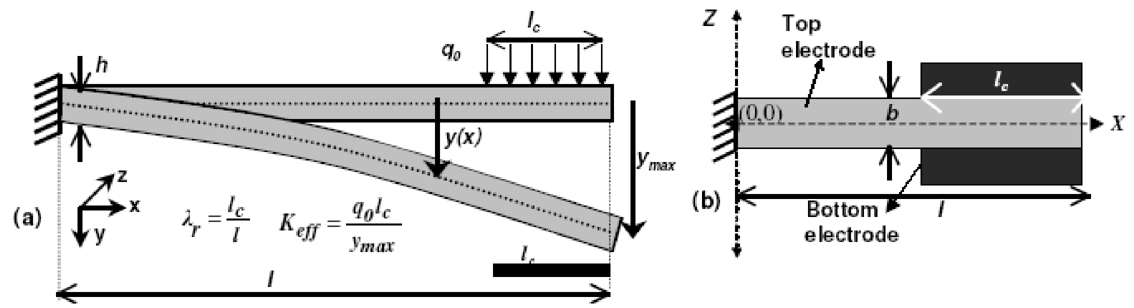

Fig. 2. (a) Cross sectional view of cantilever with load q_0 applied equally on length l_c
(b) Top view of cantilever and the region of loading [26].

See Fig. 3 for beam dimension v/s spring constant for reference. The beam dimension includes length of beam, width of beam and thickness of beam, the graph of each dimension against spring constant is plotted in Fig. 3A, Fig. 3B, and Fig. 3C respectively. These graphs can be used as ready reference. For example assume a spring constant of 10 N/m is required. For a cantilever which will be loaded with 60 % of its area from free end, with sensor material or electrode. For the said spring constant from Fig. 3A cantilever beam length can be estimated approximately 110 µm.

Now from Fig. 3B it is noted that by varying the cantilever width in 50 µm to 200 µm the net change in spring is 0.1 to 1 N/m only. Thus in sensors the required area of cantilever and in RF MEMS switch actuation electrode area governs the width. The variation of spring constant with respect to thickness of cantilever is shown in Fig. 3C. Thickness of cantilever is solely dependent on the fabrication method, but it is clear that spring constant depends on thickness non-linearly, more the thickness-higher the spring constant.

But lower the spring constant of beam the resonant frequency decreases thus increasing the switching time in case of RF MEMS switch and would reduce deflection or will not be sensitive to small change in mass on cantilever when bio-molecules adsorbs for sensing some anti-bodies. It is very clear that for lambda (λ_r) = 1 i.e. for full loading the spring constant is high and by reducing λ_r the spring constant decreases. Similarly spring constant goes on decreasing with increase in beam length and goes on increasing with increase in beam width and thickness. According to Gabrial M. Rebeiz, *et. al.* [10] the spring constant for RF MEMS switches should be between 5 to 40 N/m. This helps us selecting the beam length. Spring constant is mainly governed by the length of beam, young's modulus and the loading condition.

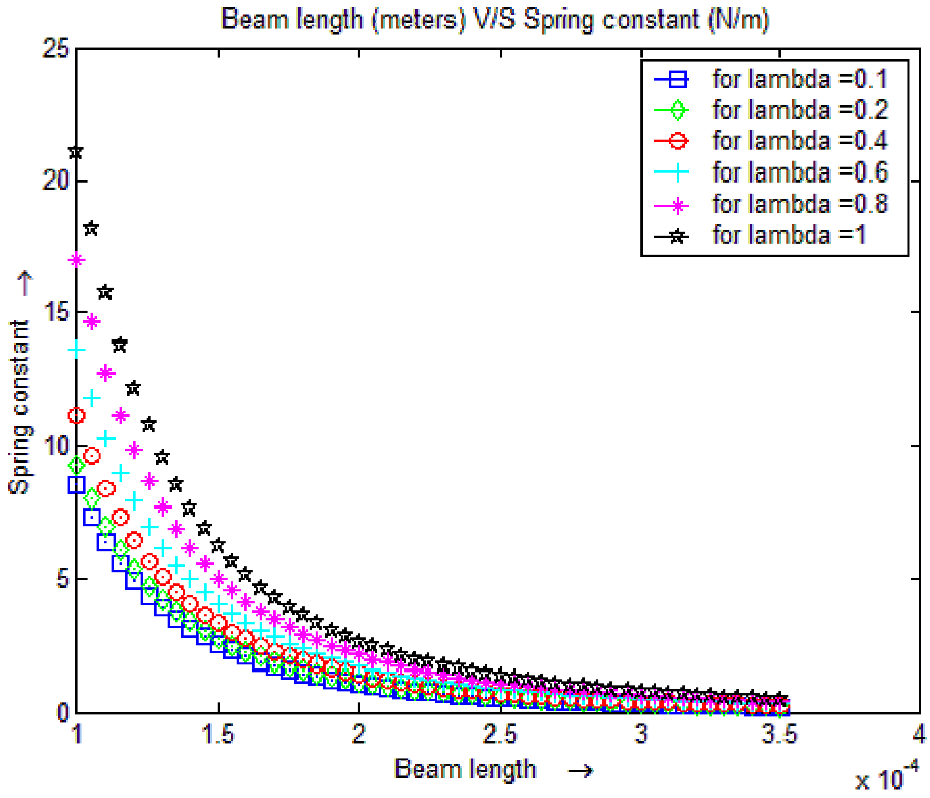

Fig. 3A. Variation of spring constant as a function of cantilever length.

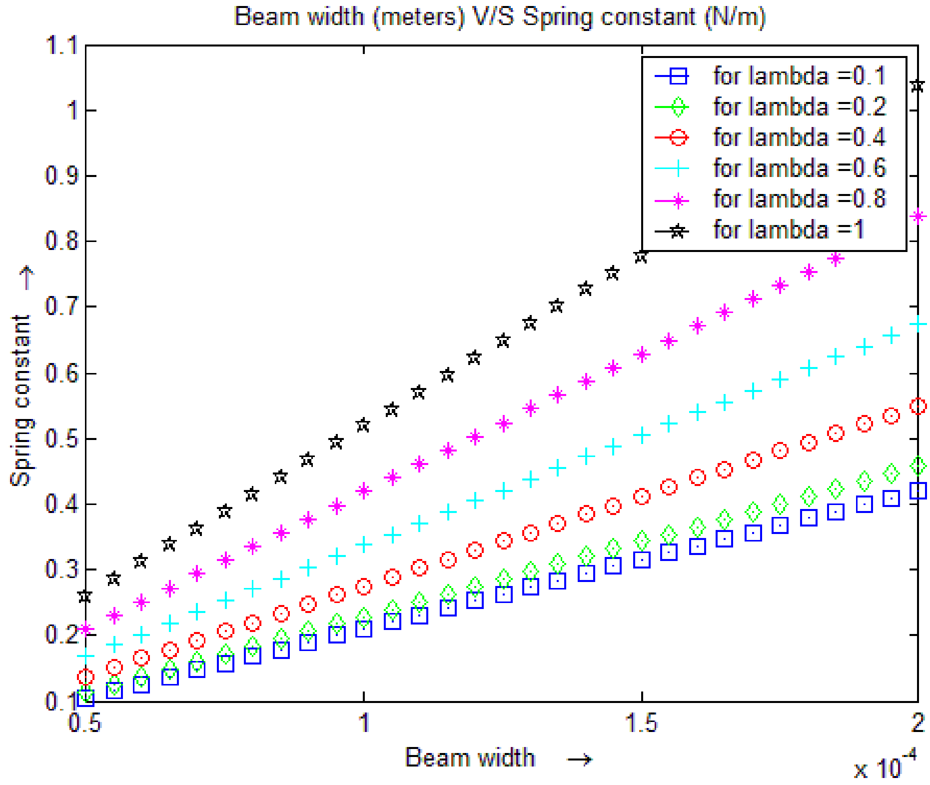

Fig. 3B. Variation of spring constant as a function of cantilever width.

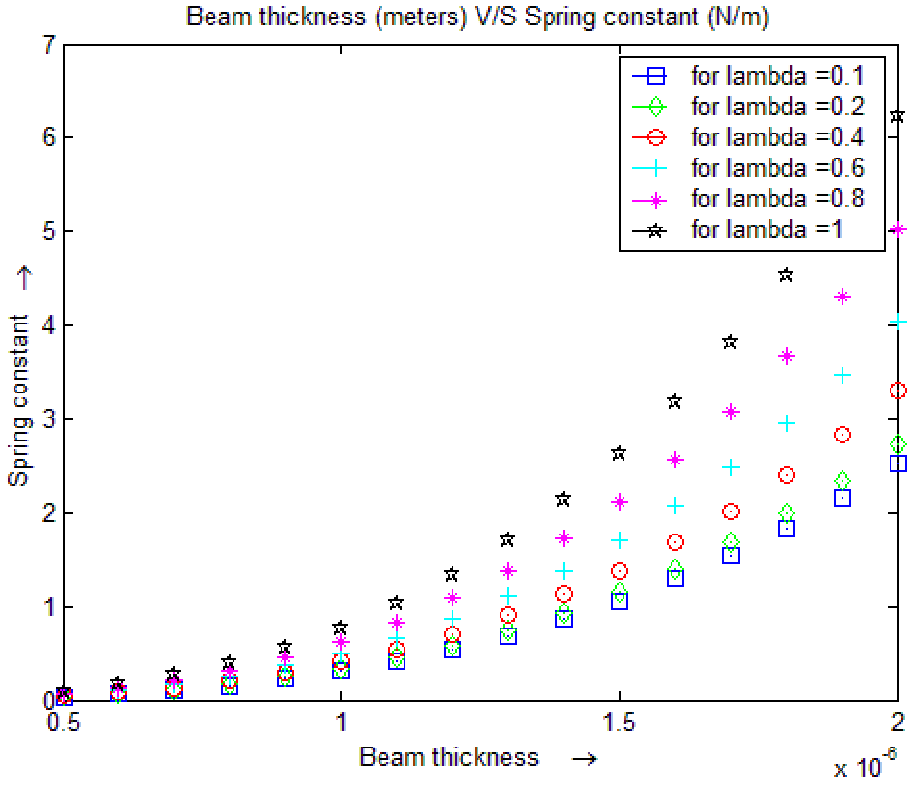

Fig. 3C. Variation of spring constant as a function of cantilever thickness.

3.2. Pull-in Voltage

Cantilever beam could be modeled considering it as a parallel plate capacitor, with beam as one electrode and substrate as second electrode and separated by air dielectric. Thus electrostatic force may be generated by applying a *DC* bias voltage (also known as polarization voltage) between the two plates of the capacitor shown in Fig. 4. Due to the electrostatic attractive force, F_{elec} the gap spacing varies with the *DC* polarization voltage, V_{DC} applied across the plates. Spring force, F_{mech} and the electrostatic force, F_{elec} are in balance, and the net force, F_{net} is equal to zero below pull-in. However, the electrostatic force increases non-linearly with decreasing gap spacing $(F_{elec}\ \alpha\ (d_0 - y_{max})^{-2})$ whereas the spring force is a linear function of the change in the gap spacing $(F_{mech} = K_{eff} y_{max})$.

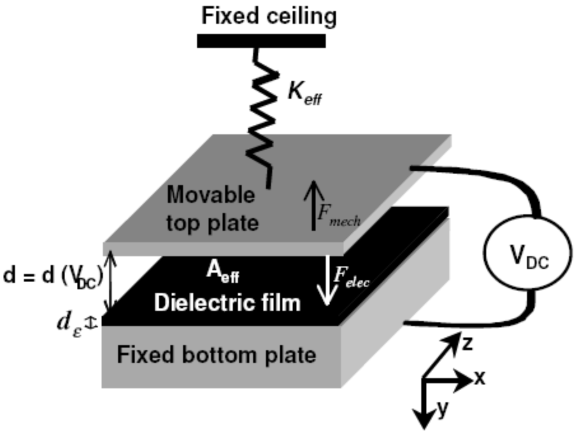

Fig. 4. Lumped parameter pull-in system [26].

As the *DC* polarization voltage, V_{DC} is increased, the electrostatic force increases at a faster rate compared to the restoring mechanical spring force. Due to this difference in the way F_{elec}, and F_{mech} are increased, the system is driven into instability with increase in polarization voltage, V_{DC}. The threshold voltage, above which the gap distance effectively closes to zero, is known as pull-in voltage, denoted by V_{PI} (when $V_{DC} = V_{PI}$). At V_{PI}, the mechanical restoring force, F_{mech} can no longer balance the electrical force, F_{elec} leading to the onset of pull-in voltage. Based on the energy considerations of the lumped spring mass system as that shown in Fig.6, pull-in voltage can be easily derived and is given by [27, 28].

$$V_{PI} = \sqrt{\frac{8K_{eff}d_o^{\,3}}{27\varepsilon_0 A_{eff}}}, \tag{4}$$

where K_{eff} is the effective spring constant computed by considering the loading. Gap between top and bottom electrode is d_0 and A_{eff} is the effective electrode area. Effective electrode area is the area which the electrode overlaps or has in common. The effective electrode area is computed as (length) X (width) X (λ_r). The plot of beam length v/s pull-in voltage is shown in Fig. 5. It shows more the length of beam lower the pull-in voltage. If we consider Eq. 4 of pull-in voltage, we note that the spring constant 'K' is directly dependent on 'V_{PI}' which implies that pull-in voltage will increase with increase in spring constant.

But looking at the Fig. 5A it is understood that least pull-in voltage is observed for all values of $\lambda_r = 1$ which had maximum spring constant as plotted in Fig. 3. This could be explained as pull-in voltage phenomenon is governed by electrostatic actuation. This makes very clear that more the effective electrode area 'A_{eff}' less the actuation voltage or pull-in voltage. Fig 5B shows pull-in voltage dependence on cantilever width, which is very less and practically zero in 50µm to 200µm range in this case. The pull-in voltage highly depends on thickness of cantilever as shown in Fig. 5C, more the thickness – higher the pull-in voltage.

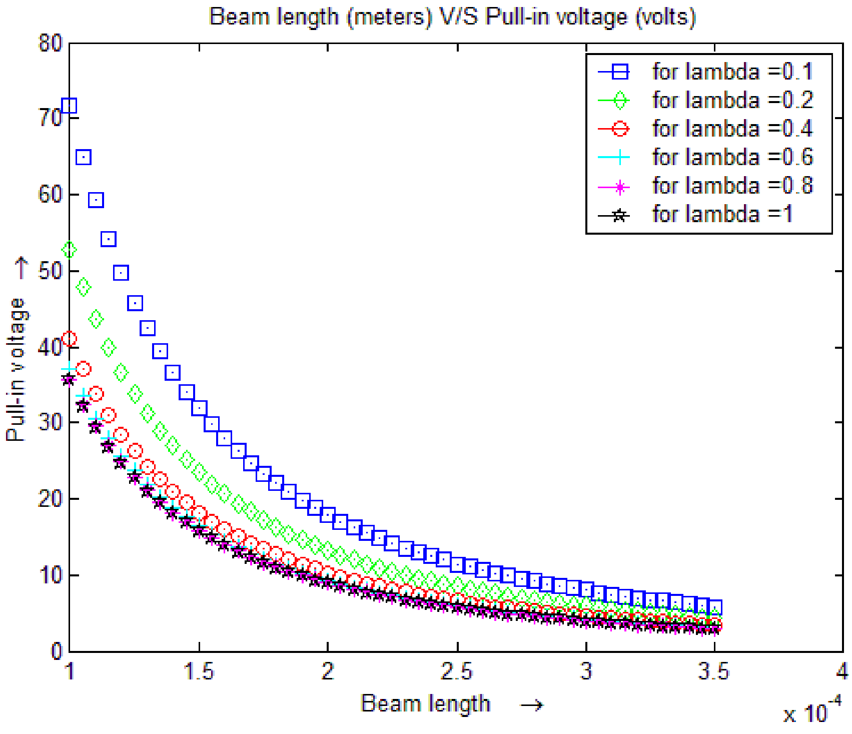

Fig. 5A. Variation of Pull-in Voltage as a function of cantilever length.

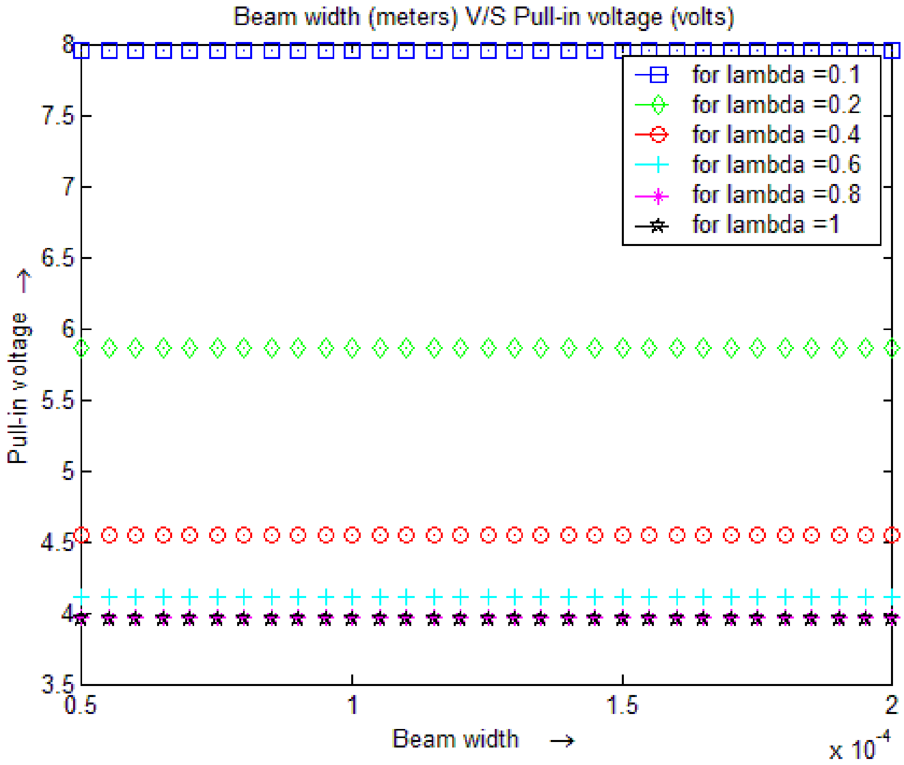

Fig. 5B. Variation of Pull-in Voltage as a function of cantilever width.

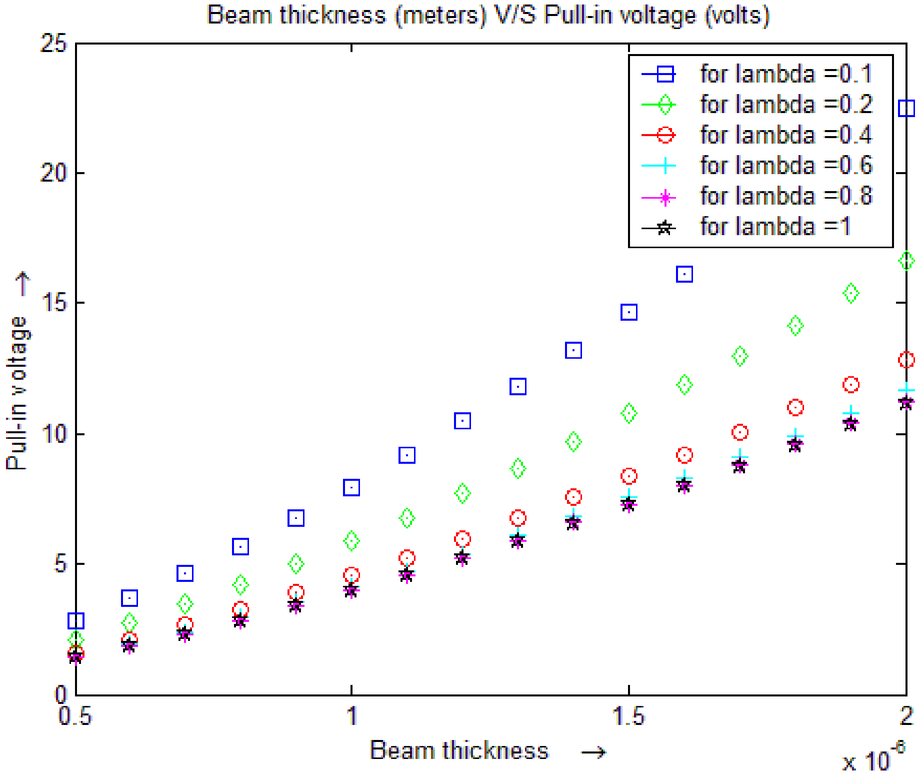

Fig. 5C. Variation of Pull-in Voltage as a function of cantilever thickness.

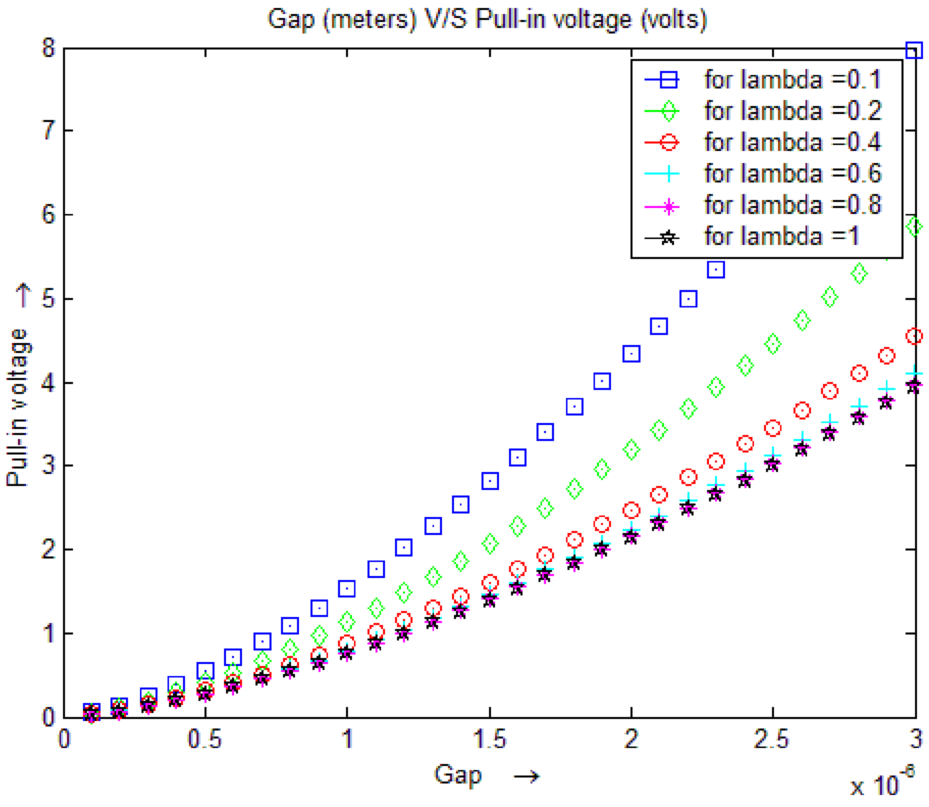

Fig. 5D. Variation of Pull-in Voltage as a function of Gap.

The gap between beam and substrate d_o is the other governing parameter of switches pull-in voltage as shown in Fig. 5D. Increasing gap between electrodes beyond 5µm increases the V_{PI} drastically and makes the switching highly impossible. Thus keeping d_o small, K less and large electrode area will decrease the pull-in voltage.

3.3. Hold Down Voltage

Once the series switch is pulled down, the gap is reduces to 0.3 to 0.5µm. For the switch in down state position, the electrostatic force should now be larger than mechanical restoring force [4, 29].

$$F = kd_o \tag{5}$$

And this is achieved for the hold voltage given by [26].

$$V_h = \sqrt{\frac{2k(d_o - d)}{\varepsilon\varepsilon_o A}\left[d + \frac{h}{\varepsilon_r}\right]^2} \tag{6}$$

Hold down voltage is much less than V_{PI}. The graph of beam length versus hold-down voltage is shown in Fig. 6. From Fig. 6A it is clear that as length of beam increases the hold-on voltage decreases. Again similar to pull-in voltage case, hold-down voltage shows no dependence on beam width in 50 µm to 200 µm range as shown in Fig. 6B. The variation of hold-down voltage against beam thickness is plotted in Fig. 6C, which shows non-linear dependence. As the beam thickness increases the hold-down voltage also increases. The variation of hold-down voltage against gap is plotted in Fig. 6D, which is different in nature than Fig. 6D. The graph intends to get saturated at some higher gap value point.

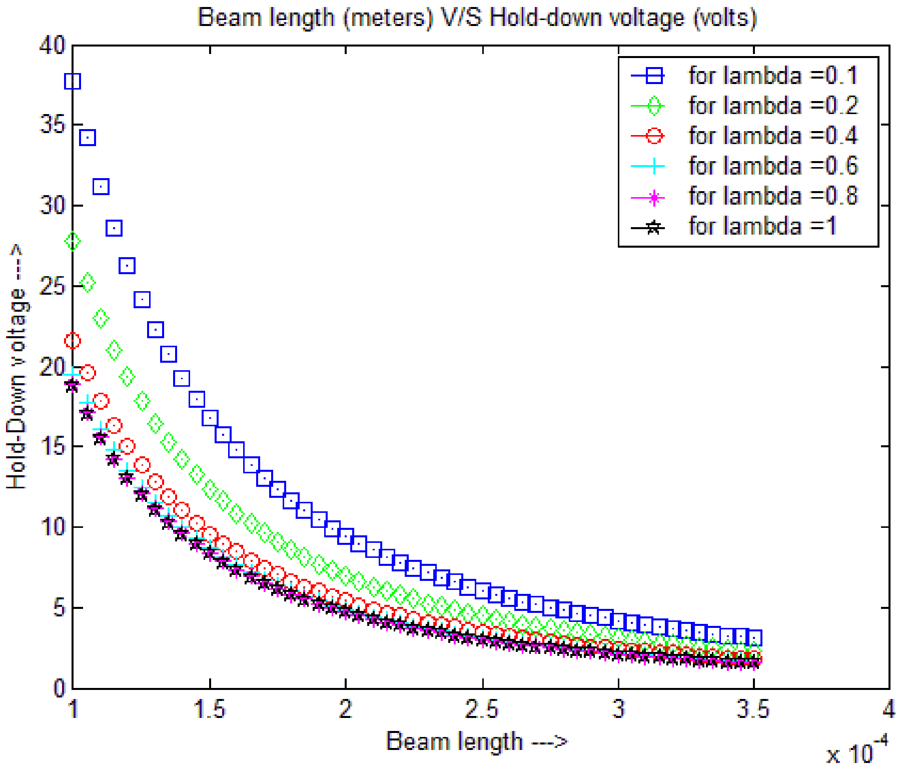

Fig. 6A. Variation of Hold-on Voltage as a function of cantilever length.

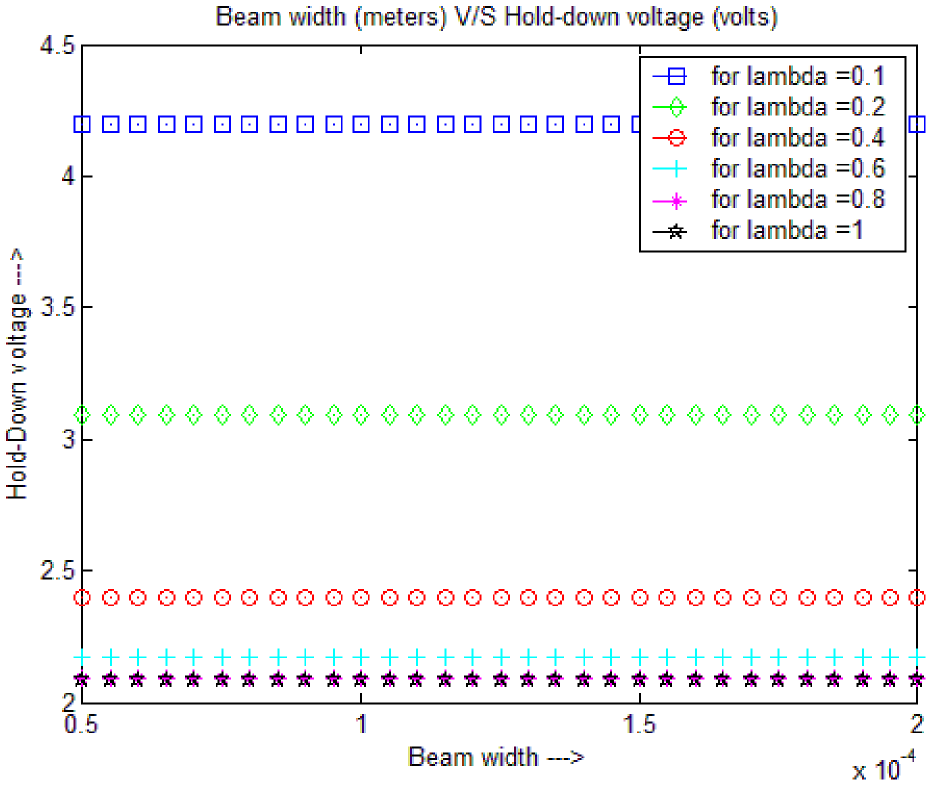

Fig. 6B. Variation of Hold-on Voltage as a function of cantilever width.

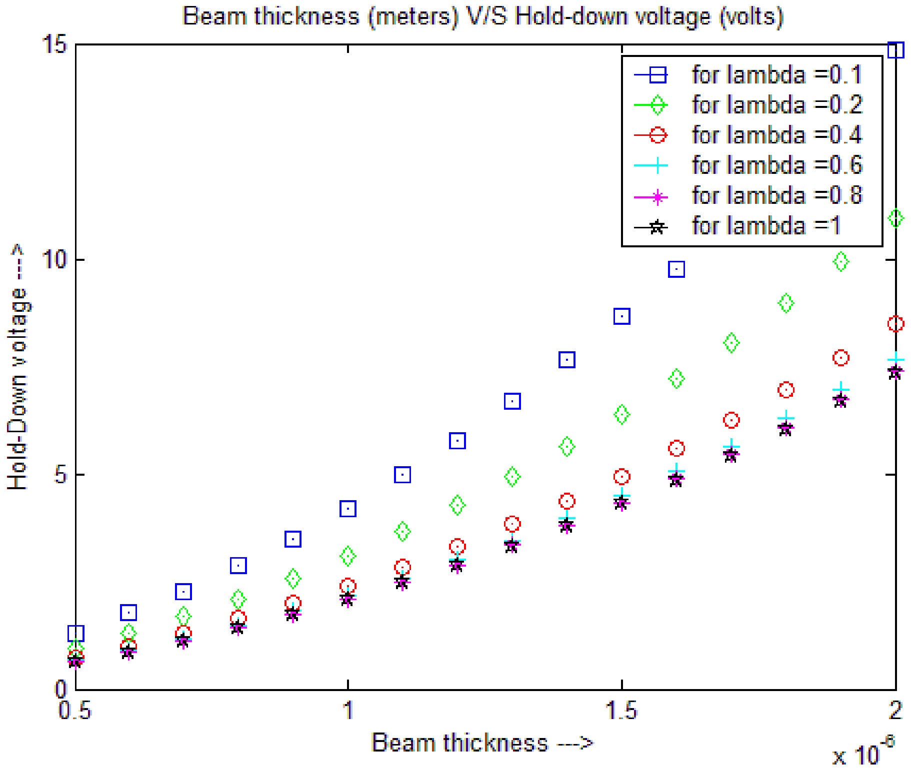

Fig. 6C. Variation of Hold-on Voltage as a function of cantilever thickness.

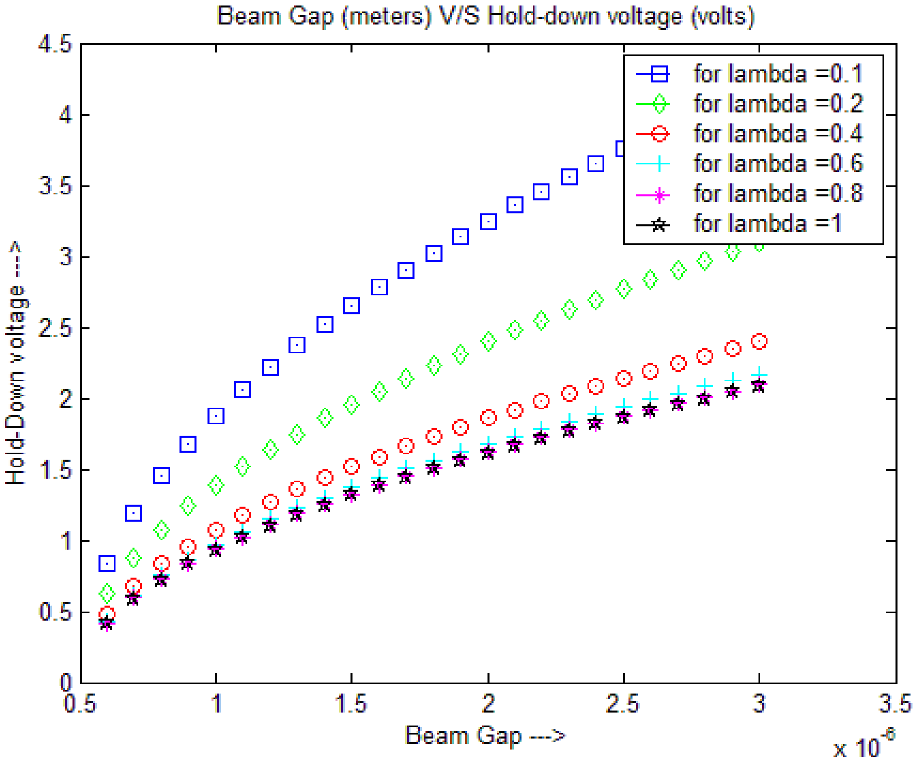

Fig. 6D. Variation of Hold-on Voltage as a function of gap.

This parameter is generally not required for designers who need switching continuously. In many applications very high continuous switching is required.

3.4. Resonant Frequency and Switching

The frequency response of the cantilever beam is useful for determining the switching time of the switch and mechanical bandwidth over which it can be used [4].

$$\omega_o = \sqrt{\frac{k}{m}}, \tag{7}$$

where k is the spring constant and m is the mass of cantilever.
Now in case of RF MEMS switches total mass of cantilever is calculated as

$$M_{total} = M_{canti} + M_{elec} + M_{tip}, \tag{8}$$

where
M_{total} is the total mass of cantilever;
M_{canti} is the mass of cantilever structure (here Si_3N_4);
M_{elec} is the mass of electrode (here gold);
M_{tip} is the mass of tip (here gold).

Here total mass should be the addition of mass of cantilever electrode and tip. For hand calculation we can find mass by knowing the density of the material.

$$\text{Mass} = \text{density X volume} \tag{9}$$

Thus we get the ideal resonant frequency of the switch. This resonant frequency defines the switching time of the switch. Higher the resonant frequency (i.e. high spring constant and low mass) better the switching time. It should be noted that though resonant frequency is high, switch is never switched at that frequency. Because this value indicates the maximum possible value above which the cantilever may break. Thus it is advisable to use switch in 50-60% of resonant frequency for long switch life. It is also important to point out that resonant frequency and the cut-off frequency of switch are not same. They define two different entities. Resonant frequency is the maximum frequency at which the cantilever can oscillate, on other hand, cut-off frequency is the maximum signal frequency that the microstrip line and tip would handle. The signal frequency is switched at resonant frequency. The plot of beam length v/s resonant frequency is shown in Fig.7A. as beam length increases the resonant frequency decreases. Thus it will increases the switching time in RF MEMS switch case and would reduce the sensor recovery time in cantilever sensor case. Resonant frequency also doesn’t show much dependence on the beam width see Fig. 7B. The graph of resonant frequency against beam thickness is almost linear as shown in Fig. 7C.

4. Conclusion

From all the graphs above it is clear that using given formulae, graph and knowing the requirement of specific application, one can get an estimate of the cantilever dimensions. Now using this as basic input for constructing model in simulation software and understanding the interdependence of each parameter, the optimization process is eased. In end we conclude with some thumb rules for good design:

1. Select the length of cantilever between 150 μm to 300 μm.
2. Select the spring constant between 5 N/m to 40 N/m (for RF MEMS switch only).
3. Select the electrode area in between 60 % to 80 % of the beam length (for RF MEMS switch only).
4. Switch the cantilever at 50 % to 60 % of the resonant frequency for long switching life.

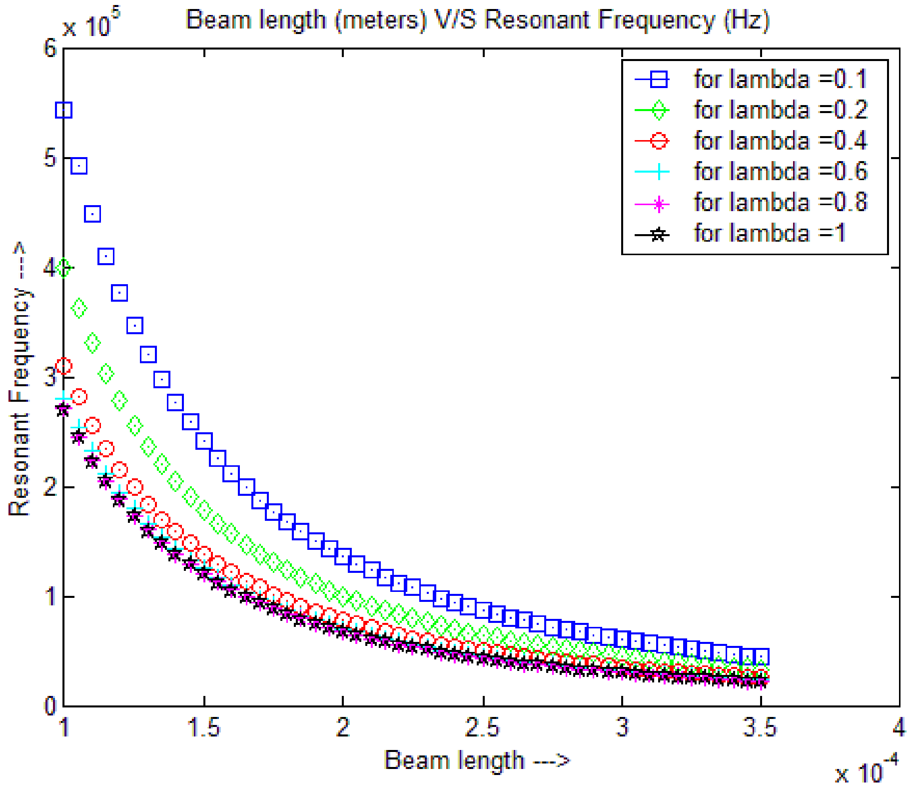

Fig. 7A. Variation of Resonant frequency as a function of cantilever length.

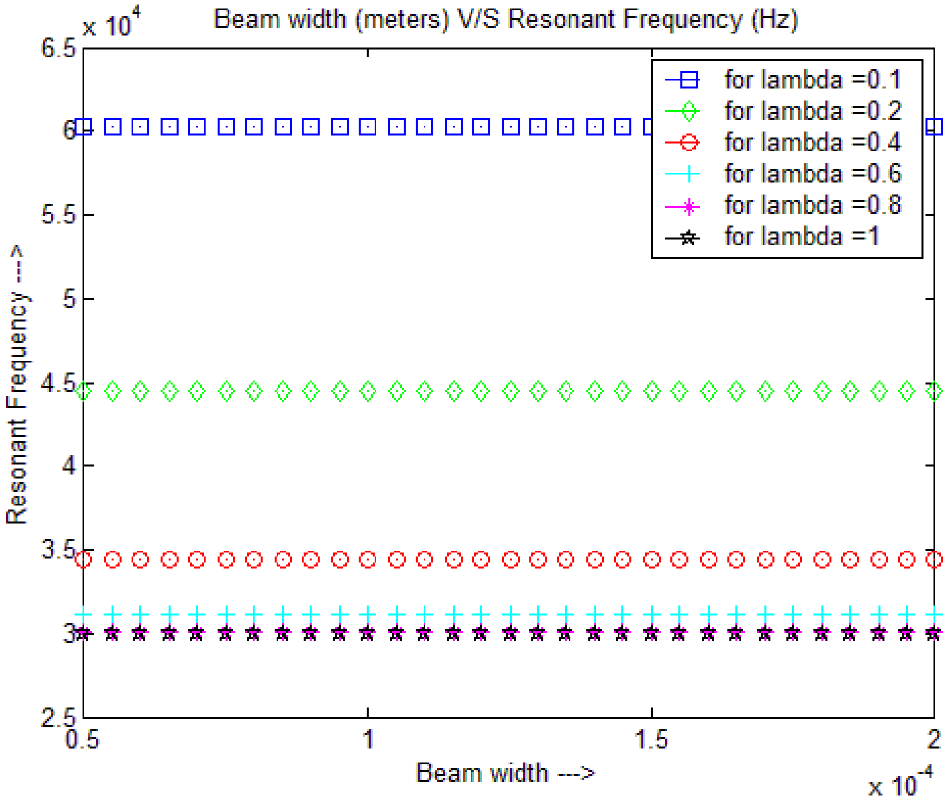

Fig. 7B. Variation of Resonant frequency as a function of cantilever width.

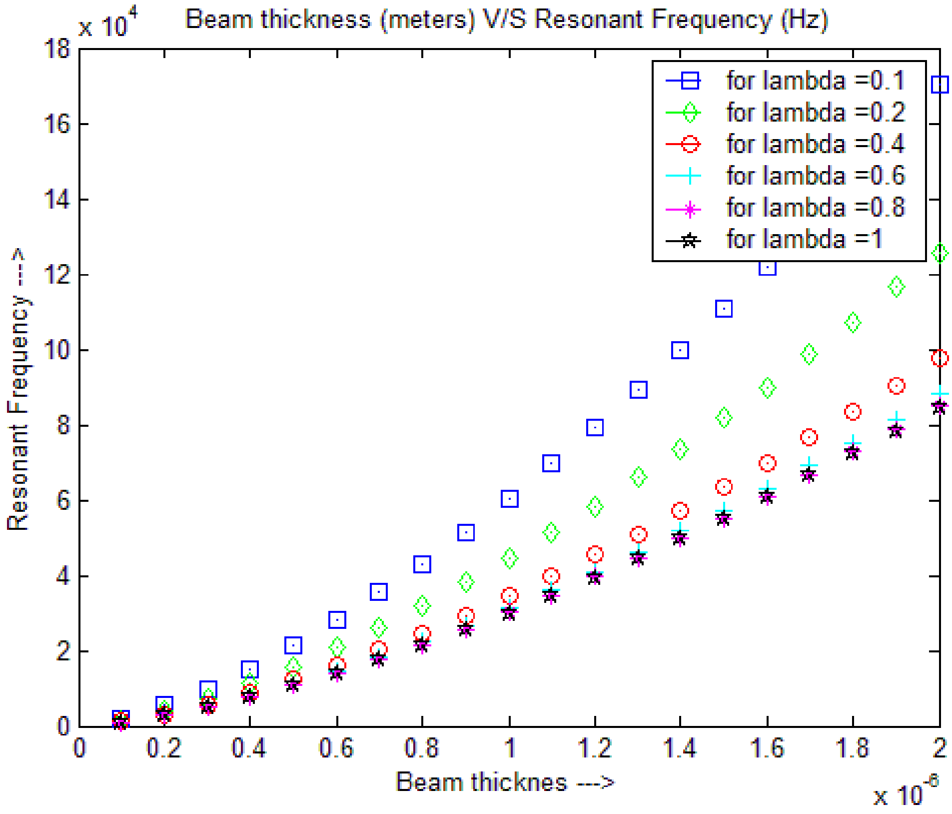

Fig. 7C. Variation of Resonant frequency as a function of cantilever thickness.

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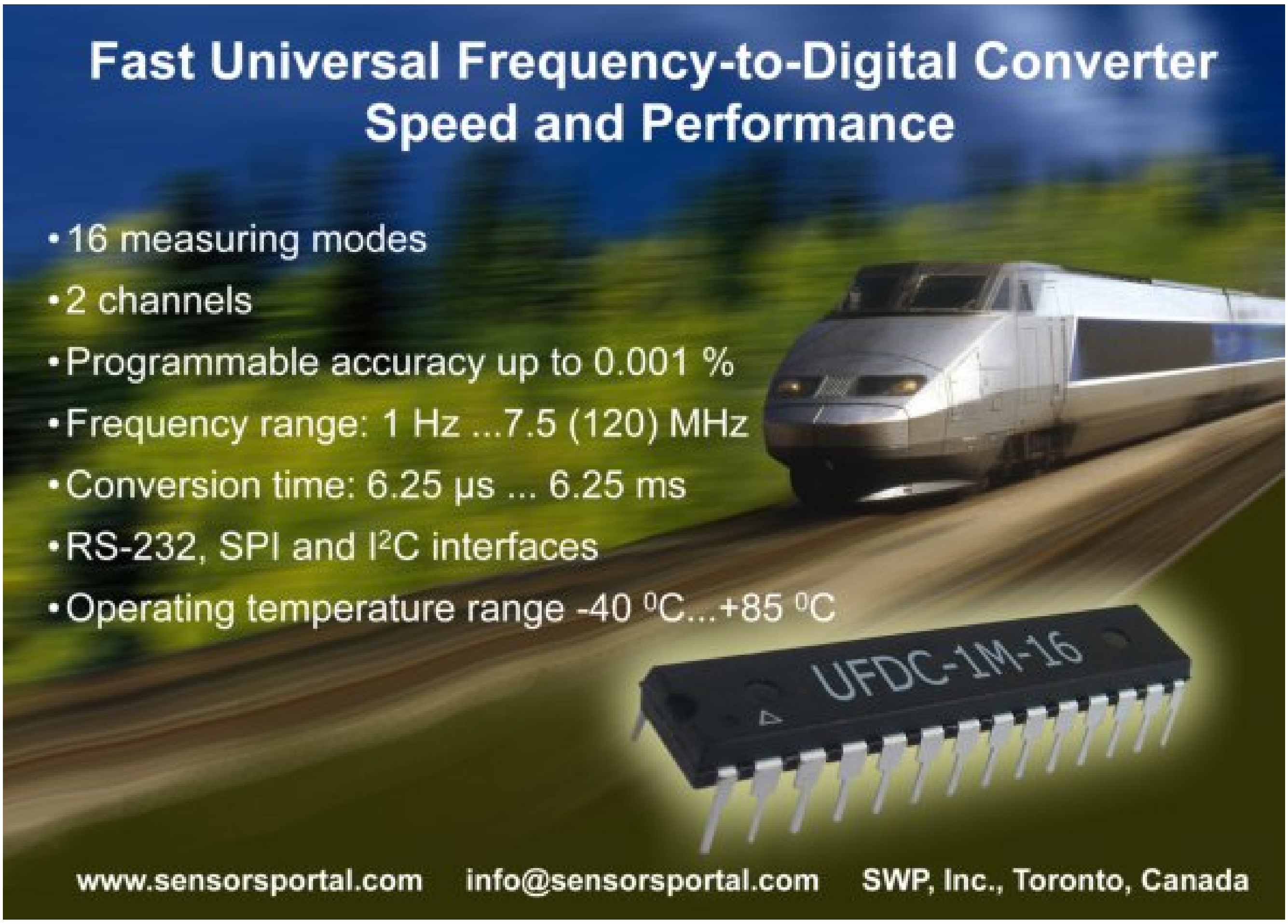

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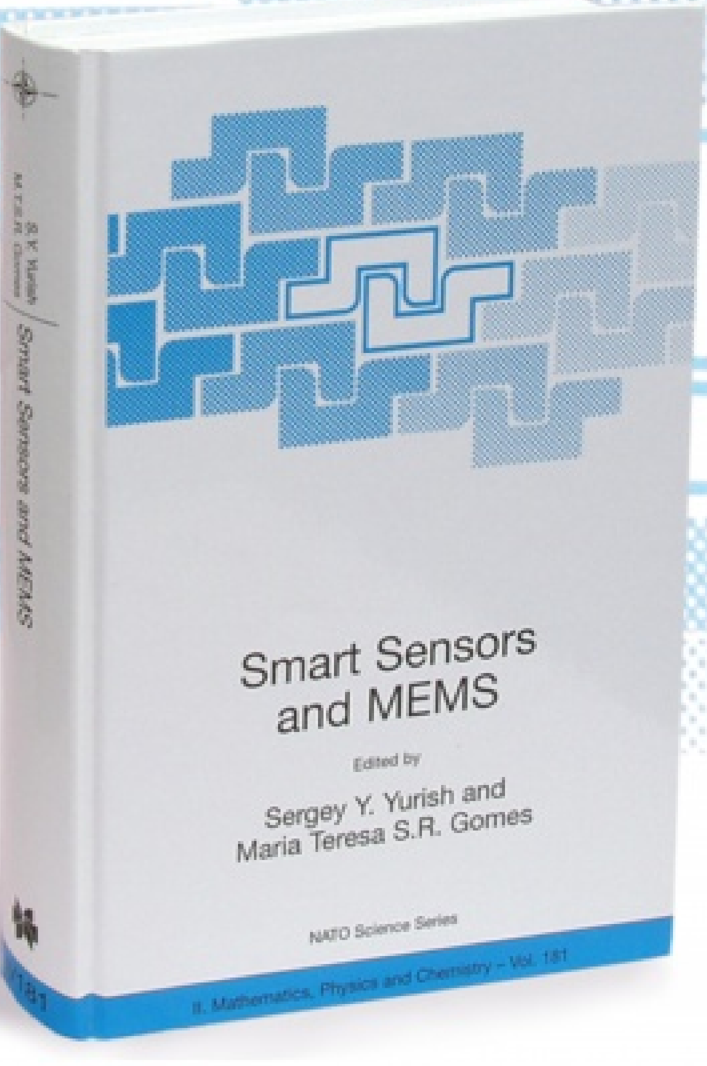
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