

## An Image Matching Method Based on Fourier and LOG-Polar Transform

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**Abstract:** This Traditional template matching methods are not appropriate for the situation of large angle rotation between two images in the online detection for industrial production. Aiming at this problem, Fourier transform algorithm was introduced to correct image rotation angle based on its rotatory invariance in time-frequency domain, orienting image under test in the same direction with reference image, and then match these images using matching algorithm based on log-polar transform. Compared with the current matching algorithms, experimental results show that the proposed algorithm can not only match two images with rotation of arbitrary angle, but also possess a high matching accuracy and applicability. In addition, the validity and reliability of algorithm was verified by simulated matching experiment targeting circular images.  
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**Keywords:** Fourier transform, Angle correction, Log-polar transform, Image matching.

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### 1. Introduction

With the fast development of information technology, image matching technology has been widely used in machine vision detection, medical diagnosis, pattern recognition, remote sensing etc. [1, 2]. It has become an important item of modern information processing, image information and computer vision field in particular, and thus a difficult issue which has been focused on. There is a random rotation between the tested image and reference image in application, encumbering matching work to some extent and affecting the accuracy of matching. Many scholars at home and abroad have been dedicated to the study of anti-rotation matching method aimed at improving the speed, precision and anti - interference of matching in

recent years. However, these proposed methods always have the limit of rotary angle range. For instance, some matching algorithms [3, 4] can only solve the problem of small-angle rotation while other methods [5, 6] are only suitable for wide-angle rotating cases, and the image matching problem can not be solved for arbitrary rotation angle. Although the problem of the tested images with arbitrary rotary angle can be solved with matching method based on moment invariants [7], the matching speed is slow and recognition rate low. Matching method based on circle projection vector [8] can resist arbitrary rotation angle but are quite sensitive to noise.

On the basis of thorough study on the existing methods, circular image matching method based on Fourier transform [9] has been proposed in this paper. Firstly, the tested image was binarized using

morphological processing approach, and then the rotational angle of the detected image was tested via fast Fourier transform, which was corrected rotationally afterwards until in the same direction with the reference image, and finally the tested and reference image were matched using the matching algorithm based on logarithmic polar transformation [10-12]. Authentic beer bottle image captured from online detection system was chosen as the research object. The algorithm can overcome arbitrary angle rotation problem of the tested images, which was validated through experiments, and the algorithm shows advantages of higher speed, recognition rate etc over other methods.

The organization of the paper is as follows. After a general introduction of the image matching technology and the research object in the paper, the method of rotational angle correction based on fast Fourier transform has been discussed in section 2. Log-polar transform was applied in section 3 for the conversion from circular image matching to rectangular image matching. In section 4, angle correction and image matching experiment was performed. The paper has been concluded in section 5.

## 2. Image Rotation Correction Based on FFT

### 2.1. Localization of Circular Area

As can be seen from Fig. 1, the acquired image contains background and the gray-scale contrast of the background and effective region of the cap (circular area) is low. In order to effectively extract circular area, it is necessary to pre-treat the image to enhance the significance of circular area. Three-stage linear transformation pre-treatment method was adopted after analyzing the image, and the corresponding calculation formula is shown as follows:

$$g(x, y) = \begin{cases} (c/a)f(x, y), & 0 \leq f(x, y) < a \\ [(d-c)/(b-a)][f(x, y) - a] + c, & a \leq f(x, y) < b \\ f(x, y), & b \leq f(x, y) < d \end{cases} \quad (1)$$

where  $g(x, y)$  stands for the gray-scale value of the image after conversion and  $f(x, y)$  that of the original

image;  $a, b, c, d$  are threshold values dividing staging areas and could be determined with several experiments:  $a = 50, b = 125, c = 0$  and  $d = 255$ . Gray-scale interval  $[a, b]$  was stretched linearly as a result of conversion while  $[0, a]$  was compressed (Fig. 1(b)). After transformation, it is easy to ascertain the coordinates of circular area boundary points. Thereby the circular area was extracted according to the three-point-orientation-round method, which is shown in Fig. 1(c).

### 2.2. Rotational Angle Correction Based on fast Fourier Transform

There is always angular deviation between the circular image after localization and the reference image while most of the current matching algorithms exhibit low recognition rate for the matching of two images with angle rotation between them, especially for the real-time matching issue of production line, which is inefficient. The rotational angle correction algorithm based on fast Fourier transform is able to angularly correct real time images to be in the exactly same direction with reference image before matching in this paper, and thus effectively solve the rotation problem that exists in subsequent matching and improve the speed and precision of matching.

1) Fourier transform method.

Fourier transform is commonly used in the field of image processing and can deal with problems of airspace and time domain. The Fourier transform used in digital image processing is usually in the form of discretion, i.e. discrete Fourier transform (DFT), and the Fourier transform  $F(u)$  of single variable discrete function  $f(x)$  is given by:

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}, \quad (2)$$

Similarly, the discrete Fourier transform of a function  $f(x, y)$  with image size of  $M \times N$  can be given by the following formula:

$$F(\mu, \nu) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{\mu x}{M} + \frac{\nu y}{N})}, \quad (3)$$

where  $x=0, 1, 2, \dots, M-1; y=0, 1, 2, \dots, N-1$ .



(a) original image



(b) gray-scale image after conversion



(c) reference image

Fig. 1. Localization of circular area.

As can be known from the rotational characteristics of image discrete Fourier transform: when discrete function  $f(x, y)$  in time domain rotate through an angle of  $\theta$ , the corresponding transform  $F(u, v)$  in frequency domain is also rotated through the same angle. Function  $f(r, \theta)$  and  $F(w, \Phi)$  are the corresponding forms of  $f(x, y)$  and  $F(u, v)$  in polar coordinate respectively, which is defined as follows:

$$\mathfrak{F}[f(r, \theta + \theta_0)] = F(w, \phi + \theta_0), \quad (4)$$

According to the above-mentioned properties, if the image in time domain shows a strong directivity, it also presents similar characteristic in the frequency domain after DFT.

2) Rotational angle correction based on Fourier transform.

It can be approximately seen from Fig. 1(c) that the direction with most abundant textures is on the horizon of the circular image and direct Fourier transform will cause too complex calculation. To simplify the calculation and highlight texture features of the image, we can binarize the image through morphology, and the processed result is shown in

Fig. 2(a). The direction with most abundant textures in the rotated image can be considered as the horizon of the original circular image. Hence, the tilt angle of the original image can be obtained as long as the direction with most abundant textures is identified. After Fourier transform of the image, the straight line on time domain and frequency domain is perpendicular to each other, as shown in Fig. 2(b).

The straight line a in Fig. 2(b) is formed by the reflection of the textures in the horizontal direction of the circular image, being called as horizontal amplitude line of which the intensity is defined as the accumulated gray scale of pixel in the radial direction of the straight line.

Considering the center of circular image as origin and half of the image width as radius, draw a circle and then count the accumulated gray scale in radial direction in the range of 0-180°, and the actual rotational angle is  $\alpha$  corresponding to the maximum value (accumulated gray scale on straight line a), which is the angle between straight line a and x-axis in Fig. 2(b). Rotate the whole image clockwise through that angle and the rotational angle will be corrected, and the result is shown in Fig. 2(c).

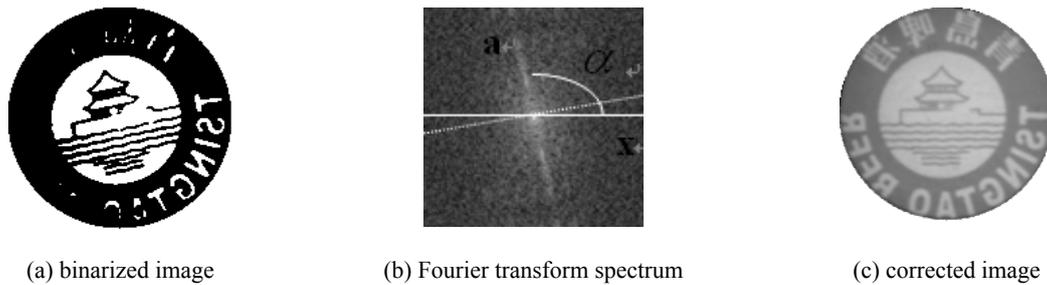


Fig. 2. Correction process based on DFT.

If the actual rotational angle of circular image exceeds 180 degrees, a  $360 - \alpha$  rotation angle will be obtained after DFT and image inversion will occur after correction according to the above method (Fig. 3(a)).

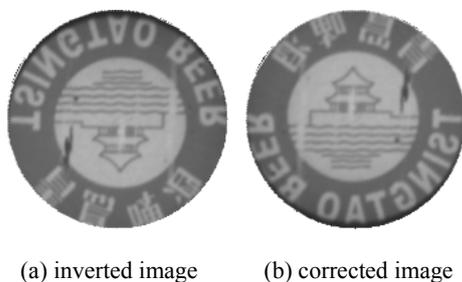


Fig. 3. Correction of inverted image.

(15 gray scale values were accumulated in this paper), and record the coordinates of these locations. Consequently, regardless of whether the image is inverted, always subtract the accumulated gray scale value at lower position from that at upper position corresponding to the coordinates, the correction of inverted image can be completed with the analysis of the subtraction result. The corrected image is shown in Fig. 3(b).

### 3. Matching Based on LOG-POLAR Transformation

After the rotational angle correction process, the measured image is corrected to be in the same direction with reference image, and therefore product quality test can be conducted via image matching method. Method based on log-polar transform was applied in this paper for the conversion from circular image matching to rectangular image matching. The image matching was performed with extracting

Aimed at this phenomenon, continuously measure and accumulate gray scale values at certain horizontal upper and lower positions of the image respectively

projection vector in log-polar plane and analyzing its similarity.

The log-polar transform of image is converting Cartesian coordinate system into log-polar coordinate system, and the scale and rotation transformation in the original coordinate system are transformed into translational transform. Log-polar transform is shown in Fig. 4, where circular and spoke lines in the left figure are mapped to Links line mapped to horizontal and vertical lines in the right log-polar coordinate plane.

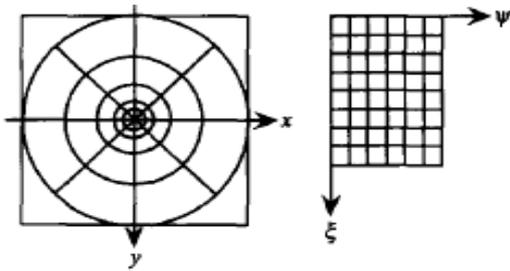


Fig. 4. Sketch map for log-polar transform.

Polar coordinate system  $(\rho, \theta)$  is expressed by Cartesian coordinate system  $(x, y)$  in formula (4), where

$$\rho = \sqrt{x^2 + y^2} \quad \theta = \text{actg}(y/x), \quad (5)$$

The correlation between log polar and polar coordinate is given by:

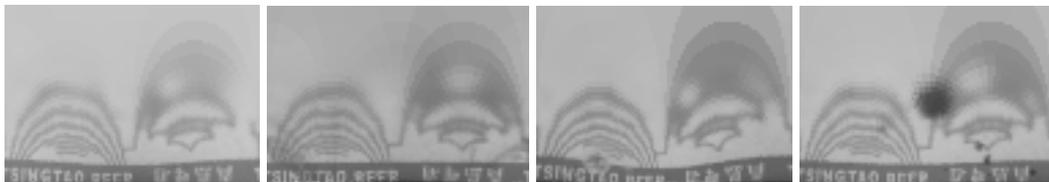
$$\xi = k \log \rho \quad \psi = k\theta, \quad (6)$$

where  $\xi$  is the distance axis;  $\psi$  is the angle axis;  $k$  is the amplification factor. Because the range of  $\xi$  and  $\psi$  is too small (both are decimals), it is necessary to amplify formulation (5). The conversion results are shown in Fig. 5.

Circular image could be converted into rectangular image through transform was applied in this paper for matching. The image matching was performed with extracting projection vector in rectangular image and analyzing its similarity.



(a) images for testing after correction



(b) images after transformation

Fig. 5. Being tested image and transformation matrix figure.

#### 4. Angle Correction and Matching Experiment

Localized circular beer bottle (a circular image) was chosen as reference image in the experiment (Fig 1(c)). The tested images of group A (i.e. real-time images) are 120 images of 120 times arbitrary rotations of the reference image, which should be qualified products in online production; The tested images of group B are circular caps involving flaws of blemishes, scratches, color deviations etc., which should be unqualified products in online production. Meeting the demands of experiment, a complete line cap detection and simulation system was designed in this paper. At first, characteristics of the reference

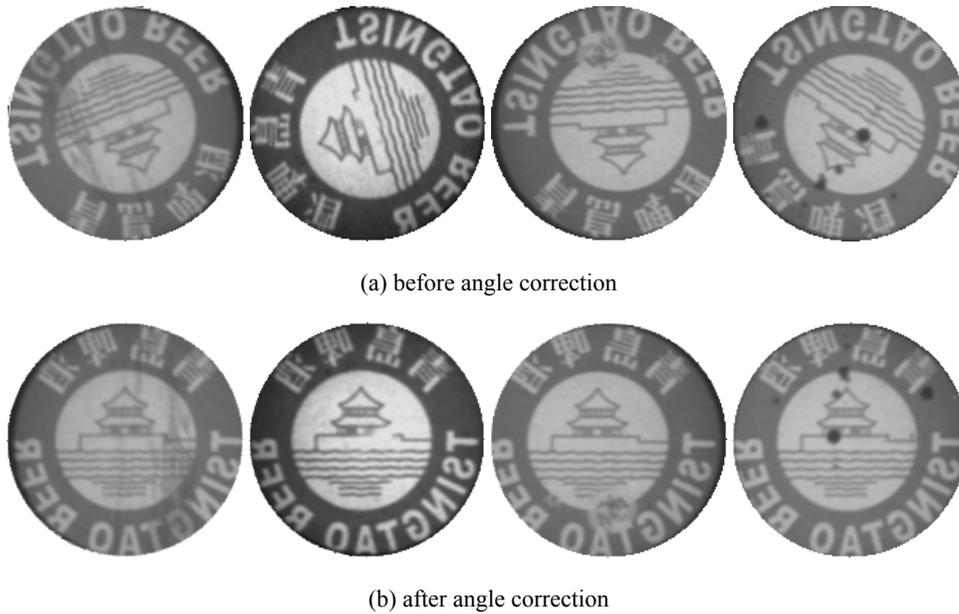
image was extracted via log-polar transform method and then recorded in the system. Read in tested image and corrected the measured image angularly applying Fourier transform. The characteristics were extracted and recorded by the system, and finally whether the tested image was a qualified image of beer bottle cup was judged with the log-polar transform matching algorithm.

##### 4.1. Angle Correction Experiment

The preciseness of angle correction algorithm directly affects the matching accuracy of the entire system, so each image was tested several times in this

paper to obtain accurate correction results. In Fig. 6, the angle correction algorithm based on Fourier transform still could correct the images accurately under situations of contrast changes and noise

pollution of the tested images, to ensure that the following matching work can be conducted smoothly. The experimental results demonstrate that the correction algorithm possesses strong robustness.



**Fig. 6.** Images of angle correction.

#### 4.2. Matching Experiment

Aiming at the images of group A and B in image library, the algorithm proposed by us, image matching method based on moment invariants and matching method based on circular projection vector were compared experimentally, and the results are shown in Table 1. FAR is called false acceptance rate, which refers to the probability of unqualified circular image being mistaken as qualified image. In other words, the system wrongly determines the beer bottle caps that do not meet specifications to be qualified products in production line. FRR is called false rejection rate, namely the probability of qualified circular image being judged as the unqualified. Obviously, the larger FRR value is, the more accurate the system will be, and the smaller the chance of unqualified products pass detection, but this will also make an amount of qualified products unacceptable due to the detection of the system. Recognition rate is the ratio of correct identification

quantity of samples accounting for the amount of tested samples.

With the aid of analysis of experimental data in the table, the algorithm in this paper not only has advantage of time but also achieves greater improvement in recognition accuracy compared to the other two algorithms. Approaches based on moment invariants and circular projection vector can resist the image rotation in matching, though it is more susceptible to noise interference, while our algorithm corrects all the images angularly before image matching and the entire correction process is isolated from any noise interference, making the entire matching algorithm possesses greater robustness. The experimental results show that the matching algorithm of circular image based on Fourier transform exhibits a high matching rate and strong adaptability, so procedure containing the identification algorithm can be embedded in the hardware system for quality detection of the beer bottle cup.

**Table 1.** Comparison of the three algorithms.

Recognition method	FAR(%)	FRR(%)	Recognition rate (%)	Matching time (s)
Moment invariants	3.7926	7.3315	88.8759	1.1989
Circular projection	4.3185	1.8791	93.8024	0.7837
This paper	0.0943	1.1543	98.7514	0.3675

## 5. Conclusion

Matching method for circular image based on Fourier transform and log-polar transform was proposed in the paper. In the situation of arbitrary angle rotation of the measured image, accurate correction of the rotated image can be achieved with Fourier transform algorithm to ensure the accuracy of subsequent matching; and the matching algorithm based on log-polar transform converts circular image into rectangular image to simplify the complexity of the algorithm, so that the entire matching system reaches a high speed. The experimental results demonstrate that the algorithm possesses strong anti-noise capability and greater robustness. In summary, the proposed algorithm is suitable to be applied to situations of arbitrary rotation and higher demanded matching accuracy. Although the experimental object in the paper is a circular image, as a matter of fact, images of arbitrary shape can be matched by the proposed method. This matching algorithm shows could find its promising applications in beer bottle cup online detection system and be applied to the online detection of other products.

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