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Contents

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Prism Based Real Time Refractometer

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Abstract:A rigorous theoretical study of prism based real time refractometer sensor where modulation in reflected light intensity with variation in ambient refractive index is the sensing mechanism is made. Effect of polarization of incident light used in such sensors is studied. Effect on emergent light intensity due to reflection and transmission at various interfaces in the sensor configuration is compared with the intensity obtained when only the prism-ambient interface is considered for normal incidence of light at the air-prism interface; result comes out to be the same for both the cases. For obliquely incident light the difference is determined by the percentage fractional loss. Theoretical expressions obtained are in good agreement with experimental observations. *Copyright © 2007 IFSA.*

Keywords: Refractometer, Prism, Oblique incidence, Fresnels Relations, Refractive index

1. Introduction

Most of the conventional refractometers are based on the measurement of the angle of deviation or critical angle at the interface. Mechanical adjustments required in these refractometers are eliminated in their opto-electronic versions but this lead to an increase in accuracy in measuring the refractive index.

Experimental investigations on liquid level sensor by Spenner et al using optical fiber have paved way for compact sized refractometers. The fragility factor due to optical fiber transducer has been removed. In the prism based opto-electronic refractometer sensor proposed by the authors earlier based on the measurement of intensity modulation is sturdy, user friendly and real time.

In our prism based real time refractometer, a parallel beam of light falls on the base which is interfaced with ambient liquid whose refractive index is to be measured. Light reflected from this interface undergoes refraction at exit face AC of the prism and the emergent light is finally detected by the power meter.

In Ref. [4], the refraction at the entry and exit faces of the prism had not been considered to calculate the reflected (I_r) light intensity. In the present study the effect of refraction at each faces of the prism has been taken into account and the therefore deviation from Ref. [4] have been obtained. Also the effect of incident beam polarization has been studied.

2. Principle of Operation

According to Fresnel relations [7] the reflected light intensity from an interface depends upon the RI of the two media interfacing each other, angle of incidence, angle of refraction and state of polarization of the incident light.

Considering all the interfaces involved i.e. μ_{air}-μ_g (interface AB), μ_g-μ_a (interface-BC) and μ_g-μ_{air} (interface AC), the expression for relative emergent light intensity is calculated for both p and s-polarizations as well as for randomly polarized light. Here μ_{air}, μ_g and μ_a are the RI's of air, glass prism and ambient liquid respectively. The relative emergent light intensity is defined as the ratio of intensity of emergent light to that of the incident light.

2.1 Relative Emergent Intensity with Unpolarized Light

As shown in Fig. 1, light ray of amplitude A is incident on μ_{air}-μ_g interface at an angle θ, is transmitted into the prism with amplitude T then reflected from μ_g-μ_a interface with amplitude R before being finally transmitted into the air at the μ_g-μ_{air} interface with amplitude T'. Using Fresnel relations for each polarization the expression for emergent amplitude T'p at μ_g-μ_{air} interface for p-polarized light is calculated to be

$$T_P^{'} = f_{P3P2P1} A_P , \tag{1a}$$

where

$$f_{P1} = \frac{2\mu_{air}\cos\theta}{\mu_g \cos\theta + \mu_{air}\cos r} \tag{1b}$$

$$f_{P2} = \frac{\mu_r \cos\theta_i - \mu_g \cos\theta_t}{\mu_a \cos\theta_i + \mu_g \cos\theta_t} \tag{1c}$$

$$f_{P3} = \frac{2\mu_g \cos r}{\mu_{air}\cos r + \mu_g \cos\theta} \tag{1d}$$

Here A_p is the amplitude of p-polarized light incident at μ_{air} - μ_g interface, r is the angle of refraction at this interface is given by

$$r = \sin^{-1}\left(\frac{\mu_{air}\sin\theta}{\mu_g}\right) \tag{1e}$$

$$\theta_i = \alpha - r, \tag{1f}$$

where α is the base angle of the isosceles prism. θ_i and θ_t are the angles of incidence and refraction at μ_g-μ_a interface.

Similarly amplitude of the emergent light at μ_g-μ_{air} interface is

$$T_s' = f_{s3} f_{s2} f_{s1} A_s, \tag{2a}$$

where

$$f_{s1} = \frac{2\mu_{air}\cos\theta}{\mu_{air}\cos\theta + \mu_g\cos r} \tag{2b}$$

$$f_{s2} = \frac{\mu_g\cos\theta_i - \mu_a\cos\theta_t}{\mu_g\cos\theta_i + \mu_a\cos\theta_t} \tag{2c}$$

$$f_{s3} = \frac{2\mu_g\cos r}{\mu_g\cos r + \mu_{air}\cos\theta} \tag{2d}$$

and A_s is the amplitude of s-polarized light incident at μ_{air}-μ_g interface.

Using Eqs.(1) and (2) and averaging over the angle which electric vector of the incident field makes with the plane of incidence , the relative emergent intensity for incident unpolarized light is deduced as

$$\left(\frac{I^t}{I^t_{TIR}}\right)_{un-pol} = \frac{\left(f_{P3} f_{P2} f_{P1}\right)^2 + \left(f_{s3} f_{s2} f_{s1}\right)^2}{\left(f_{P3} f_{P1}\right)^2 + \left(f_{s3} f_{s1}\right)^2} \tag{3}$$

Here I^t is the intensity of the light emerging from the face AC and I^t_{TIR} is the light intensity when TIR takes place at μ_g-μ_a interface.

For normal incidence (θ =0) on the face AB of the prism

$$f_{P1} f_{P3} = f_{s1} f_{s3} \tag{4}$$

Hence Eq. (3) becomes

$$\left(\frac{I^t}{I^t_{TIR}}\right)_{un-pol} = \frac{1}{2}\left(f_{P2}{}^2 + f_{s2}{}^2\right) = \frac{I_r}{I_i}^{Ref\,4}, \tag{5a}$$

which can explicitly be written as

$$\left(\frac{I^t}{I^t_{TIR}}\right) = \frac{1}{2}\left[\left(\frac{\mu_a\cos\theta_i - \mu_g\cos\theta_t}{\mu_a\cos\theta_i + \mu_g\cos\theta_t}\right)^2 + \left(\frac{\mu_g\cos\theta_i - \mu_a\cos\theta_t}{\mu_g\cos\theta_i + \mu_a\cos\theta_t}\right)^2\right] \tag{5b}$$

$$\left(\frac{I^t}{I^t_{TIR}}\right) = f(\mu_a, \mu_g, \theta_i) \tag{5c}$$

After a little algebraic manipulations Eq. (3) can be written as

$$\left(\frac{I^t}{I^t_{TIR}}\right)_{un-pol} = \frac{f_{P2}^{\ 2} + Kf_{s2}^{\ 2}}{1+K}, \tag{6}$$

where

$$K = \left(\frac{f_{s3}f_{s1}}{f_{P3}f_{P1}}\right)^2 = \left(\frac{\mu_g\cos\theta + \mu_{air}\cos r}{\mu_{air}\cos\theta + \mu_g\cos r}\right)^2 \tag{7a}$$

thus,

$$K = f(\theta, \mu_g, \mu_{air}) \tag{7b}$$

The parameter K represents the obliquity factor and can have values less than or equal to 1. For normal incidence at face AB of the prism i.e. for θ =0, K=1 as can be seen from Eq. (7a). Under this condition Eq. (6) reduces to Eq. (5a).

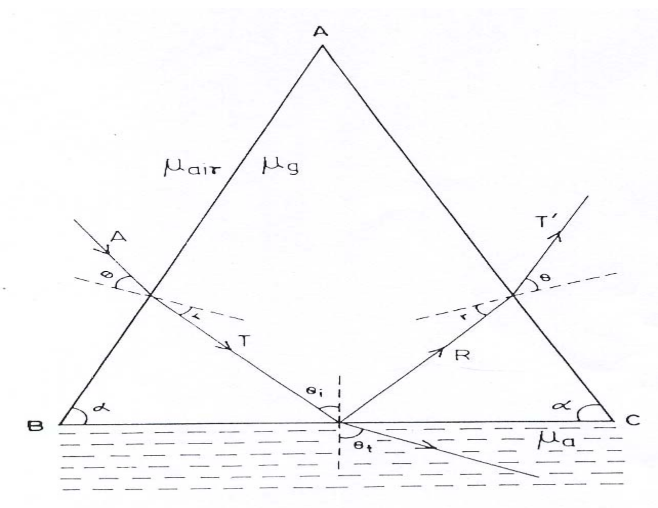

Fig. 1. Experimental set-up for observing variation in reflected light intensity with those in refractive index of ambient.

Thus for normal incidence, the relative emergent intensity evaluated in the present treatment comes out to be the same as the ratio of reflected to incident light intensity given in Ref. [4]. The parameter K having values other than 1 is a measure of departure from normal incidence at face AB of the prism. The variation of K with varying angle of incidence θ is shown in Fig. 2. It is seen that as θ is increased from 0-30˚, K decreases from 1-0.948. θ can be even larger depending upon the prism parameters and the RI of the liquid to be measured.

It is important to mention here that the refractometer sensitivity (S) defined as

$$S = \frac{\Delta\left(\dfrac{I^t}{I^t_{TIR}}\right)}{\Delta\mu_a} \tag{7c}$$

is highest in the region close to TIR. Initially up to θ =10˚, change in K is negligible but for larger angles of incidence, K decreases sharply. In our experimental set up, θ is varied from 0-29.1˚.

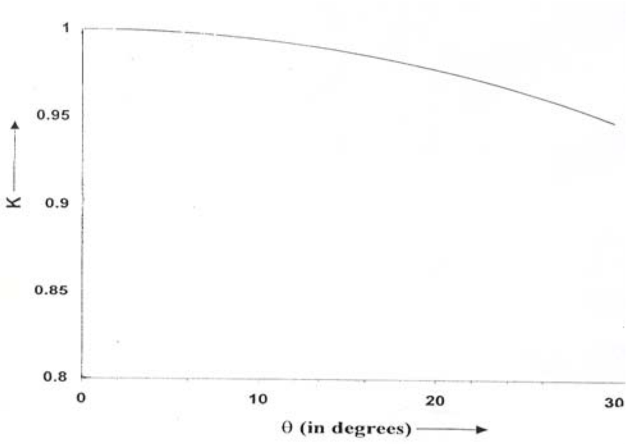

Fig. 2. Variation of obliquity factor K against angle of incidence θ for μ_g=1.517 and α=67.6˚.

2.2 Relative Emergent Intensity with Polarized Light

For incident light polarized parallel to plane of incidence, the expression for relative emergent intensity Eq. (3) reduces to

$$\left(\frac{I^t}{I^t_{TIR}}\right)_P = f_{P2}^2 = \left(\frac{\mu_g\cos\theta_t - \mu_a\cos\theta_i}{\mu_g\cos\theta_t + \mu_a\cos\theta_i}\right)^2 \tag{8}$$

and that for s polarized light Eq. (3) reduces to

$$\left(\frac{I^t}{I^t_{TIR}}\right)_s = f_{s2}^2 = \left(\frac{\mu_g\cos\theta_t - \mu_a\cos\theta_i}{\mu_g\cos\theta_t + \mu_a\cos\theta_i}\right)^2 \tag{9}$$

For representative case of an isosceles glass prism of RI μ_g=1.517 and base angle α=45˚ each, on which a parallel beam of light falls on the face AB at an angle θ =25.51˚, the variation in $(I^t/I^t_{TIR})_{un\text{-}pol}$, $(I^t/I^t_{TIR})_p$ *and* $(I^t/I^t_{TIR})_s$ with ambient RI is plotted in Fig. 3. Curve a is for unpolarized light and is obtained from Eq. (3). Curve b obtained from Eq. (8) and curve c from Eq. (9) show the variation of such ratio when incident beam is polarized parallel and perpendicular to the plane of incidence respectively.

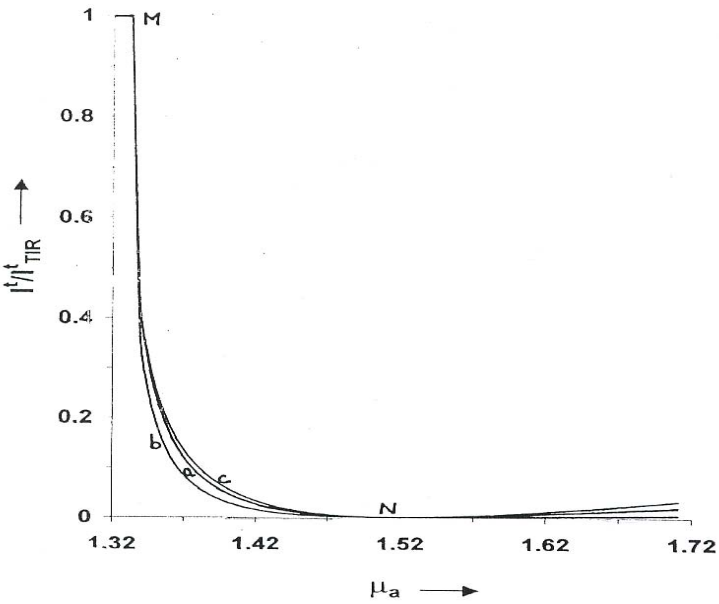

Fig. 3. Relative emergent intensity I^t/I^t_{TIR} against the ambient refractive index μ_a. Curve a, b and c corresponds to unpolarized, p-polarized and s-polarized respectively. Point M correspond to $\mu_a=\mu_{ac}$ and point N corresponds to $\mu_a=\mu_g$.

Up to $\mu_a=\mu_{ac}=1.333$, there is TIR for s-polarized, p-polarized and unpolarized light, hence relative emergent intensity is equal to one. As μ_a increases beyond μ_{ac}, I^t/I^t_{TIR} starts decreasing in all the cases. The decrease is fastest in p-polarised light (curve b) comparatively slower in unpolarized light (curve a) and slowest for s-polarized light (curve c). I^t/I^t_{TIR} becomes zero for all three cases at $\mu_a=\mu_g=1.517$ since at this point no reflection takes place and whole of the light is transmitted into the ambient.

Here it should be noted that in the present configuration minima in the I^t/I^t_{TIR} vs. μ_a curve is determined solely by the value of μ_g. Hence curves a, b and c in Fig. 3 overlap up to $\mu_a=\mu_{ac}$ (point M), separate out in between and again met at $\mu_a=\mu_g$ (point N). These curves separate out again for $\mu_a>\mu_g$.

When only one i.e. μ_g-μ_a interface is considered, Eqs. (1a) and (2a) reduces to

$$T_P^{'} = f_{p2} A_P \tag{10a}$$

and

$$T_s^{'} = f_{s2} A_s \tag{10b}$$

respectively.

Thus for polarized incident light although the emergent amplitude and subsequently emergent intensity changes but the relative emergent intensity remains unchanged whether refraction at all the three or only one interface is taken into account.

3. Experimental Details

Fig. 1 describes the experimental setup. The isosceles glass prism used here has μ_g=1.517. The base of this prism is adjoined with a temperature controlled steel bath maintained at 35˚C. It is filled with ambient medium. A parallel beam of unpolarized light is allowed to fall on the first refracting surface AB of the prism. The experiment is performed for the normal and oblique incidence. Parallel beam used here has been produced by randomly polarized He-Ne laser of wavelength 630 nm (Aerotech Laser Model LSR 5R).

Water (RI=1.333) and glycerin (RI=1.473) have been mixed together to obtain ambients with RI increasing from 1.333-1.473. Still higher values of RI in the range 1.473-1.520 are obtained by mixing acetone in aniline. Steel bath is filled with above mentioned ambient samples one by one and emergent intensity I^t is noted using the optical power meter. I^t_{TIR} is the observed value of I^t when the ambient is air. It is the same for all the cases as it corresponds to total internal reflection. The ratio I^t/I^t_{TIR} is obtained from the observed values of I^t/I^t_{TIR}. The least count of Abbe refractometer is 0.001 and that of optical power meter is 0.1 dBm.

Variation in I^t/I^t_{TIR} with varying μ_a is plotted in Fig. 4 for prism 1 (α=45˚) and in Fig.5 for prism 2 (α=67.6˚) for different values of θ. Curves a, b, c, d and e of Fig.4 and 5 correspond to θ=0, θ=5.66˚, θ=15.45˚, θ=25.51˚ and θ=29.1˚ respectively. The solid line in curves a of both the figures is described by Eq.(5a).The solid lines in other curves show the theoretical value given by Eq.(3) in which losses occurring at all the faces are taken into account, and are in good agreement with the experimentally observed values shown by dots.

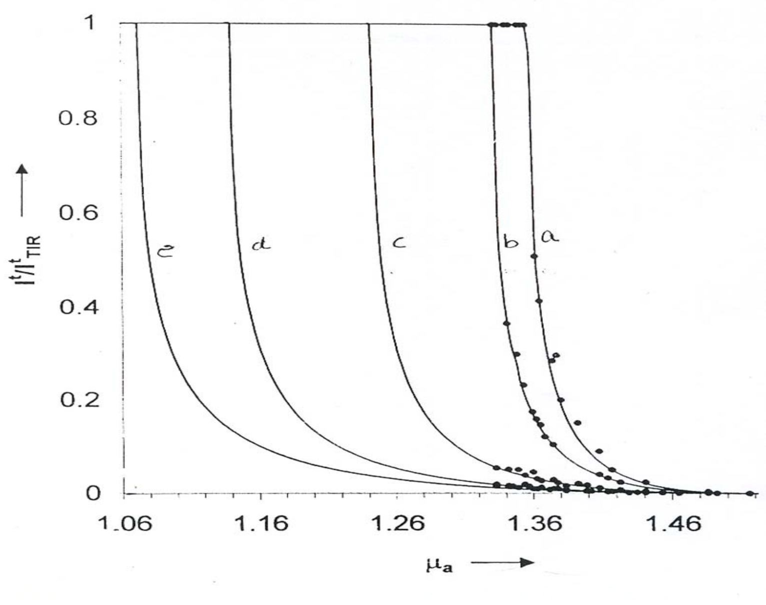

Fig. 4. Relative emergent intensity I^t/I^t_{TIR} against the ambient refractive index μ_a for unpolarized incident light at different angles of incidence θ at the first refracting surface of the prism μ_g=1.517 and α=45˚. Curve a corresponds to normal incidence (θ =0). Curves b, c, d and e correspond to θ =5.66˚, 5.45˚, 25.51˚ and 29.1˚ respectively.

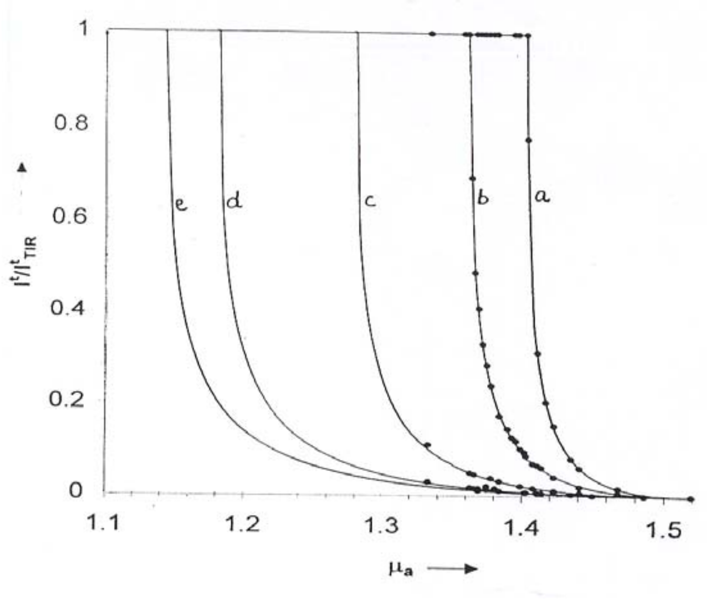

Fig. 5. Relative emergent intensity I^t/I^t_{TIR} against the ambient refractive index μ_a for unpolarized incident light at different angles of incidence θ at the first refracting surface of the prism μ_g=1.517 and α=67.6˚. Curve a corresponds to normal incidence (θ =0). Curves b, c, d and e correspond to θ =5.66˚, 5.45˚, 25.51˚ and 29.1˚ respectively.

4. Result and Discussion

In the present configuration, there are three interfaces i.e. μ_{air}-μ_g , μ_g-μ_a and μ_g-μ_{air} at which refraction and reflection of incident light occurs. Such phenomenon affects the intensity of emergent light.

Here, the emergent light intensity is calculated considering all the three interfaces and the ratio of emergent light intensity to that when total internal reflection takes place i.e. I^t/I^t_{TIR} termed as relative emergent intensity is given by Eq.(3) or Eq.(6). Experiments are carried out for different values of θ, the angle of incidence at interface AB. The experimental values are in good agreement of those calculated theoretically using Eq.(3) and are shown in Fig. 4 (prism 1) and Fig. 5 (prism 2).These results are compared with that reported in Ref.[4] by determining fractional loss, which is given by

$$Percentage\ Fractional\ Loss = \frac{\dfrac{I^t}{I^t_{TIR}} - \dfrac{I_r}{I_i}}{\dfrac{I_r}{I_i}} \times 100 \tag{11}$$

For normal incidence such fractional loss is zero and results through both the treatments come out to be the same. Fractional loss gradually decreases as K value departs from 1 i.e. angle of incidence increases. Fig.6 shows the variation of percentage fractional loss with varying θ for four values of μ_a i.e. μ_a=1.35 (curve a), 1.40 (curve b), 1.45 (curve c) and 1.50 (curve d) and α=67.6˚. Curves a, b, c and d starts from zero fractional loss, which corresponds to normal incidence. Thereafter fractional loss increases sharply with increase in θ for a given ambient. For a particular angle of incidence θ, fractional loss is higher for higher μ_a.

Similar inferences are derived from Fig.7 which shows the variation of percentage fractional loss with varying μ_a for four different values of θ. Curve a corresponds to normal incidence. Curve b, c and d correspond to θ=5.66˚, 15.45˚ and 29.1˚.For each θ there is a value of μ_a=μ_{ac} up to which TIR takes place and the percentage fractional loss remains zero. The values (θ, μ_{ac}) for curves a, b, c and d are (0,1.402) (5.66˚,1.361) (15.45˚,1.279) and (29.1˚,1.143) respectively. Beyond such μ_{ac}, percentage fractional loss comes into play and increases with increase in μ_a for a particular value of angle of incidence. For larger θ, as μ_a increases, percentage fractional loss increases sharply in beginning and slowly thereafter. For smaller θ, it increases slowly.

In Ref. [4], a refractometer configuration using divergent incident beam of light had been used where the theoretical evaluation for emergent intensity ratio was done using Eq. (5a). In the light of present calculation where all the interfaces involved are considered, it is found that such ratio should be calculated using Eq. (3), since for θ>10˚ like those occurring in [4], where conical beam of divergence 18˚ or 23˚ are used, fractional loss is not negligible.

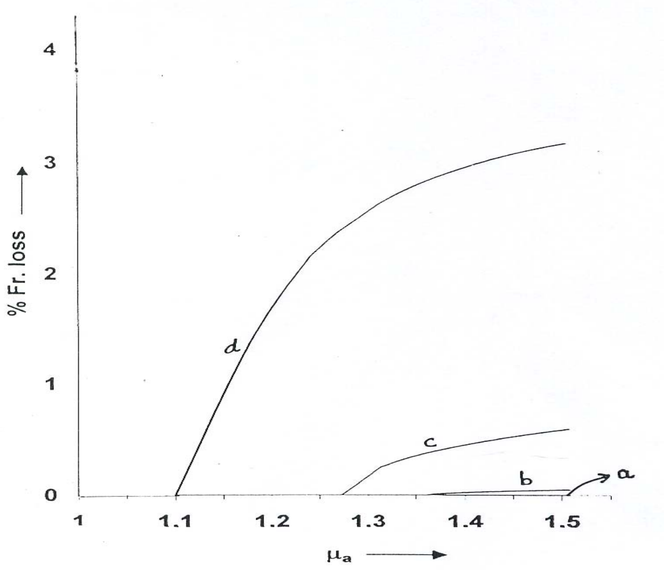

Fig. 6. Percentage fractional loss for unpolarized incident light against angle of incidence θ for μ_a =1.35 (curve a), μ_a =1.40 (curve b), μ_a =1.45 (curve c) and μ_a =1.50 (curve d). μ_g=1.517 and α=67.6˚.

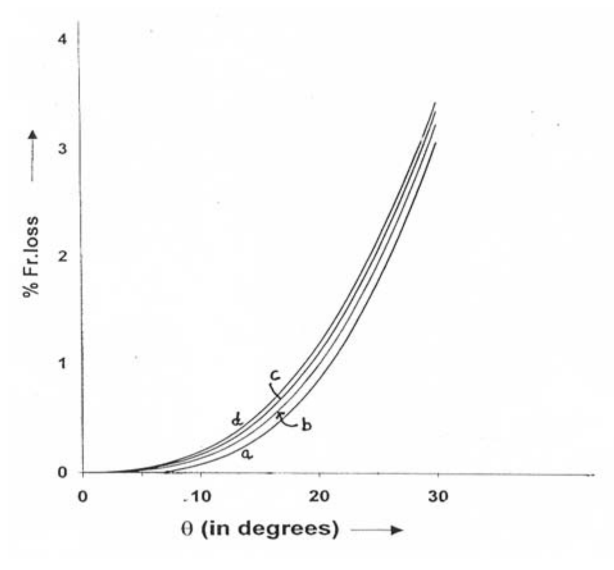

Fig. 7. Percentage fractional loss for unpolarized incident light against μ_a for different angles of incidence θ. Curve a corresponds to normal incidence (θ=0). Curves b, c and d correspond to oblique incidence θ=5.66˚, 15.45˚ and 29.1˚ respectively.

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Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726- 5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually.

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- Sensor instrumentation;
- Virtual instruments;
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- Signal processing;
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- Nanosensors;
- Microsystems;
- Applications.

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