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Kinematics and Application of a Hybrid Industrial Robot – Delta-RST

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Abstract: Serial robots and parallel robots have their own pros and cons. While hybrid robots consisting of both of them are possible and expected to retain their merits and minimize the disadvantages. The Delta-RST presented here is such a hybrid robot built up by integrating a 3-DoFs traditional Delta parallel structure and a 3-DoFs RST robotic wrist. In this paper, we focus on its kinematics analysis and its applications in industry. Firstly, the robotic system of the Delta-RST will be described briefly. Then the complete and systemic kinematics of this kind of robot will be presented in detail, followed by simulations and applications to demonstrate the correctness of the analysis, as well as the effectiveness of the developed robotic system. The closed-form kinematic analysis results are universal for similar hybrid robots constructing with the Delta parallel mechanism and serial chains. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Hybrid robots, Industrial robots, Kinematics analysis, Control and application.

1. Introduction

In general, most currently-existing industrial robots designed are either built up of serial or parallel kinematic chains [1]. Serial robots consist of a series of active joints connecting the base to the end effector, while parallel robots are composed of a set of parallel chains (legs) with active and passive joints. Due to their open-loop and cantilever-type kinematic configuration, serial robots feature large workspaces and high dexterity but suffer from lack of stiffness and from relatively large positioning errors. On the other hand, as each independent leg forming a closed loop and connecting the base and the moving platform, parallel robots are able to achieve high stiffness and high force-to-weight ratio. However, parallel robots are known for a restricted workspace and low dexterity. In a word, serial robots and parallel robots have their own advantages and disadvantages.

A general manipulating task in industry can be divided into a position sub-task (position mechanism) and an orientation sub-task (orientation mechanism) [2], respectively. The position mechanism controls the position whereas the orientation mechanism manipulates the orientation of the end effecter. It is natural to combine the serial and parallel chains together to obtain a hybrid industrial robot, so as to retain the merits of serial configurations, i.e. large workspace and high dexterity, and parallel structures, i.e. high stiffness and high force-to-weight ratio, while their disadvantages are minimized. A simple way to do this is to utilize a parallel mechanism to position the end-effecter and connect serial joints to adjust the orientation, and vice versa.

Owing to the above observation, hybrid robot manipulators have attracted more and more attention

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in the field of robotics, although comparatively little literature on these robots is available currently. Some researches have dealt with the design of hybrid robots [3], resulting in novel but well-known robots like Tricept [4], TriVariant [5] and Exechon [6]. These three robots could be treated as serial combination (connection) of parallel and serial mechanisms both with few degrees of freedom. They have found various commercial applications such as high-speed milling, welding and component assembling in an aeronautical and automotive industry [7, 8]. In addition, lots of researches have focused on the analysis and modeling of several hybrid robots [9-13]. Literatures [10] and [11] investigated the kinematics and dynamics of a type of hybrid robots constructed by serially connected non-redundant parallel modules, respectively; while [12] and [13] aimed to propose a universal velocity-based model for the serial connection type (e.g. parallel-parallel, parallel-serial and serial-parallel) of hybrid robots, which certainly results in comparatively low efficiency and high complexity.

Since invented by Clavel in 1980s [14], the Delta parallel mechanism has been successfully and widely used in many fields [15] like rapid picking and packaging, thanks to its outstanding features including light-structure, high-speed, high-accuracy and simple-control. Researches on the Delta parallel mechanism include almost all aspects [16-18]. However, as a translational parallel manipulator, its application is limited because of lacking capability to change the orientation of the manipulated object. As a potential solution, Delta with orientation is considered as one of the tendencies [19], resulting in commercial robots like FANUC M-1iA/M-3iA serials [20] and ABB IRB 340/360 FlexPicker [21]. However, to the best of our knowledge, there is not complete and systemic kinematic analysis reported for this class of robots in the literatures.

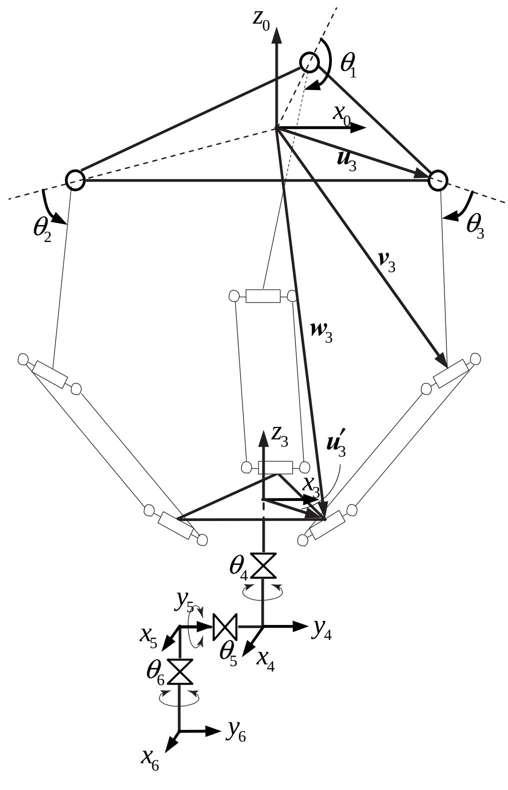
In this paper, we present the closed-form kinematics solutions for a parallel-serial hybrid robot – the Delta-RST. To some extents, the Delta-RST may be regarded as a hybrid robot built up through connecting a Delta parallel structure with a serial robotic wrist. However, to completely analyze its kinematics and dynamics, the Delta-RST robot should be treated as a whole to obtain closed-form relationship between the base and the end-effector, which is critical and fundamental for further researches, such as workspace analysis, dimensional synthesis and layout optimization, and so on. The presented analysis and methods are able to be applied to similar robots with Delta parallel structure and serial chain fixed to the moving platform.

The rest of this paper is organized as follows. Section 2 describes the Delta-RST robotic system. Section 3 presents the complete kinematics analysis of Delta-RST-like robots. Experiments and applications are provided in Section 4, followed by a conclusion in Section 5.

2. Robotic System

To improve its dexterity to satisfy more industrial applications' demands, the RST robotic wrist is integrated to the moving platform of the Delta parallel mechanism in our group, as shown in Fig. 1.

(a) The Delta-RST prototype.



(b) The kinematic diagram of Delta-RST.

Fig. 1. The Delta-RST robot.

The RST robotic wrist can be driven through three motors mounted on the fixed platform of the Delta mechanism and three corresponding axes with universal couplings. An alternative is to configure each driver with its corresponding axis on the moving platform, respectively. Surely, the former one may achieve high-speed and fast-response since the inertia of the moving part is less, while the latter option is entitled to simple structure and control.

Using the Kutzbach-Grubler formula [22], it is convenient to obtain the degrees of freedom of the Delta-RST robot, as $F = 6(n-g-1)+\sum_{i=1}^{g} f_i$, where n refers to the total number of moving links, g to the number of joints, and f_i to the DoFs of the i-th joint. As regards to Delta-RST, we have, $F = 6(20-24-1)+3\times 6+1\times 18 = 6$.

Note that the Delta parallel structure has no rotational degree of freedom. Therefore, Delta-RST is a hybrid robot with six degrees of freedom: three-DoFs Delta parallel mechanism to position the end-effector while three-DoFs serial RST robotic wrist responsible for controlling the orientation. Owing to the different roles played by the two sub-structures, it is possible to derive the kinematics and dynamics of them separately, and then combine them with the general knowledge of robotics, to obtain that of the entire hybrid robot.

3. Kinematic Analysis

In this Section, we analyze the kinematics of the Delta-RST robot using the vector method and the configuration matrix expressing approach. Without loss of generality, denote $(x,y,z,\alpha,\beta,\gamma)$ the configuration of the end-effector and θ_i the rotational angle of each active joint.

3.1. Forward Kinematics

Forward kinematics is to compute the configuration of the end-effector $(x,y,z,\alpha,\beta,\gamma)$ from given joint angles θ_i. Setting up coordinate frames as shown in Fig. 1(b), the position of Point A_i (I = 1, 2, 3) with respect to the base frame {0} can be expressed as,

$$\boldsymbol{u}_i = \begin{bmatrix} l_0 \cos\gamma_i & l_0 \sin\gamma_i & 0 \end{bmatrix}^T, \tag{1}$$

where $\gamma_i = \frac{4i-1}{6}\pi$. Similarly, Point C_i relative to Frame {3} is $\boldsymbol{u}_i' = \begin{bmatrix} l_3 \cos\gamma_i & l_3 \sin\gamma_i & 0 \end{bmatrix}^T$.

Describing Point B_i with respect to Frame {0}, we obtain,

$$\boldsymbol{v}_i = \begin{bmatrix} (l_0 + l_1\cos\theta_i)\cos\gamma_i & (l_0 + l_1\cos\theta_i)\sin\gamma_i & -l_1\sin\theta_i \end{bmatrix}^T \tag{2}$$

Supposing the origin of Frame {3} relative to Frame {0} is $\boldsymbol{e} = \begin{bmatrix} x_3 & y_3 & z_3 \end{bmatrix}^T$, C_i can be expressed also respect to Frame {0} as $\boldsymbol{w}_i = \boldsymbol{e}_i + \boldsymbol{u}_i'$.

Considering the geometry of the Delta robot, we have the following constrain,

$$|\boldsymbol{v}_i - \boldsymbol{w}_i| = l_2, \tag{3}$$

which can be further simplified as,

$$x_3^2 + y_3^2 + z_3^2 + a_i \cdot x_3 + b_i \cdot y_3 + c_i \cdot z_3 + d_i = 0, \tag{4}$$

where

$a_i = -2(l_1\cos\theta_i + l_0 - l_3)\cos\gamma_i$,

$b_i = -2(l_1\cos\theta_i + l_0 - l_3)\sin\gamma_i$,

$c_i = 2l_1\sin\theta_i$,

$d_i = l_1^2 - l_2^2 + (l_0 - l_3)^2 + 2l_1l_0\cos\theta_i - 2l_1l_3\cos\theta_i$.

Eq. (4) leads to two linear equations as,

$$\begin{cases} (a_1-a_2)\cdot x_3 + (b_1-b_2)\cdot y_3 + (c_1-c_2)\cdot z_3 + (d_1-d_2) = 0 \\ (a_2-a_3)\cdot x_3 + (b_2-b_3)\cdot y_3 + (c_2-c_3)\cdot z_3 + (d_2-d_3) = 0 \end{cases} \tag{5}$$

As $z_3 \neq 0$ in practice, x_3 and y_3 can be represented by z_3, resulting in,

$$\begin{cases} x_3 = m_1 z_3 + n_1 \\ y_3 = m_2 z_3 + n_2 \end{cases}, \tag{6}$$

where

$$m_1 = \frac{(c_2-c_3)(b_1-b_2)-(c_1-c_2)(b_2-b_3)}{(a_1-a_2)(b_2-b_3)-(a_2-a_3)(b_1-b_2)},$$

$$n_1 = \frac{(d_2-d_3)(b_1-b_2)-(d_1-d_2)(b_2-b_3)}{(a_1-a_2)(b_2-b_3)-(a_2-a_3)(b_1-b_2)},$$

$$m_2 = \frac{(c_2-c_3)(b_1-b_2)-(c_1-c_2)(b_2-b_3)}{(a_1-a_2)(b_2-b_3)-(a_2-a_3)(b_1-b_2)},$$

$$n_2 = \frac{(d_2-d_3)(b_1-b_2)-(d_1-d_2)(b_2-b_3)}{(a_1-a_2)(b_2-b_3)-(a_2-a_3)(b_1-b_2)}.$$

Substituting Eq. (6) into Eq. (4), we get

$$a'z_3^2 + b'\cdot z_3 + c' = 0, \tag{7}$$

where

$$\begin{aligned} a' &= m_1^2 + m_2^2 + 1 \\ b' &= 2m_1n_1 + 2m_2n_2 + a_i\cdot m_1 + a_i\cdot m_2 \\ c' &= n_1^2 + n_2^2 + a_i\cdot n_1 + a_i\cdot n_2 + c_i\cdot z_3 + d_i \end{aligned}.$$

Solving Eq. (7), we have

$$z_3 = \frac{-b' - \sqrt{b'^2 - 4a'c'}}{2a'} \tag{8}$$

Then x_3 and y_3 can be computed with Eq. (6). Using the homogeneous transformation matrix, the configuration of the moving platform with respect to the base frame is,

$$ {}^{0}T_3 = \begin{bmatrix} \boldsymbol{I} & \begin{bmatrix} x_3 & y_3 & z_3 \end{bmatrix}^T \\ 0 & 1 \end{bmatrix}, \qquad (9) $$

where $\boldsymbol{I}$ is the 3×3 identity matrix.

$$ \begin{aligned} {}^{3}T_6 &= Rot(z,\theta_4')Trans(z,-l_4)\cdot Rot(y,\theta_5)Trans(y,-l_5)\cdot \\ &\quad Rot(z,\theta_6)Trans(z,-l_6) \\ &= \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ 0 & 1 & 0 & -l_5 \\ -s_5 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_6 & -s_6 & 0 \\ s_6 & c_6 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & s_4l_5 - c_4s_5l_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & -c_4l_5 - s_4s_5l_6 \\ -s_5c_6 & s_5s_6 & c_5 & -l_4 - c_5l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \end{aligned} \qquad (10) $$

where $\theta_4' = \theta_4 - 90$, $c_i \triangleq \cos\theta_i$, $s_i \triangleq \sin\theta_i$. Hereafter, similar simplified representations will be used.

As regards to the RST robotic wrist, using the Z-Y-Z Euler angle notation (that is to say, rotate around the Z$_4$-axis with an angle of $\theta_4 - 90^{\circ}$ first, then rotate θ_5 around the Y$_5$-axis, finally rotate θ_6 around the Z$_6$-axis, referring to Fig. 1 (b)), the homogeneous transformation matrix can be computed as Eq. (2).

Therefore, the forward kinematics of the Delta-RST is

$$ \begin{aligned} {}^{0}T_6 &= {}^{0}T_3\,{}^{3}T_6 \\ &= \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 & x_3 + s_4l_5 - c_4s_5l_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & y_3 - c_4l_5 - s_4s_5l_6 \\ -s_5c_6 & s_5s_6 & c_5 & z_3 - l_4 - c_5l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \qquad (11) $$

3.2. Inverse Kinematics

Inverse kinematics is to solve the joint angles θ_i when giving the target configuration of the robot's end-effecter, $(x, y, z, \alpha, \beta, \gamma)$. As a Delta parallel mechanism has only the translational degrees of freedom, the orientation of the end-effecter is depending on the RST robotic wrist. Hence, according to Eq. (10), we simply have,

$$ \begin{cases} \theta_4 = 90 + \alpha \\ \theta_5 = \beta \\ \theta_6 = \gamma \end{cases} \qquad (12) $$

From Eq. (11), the center of the moving platform (also the origin of Frame {3}) can be computed as,

$$ \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} x - s_4l_5 + c_4s_5l_6 \\ y + c_4l_5 + s_4s_5l_6 \\ z + l_4 + c_5l_6 \end{bmatrix} \qquad (13) $$

Recalling Eq. (4), taking $\theta_i\ (i = 1,2,3)$ as the unknowns, it can be transformed to,

$$ a_i \cdot \sin\theta_i + b_i \cdot \cos\theta_i + c_i = 0, \qquad (14) $$

where

$$ \begin{aligned} a_i &= 2z_3l_1 \\ b_i &= -2l_1\left(x_3\cos\gamma_i + y_3\sin\gamma_i - l_0 + l_3\right) \\ c_i &= x_3^2 + y_3^2 + z_3^2 + l_1^2 - l_2^2 + \left(l_0 - l_3\right)^2 - \\ &\quad 2\left(x_3\cos\gamma_i + y_3\sin\gamma_i\right)\left(l_0 - l_3\right) \end{aligned} . $$

Solving Eq. (14) yields,

$$ \theta_i = 2\arctan\frac{-a_i + \sqrt{a_i^2 + b_i^2 - c_i^2}}{c_i - b_i} \qquad (15) $$

3.3. Jacobian Matrix

To compute the Jacobian Matrix of the Delta-RST robot, we can calculate that of the Delta parallel mechanism and of the RST robotic wrist respectively, and then combine them together with recursive formula.

Getting derivative with respect to $\theta_i\ (i = 1,2,3)$ of Eq. (14) results in

$$ \boldsymbol{J}_3 \cdot \dot{\boldsymbol{\theta}}_3 = \boldsymbol{G}_3 \cdot \boldsymbol{v}_3, \qquad (16) $$

where $\dot{\boldsymbol{\theta}}_3 = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$, $\boldsymbol{v}_3 = \begin{bmatrix} \dot{x}_3 & \dot{y}_3 & \dot{z}_3 \end{bmatrix}^T$,

$\boldsymbol{J}_3 = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix}$, $\boldsymbol{G}_3 = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$, and

$$ \begin{aligned} J_{ii} &= l_1z_3 \cdot \cos\theta_i + l_1\left(x_3\cos\gamma_i + y_3\sin\gamma_i - l_{03}\right)\sin\theta_i \\ G_{i1} &= x_3 - l_{03}\cos\gamma_i - l_1\cos\theta_i\cos\gamma_i \\ G_{i2} &= y_3 - l_{03}\sin\gamma_i - l_1\cos\theta_i\sin a\gamma_i \\ G_{i3} &= z_3 + l_1\sin\theta_i \end{aligned} . $$

If $\boldsymbol{G}_3$ is not singular, we obtain

$$ \boldsymbol{v}_3 = \boldsymbol{G}_3^{-1}\boldsymbol{J}_3 \cdot \dot{\boldsymbol{\theta}}_3 \qquad (17) $$

As regards to the RST robotic wrist, its velocity can be expressed as

$$ \boldsymbol{v}_6 = \boldsymbol{J}_6 \cdot \dot{\boldsymbol{\theta}}_6, \qquad (18) $$

where $\boldsymbol{v}_6$ is the 6×1 vector, $\dot{\boldsymbol{\theta}}_6 = \begin{bmatrix}\dot{\theta}_4 & \dot{\theta}_5 & \dot{\theta}_6\end{bmatrix}^T$ and $\boldsymbol{J}_6$ is the 6×3 matrix having the form,

$$\boldsymbol{J}_6 = \begin{bmatrix}\vec{M}_4 & \vec{M}_5 & \vec{M}_6 \\ \vec{S}_4 & \vec{S}_5 & \vec{S}_6\end{bmatrix} = \begin{bmatrix}\boldsymbol{M}_6 \\ \boldsymbol{S}_6\end{bmatrix}, \quad (19)$$

where $\vec{M}_j = \vec{S}_j \times \left(\vec{P} - \vec{R}_j\right)$, $\vec{S}_j$ ($j = 4,5,6$) stands for the axial unit vector of the j-th joint, $\vec{P}$ refers to the position of the end-effector with respect to Frame {3} (also the base frame of the RST robotic wrist), $\vec{R}_j$ to the position vector of the j-th joint relative to its own coordinate. Specifically, they are

$$\vec{S}_4 = \begin{bmatrix}0\\0\\1\end{bmatrix},\ \vec{S}_5 = \begin{bmatrix}c_4 s_5\\ s_4 s_5\\ c_5\end{bmatrix},\ \vec{S}_6 = \begin{bmatrix}c_4 s_5\\ s_4 s_5\\ c_5\end{bmatrix},$$

$$\vec{R}_4 = \begin{bmatrix}0\\0\\-l_4\end{bmatrix},\ \vec{R}_5 = \begin{bmatrix}s_4 l_5\\ -c_4 l_5\\ -l_4\end{bmatrix},$$

$$\vec{P} = \vec{R}_6 = \begin{bmatrix}s_4 l_5 - c_4 s_5 l_6\\ -c_4 l_5 - s_4 s_5 l_6\\ -l_4 - c_5 l_6\end{bmatrix},$$

$$\vec{M}_4 = \begin{bmatrix}c_4 l_5 + s_4 s_5 l_6\\ s_4 l_5 - c_4 s_5 l_6\\ 0\end{bmatrix},\ \vec{M}_5 = \vec{M}_6 = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

Hence we have

$$\boldsymbol{M}_6 = \begin{bmatrix}c_4 l_5 + s_4 s_5 l_6 & 0 & 0\\ s_4 l_5 - c_4 s_5 l_6 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \boldsymbol{S}_6 = \begin{bmatrix}0 & c_4 s_5 & c_4 s_5\\ 0 & s_4 s_5 & s_4 s_5\\ 1 & c_5 & c_5\end{bmatrix}.$$

On the basis of the above analysis, building up the recurrence formula for the velocity of the RST robotic wrist from the perspective of the entire Delta-RST robot, we may obtain the following equations,

$$\begin{cases} {}^{i+1}\boldsymbol{\omega}_{i+1} = {}^{i+1}_{i}R\, {}^{i}\boldsymbol{\omega}_i + \dot{\theta}_{i+1}\, {}^{i+1}\hat{\mathbf{Z}}_{i+1} \\ {}^{i+1}\mathbf{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\mathbf{v}_i + {}^{i}\boldsymbol{\omega}_i \times {}^{i}\mathbf{P}_{i+1}\right)\end{cases} \quad (20)$$

As we know that

$${}^4_3R = \begin{bmatrix}c_4 & s_4 & 0\\ -s_4 & c_4 & 0\\ 0 & 0 & 1\end{bmatrix},\ {}^5_4R = \begin{bmatrix}c_5 & 0 & -s_5\\ 0 & 1 & 0\\ s_5 & 0 & c_5\end{bmatrix},$$

$${}^6_5R = \begin{bmatrix}c_6 & s_6 & 0\\ -s_6 & c_6 & 0\\ 0 & 0 & 1\end{bmatrix},\ {}^3\boldsymbol{P}_4 = \begin{bmatrix}0\\0\\-l_4\end{bmatrix},$$

$${}^4\boldsymbol{P}_5 = \begin{bmatrix}0\\-l_5\\0\end{bmatrix},$$

and as well as ${}^5\boldsymbol{P}_6 = \begin{bmatrix}0\\0\\-l_6\end{bmatrix}$, according to Eq. (20), we obtain the angular velocity and linear velocity of the j-th (j=4, 5, 6) joint, as

$$\begin{cases}{}^3\boldsymbol{\omega}_3 = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T\\ {}^3\boldsymbol{v}_3 = \boldsymbol{v}_3\end{cases} \quad (21)$$

$$\begin{cases}{}^4\boldsymbol{\omega}_4 = \begin{bmatrix}0 & 0 & \dot{\theta}_4\end{bmatrix}^T\\ {}^4\boldsymbol{v}_4 = {}^4_3R\cdot\boldsymbol{v}_3 = \begin{bmatrix}v_{3x}c_4 + v_{3y}s_4\\ -v_{3x}s_4 + v_{3y}c_4\\ v_{3z}\end{bmatrix}\end{cases} \quad (22)$$

$$\begin{cases}{}^5\boldsymbol{\omega}_5 = \begin{bmatrix}-s_5\dot{\theta}_4\\ \dot{\theta}_5\\ c_5\dot{\theta}_4\end{bmatrix}\\ {}^5\boldsymbol{v}_5 = {}^5_4R\,{}^4_3R\cdot\boldsymbol{v}_3 + \begin{bmatrix}-l_5c_5\dot{\theta}_4\\ 0\\ -l_5s_5\dot{\theta}_4\end{bmatrix}\end{cases} \quad (23)$$

$$\begin{cases}{}^6\boldsymbol{\omega}_6 = \begin{bmatrix}-s_5c_6\dot{\theta}_4 + s_6\dot{\theta}_5\\ s_5s_6\dot{\theta}_4 + c_6\dot{\theta}_5\\ c_5\dot{\theta}_4 + \dot{\theta}_6\end{bmatrix}\\ {}^6\boldsymbol{v}_6 = {}^6_3R\cdot\boldsymbol{v}_3 + \begin{bmatrix}-l_5c_5c_6\dot{\theta}_4 - l_6s_5s_6\dot{\theta}_4 - l_6c_6\dot{\theta}_5\\ l_5c_5s_6\dot{\theta}_4 - l_6s_5c_6\dot{\theta}_4 + l_6s_6\dot{\theta}_5\\ -l_5s_5\dot{\theta}_4\end{bmatrix}\end{cases} \quad (24)$$

where

$${}^6_3R = \begin{bmatrix}c_4c_5c_6 - s_4s_6 & s_4c_5c_6 + c_4s_6 & -s_5c_6\\ -c_4c_5s_6 - s_4c_6 & -s_4c_5s_6 + c_4c_6 & s_5s_6\\ c_4s_5 & s_4s_5 & c_5\end{bmatrix}$$

Finally, we get

$$\boldsymbol{v} = \begin{bmatrix}{}^6\boldsymbol{v}_6\\ {}^6\boldsymbol{\omega}_6\end{bmatrix} = \boldsymbol{J}\cdot\dot{\boldsymbol{\theta}} = \begin{bmatrix}\boldsymbol{J}_{11} & \boldsymbol{J}_{12}\\ 0 & \boldsymbol{J}_{22}\end{bmatrix}\cdot\dot{\boldsymbol{\theta}}, \quad (25)$$

where

$$\boldsymbol{J}_{11} = {}^6_3R\cdot\boldsymbol{G}_3^{-1}\boldsymbol{J}_3$$

$$\boldsymbol{J}_{12} = \begin{bmatrix}-l_5c_5c_6 - l_6s_5s_6 & -l_6c_6 & 0\\ l_5c_5s_6 - l_6s_5c_6 & l_6s_6 & 0\\ -l_5s_5 & 0 & 0\end{bmatrix},$$

$$J_{22} = \begin{bmatrix} -s_5c_6 & s_6 & 0 \\ s_5s_6 & c_6 & 0 \\ c_5 & 0 & 1 \end{bmatrix}$$

Therefore, the velocity of each active joint can be solved by computing the inverse of Jacobian matrix, as

$$\dot{\theta} = J^{-1} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (26)$$

4. Experiments and Applications

4.1. Simulations

We have analyzed the kinematics of the entire Delta-RST robot in the previous section. In order to verify the developed robotic prototype and to validate the theoretical analysis of the closed-form kinematics of the Delta-RST robot, simulations and experiments are conducted in this section.

In the simulation, we utilize the Delta-RST robot to machine the slope edge of a heart-shaped part. Fig. 2 shows us some snapshots of the machining procedure.

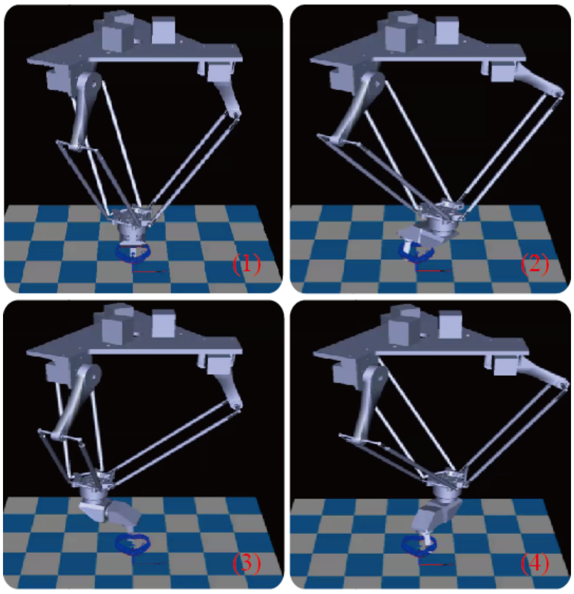


Fig. 2. Snapshots of manufacturing simulation with the Delta-RST robot.

To generate the machining trajectory, we have developed a CAD-model based off-line programming and planning algorithm for the Delta-RST robot. It firstly generates the trajectory of the machining tool (also the end-effector) in the Cartesian workspace according to the given CAD-model, then utilizes the kinematics presented in Section III to perform motion planning of the Delta-RST robot, and finally gets the trajectory of each active joint.

From the figure, we can see that the Delta parallel mechanism position the machining tool accurately during the manufacturing process. The orientation of the machining tool is also adjusted appropriately by the RST robotic wrist, to follow the slope edge of the part.

4.2. Experiments

The Delta parallel mechanism has been wildly used in the rapid picking procedure in industry. However, one of its drawbacks is that the orientation of the manipulated object cannot be changed by this robot, limited by its translational- only degrees of freedom.

This drawback can be easily overcome by the Delta-RST robot. Fig. 3 shows us serials of snapshots of an experiment, in which the Delta-RST robot is performing a rapid pick-and-place task. In this application, the robot needs to pick up some T-shaped parts put randomly in its workspace, and stack them up one by one with the right orientation on the truss. Obviously, this task could not be finished by a traditional Delta Robot.

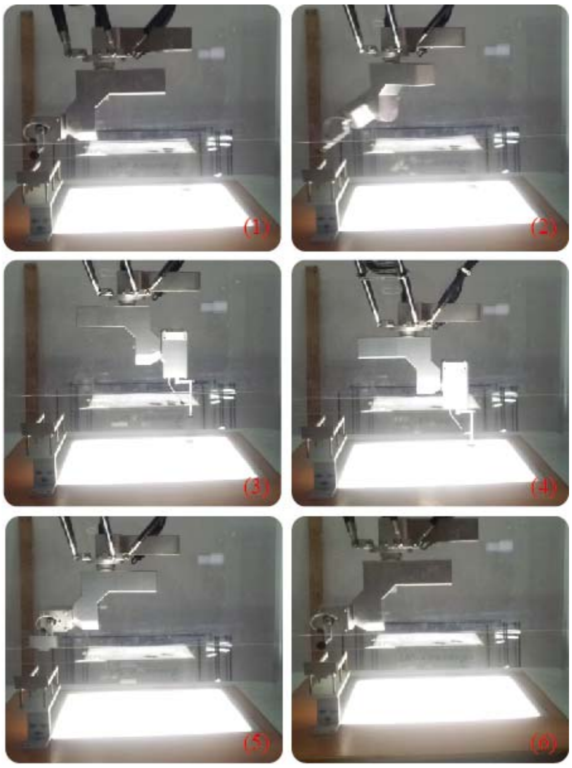


Fig. 3. Snapshots of performing the rapid pick-and-place task with the Delta-RST robot.

From Fig. 3, the Delta-RST robot firstly goes to the right position of the to-be-manipulated object and pre-adjusts the orientation of the vacuum gripper by the guidance of machine vision. This orientation pre-adjusting procedure aims at facilitating the placing and stacking process, as well as being benefit to avoid collisions with the environment. After the robot reaches its right status, the vacuum gripper picks up the part. Then the Delta-RST robot handles this part to the target stack place, simultaneously adjusts its orientations during move, and finally puts down the manipulated part. Owing to the Delta parallel mechanism, the pick-and-place task can be conducted very fast.

Therefore, the Delta-RST robot has demonstrated its potential to the applications in industry. The proposed kinematic analysis is verified in both simulations and experiments with the Delta-RST robot.

5. Conclusions

Hybrid robots merge the advantages from serial mechanisms and parallel structures. Therefore, they have great potentials in the field of industry. The treated robot, Delta-RST, in this paper, is constructed by the combination of a 3-DoFs translational Delta parallel mechanism and a 3-DoFs serial RST robotic wrist. The Delta mechanism takes charge in positioning the end-effector, while the RST robotic wrist is responsible for orientating the end-effector. Hence it features not only high-speed and high-dexterity but also relatively big-workspace.

This paper has briefly introduced the mechanical system and the control system of the Delta-RST robot developed in our group. Closed-form kinematics solutions of the Delta-RST robot have been deduced and presented in detail, which is the fundament of higher-lever research. Simulations and experiments in the realistic industrial production are carried out to verify the effectiveness of the analysis and the robotic prototype. The methodology and the analysis results are universal to similar hybrid robots based on Delta Parallel structure and serial chains.

Recently, we are conducting workspace and manipulability analysis with the Delta-RST robot to study the task-oriented layout optimization problem in the industrial production line. In the near future, the dimensional synthesis will also be considered on the basis of specific manipulation task.

References

[1]. B. Siciliano, K. Oussama, Eds., Springer handbook of robotics, *Springer*, 2008.

[2]. C. Gosselin, Determination of the workspace of 6-DOF parallel manipulators, *Journal of Mechanical Design*, 112, 3, 1990, pp. 331-336.

[3]. Campos, C. Budde, J. Hesselbach, A type synthesis method for hybrid robot structures, *Mechanism and Machine Theory*, 43, 8, 2008, pp. 984-995.

[4]. Siciliano, The Tricept robot: Inverse kinematics, manipulability analysis and closed-loop direct kinematics algorithm, *Robotica*, 17, 4, 1999, pp. 437-445.

[5]. T. Huang, M. Li, X. M. Zhao, *et al.*, Conceptual design and dimensional synthesis for a 3-DOF module of the TriVariant — a novel 5-DOF reconfigurable hybrid robot, *IEEE Transactions on Robotics*, 21, 3, 2005, pp. 449-456.

[6]. Z. M. Bi, Y. Jin, Kinematic modeling of Exechon parallel kinematic machine, *Robotics and Computer-Integrated Manufacturing*, 27, 1, 2011, pp. 186-193.

[7]. K. E. Neumann, Tricept applications, in *Proceedings of the 3rd Chemnitz Parallel Kinematics Seminar Development Methods and Application Experience of Parallel Kinematics (PKS)*, 2002, pp. 547-551.

[8]. Exechon, http://www.exechon.com/, 2014.

[9]. M. K. Lee, K. W. Park, B. O. Choi, Kinematic and dynamic models of hybrid robot manipulator for propeller grinding, *Journal of Robotic Systems*, 16, 3, 1999, pp. 137-150.

[10]. T. K. Tanev, Kinematics of a hybrid (parallel-serial) robot manipulator, *Mechanism and Machine Theory*, 35, 9, 2000, pp. 1183-1196.

[11]. Ibrahim W. Khalil, Inverse and direct dynamic models of hybrid robots, *Mechanism and Machine Theory*, 45, 4, 2010, pp. 627-640.

[12]. S. Lee, S. Kim, Efficient inverse kinematics for serial connections of serial and parallel manipulators, in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 3, 1993, pp. 1635-1641.

[13]. G. B. Chung, B. J. Yi, D. J. Lim, *et al.*, An efficient dynamic modeling methodology for general type of hybrid robotic systems, in *Proceedings of the IEEE International Conference on Robotics and Automation*, 2, 2004, pp. 1795-1802.

[14]. R. Clavel, Device for the movement and positioning of an element in space, *US Patent No. 4976582*, Dec. 11, 1990.

[15]. Bonev, Delta parallel robot-the story of success, *Newsletter*, 2001.

[16]. P. Vischer, R. Clavel, Kinematic calibration of the parallel Delta robot, *Robotica*, 16, 02, 1998, pp. 207-218.

[17]. S. Staicu, D. C. Carp-Ciocardia, Dynamic analysis of Clavel's delta parallel robot, in *Proceedings of the IEEE International Conference on Robotics and Automation*, 2003, 3, pp. 4116-4121.

[18]. M. A. Laribi, L. Romdhane, S. Zeghloul, Analysis and dimensional synthesis of the DELTA robot for a prescribed workspace, *Mechanism and Machine Theory*, 42, 7, 2007, pp. 859-870.

[19]. V. Poppeova, J. Uricek, V. Bulej, The development of mechanism based on hybrid kinematic structure, *Development*, 2011, pp. 228-229.

[20]. Fanuc, M-1iA Delta Robot Introduction, http://www.fanucrobotics.com/robot-applications/M-1iA_Delta_Robot_Introduction.aspx, 2014.

[21]. ABB, IRB 360 FlexPicker, http://new.abb.com /products/robotics/industrial-robots/irb-360, 2014.

[22]. K. J. Waldron, G. L. Kinzel, G. Kinzel, Kinematics, dynamics, and design of machinery, *Wiley*, New York, 1999.