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On Kalman Information Fusion for Multiple Wireless Sensors Networks Systems with Multiplicative Noise

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Abstract: Kalman information fusion filter is dealt with for WSNs (Wireless sensor networks), where multiplicative noises occur in the system model. There are $l > 2$ sub-systems, and for each sub-system, there are m > 2 observations that are sent through different wireless channel. Packet loss is inevitable for the wireless channel, and the optimal information fusion filter is given by the projection theorem and the scalar weighting criterion. Numerical example shows that the proposed approach is efficient. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Information fusion, Wireless sensor networks, Packet loss, Multiplicative noises.

1. Introduction

The standard Kalman filter is usually calculated by Ricatti equations and the used measurements are supplied by one sensor. However, the measurements from one sensor may be error owing to different reasons, while it isn't detected in time, so Kalman filter may be wrong from one sensor [1-2]. Different from one (or single) sensor, multi sensor information fusion problem [3, 4] can overcome the above wrong case, and has attracted much attention recent years [4–6].

The above optimal multi-sensor information fusion problem has been dealt with well for normal systems [5, 7, 8]. Particularly, [5] proposes information fusion Kalman filter fast algorithms, and the optimal information fusion Kalman filter is given under the LMSE sense in [8]. However, the above references assume that the measurements can be used, which may be impractical in application, for example, for wireless channel, the above standard information fusion result can't be used any more. Recently, Kalman filter for wireless sensor networks systems has attracted much attention [9, 10, 16]. [9] is an important result where two bounds of the steady error covariance are given for wireless sensor systems with the loss of measurements. In [10], the observation is partitioned into two parts where each part can be lost independently. The above references only consider the case of systems with additive noises, for the case of systems with multiplicative noises, the above methods can't be used. Moreover, they can't be used for the information fusion case, for WSNs with both additive noises and multiplicative noises, only [15] has considered the case of $l > 2, m = 2$.

In the paper, the general case for the information fusion for multiple WSNs with multiplicative and additive noises will be given, which extends the result of [15], i.e., there are l > 2 sub-systems, and for each sub-system, there are m > 2 observations. The information fusion filter also extends our previous works [11, 13, 14] where multiplicative noises are considered. The proposed numerical example shows that the proposed approach is efficient.

2. Problem Statements

Consider the WSN systems

$$\mathbf{x}_i(t+1) = A_i\mathbf{x}_i(t) + B_i\mathbf{x}_i(t)\mathbf{w}_i(t) + \mathbf{u}_i(t), \tag{1}$$

$$\mathbf{y}_{ij}(t) = C_{ij}x_i(t) + D_{ij}\mathbf{x}_i(t)\mathbf{w}_i(t) + \mathbf{v}_{ij}(t), \tag{2}$$

$$\bar{\mathbf{y}}_{ij}(t) = \gamma_{ij}(t)\mathbf{y}_{ij}(t), i = 1,...,l;\ j = 1,...,m, \tag{3}$$

where l, m > 2. $\mathbf{x}_i(t) \in R^n$, $\mathbf{y}_{ij}(t) \in R^{Pi}$, $\bar{\mathbf{y}}_{ij}(t)$ are state, measurements for origin and via WSNs. $\mathbf{u}_i(t) \in R^n$, $\mathbf{w}_i(t) \in R^1$, $\mathbf{v}_{ij}(t) \in R^{Pi}$ are input and noises. $\mathbf{x}_i(t)\big|_{t=0} = \mathbf{x}_{i0}$, $\langle \mathbf{v}_{ij}(t), \mathbf{v}_{kq}(s)\rangle = R_{ij}\delta_{i,k}\delta_{j,q}\delta_{t,s}$,

$$E\left\langle \begin{bmatrix}\mathbf{u}_1(t)\\ \dots \\ \mathbf{u}_l(t)\end{bmatrix}, \begin{bmatrix}\mathbf{u}_1(s)\\ \dots \\ \mathbf{u}_l(s)\end{bmatrix}\right\rangle = \begin{bmatrix} Q_1 & \cdots & Q_{1l} \\ \vdots & \ddots & \vdots \\ Q_{l1} & \dots & Q_l \end{bmatrix}\delta_{t,s},$$

$\langle \mathbf{w}_i(t), \mathbf{w}_j(t)\rangle = M_i\delta_{i,j}\delta_{t,s}$ and $\langle \mathbf{x}_{i0}, \mathbf{x}_{j0}\rangle = \prod_{i0}\delta_{i,j}$. More assumptions can be referred to [15].

The information fusion Kalman filter problem for multiple wireless sensors networks systems with multiplicative and additive noises can be given as follows,

P: For the known observations $\{\bar{\mathbf{y}}_{ij}(0),\cdots,\bar{\mathbf{y}}_{ij}(t)\}$, and scalars $\{\gamma_{ij}(0),\cdots,\gamma_{ij}(t)\}$, seek a Kalman filter $\hat{\mathbf{x}}_0(t|t)$ of $\mathbf{x}(t)$ satisfying the indexes below:

i) Unbiasedness, i.e., $E\hat{\mathbf{x}}_0(t|t) = E\mathbf{x}(t)$.

ii) Optimality, i.e., to seek the matrix weights $c_1(t),\cdots,c_l(t)$, such that

$$\hat{\mathbf{x}}_0(t|t) = \sum_{i=0}^{l} c_i(t)\hat{\mathbf{x}}_i(t|t), \tag{4}$$

wherein $\hat{\mathbf{x}}_1(t|t),\cdots,\hat{\mathbf{x}}_l(t|t)$ denote l unbiasedness filter of x(t), in order to minimize the trace of $P(t|t)$, i.e., $tr\{P_0(t|t)\} = \min\{trP(t|t)\}$.

3. Main Results

The main results for multiple wireless sensors networks systems with multiplicative and additive noises will be presented in the section.

3.1. Scalar Weighting Criterion

Lemma 1: [7, 12, 15] Denote $\mathbf{e}_i(t|t) \triangleq \mathbf{x}(t) - \hat{\mathbf{x}}_i(t|t)$. The covariance matrix is $\wp_i(t|t)$ and the cross-covariance matrix is $\wp_{ij}(t|t)$ (i,j=1,2,$\cdots$,l). Then the information fusion Kalman filter $\hat{\mathbf{x}}_0(t|t)$ is

$$\hat{\mathbf{x}}_0(t|t) = \sum_{i=1}^{l} c_i(t)\hat{\mathbf{x}}_i(t|t), \tag{5}$$

where

$$\begin{aligned}&[c_1(t) \quad c_2(t) \quad \cdots \quad c_l(t)] \\ &= (\mathbf{e}^T trP^{-1}(t|t)\mathbf{e})^{-1}\mathbf{e}^T trP^{-1}(t|t)\end{aligned} \tag{6}$$

$$\begin{aligned} P(t|t) &= \begin{bmatrix} P_1(t|t) & \cdots & \cdots & P_{1l}(t|t) \\ \vdots & & & \vdots \\ P_{l1}(t|t) & \cdots & \cdots & P_l(t|t)\end{bmatrix}, \\ \mathbf{e}^T &= [1 \quad \cdots \quad 1]. \end{aligned} \tag{7}$$

the error covariance matrix is

$$P_0(t|t) = \sum_{i,j=1}^{l} c_i(t)c_j(t)P_{ij}(t|t), \tag{8}$$

and $tr\wp(t|t) \le tr\wp_{ij}(t|t)$ (i,j=1,2,$\cdots$,l). For the convenience of discussion, $P_{ij}(\cdot)$ is replaced by $P_i(\cdot)$.

3.2. Some Important Cases

Some cases of Kalman filter for each fixed–i subsystem will be given in this part, we can use the similar definitions and denotations as in [15].

For the system (1)-(3), when i is fixed, there are 2^m cases since γ_{ij} can be equal to 0 or 1 for every j. When no packet loss occurs, we have the following.

Lemma 2: Consider the state-space description (1)-(3) for fixed-i, if $\gamma_{i1}(t) = \cdots = \gamma_{im}(t) = 1$, and the system will be the standard Kalman filter as,

$$\begin{aligned}\hat{\mathbf{x}}_i(t+1) &= A_i\hat{\mathbf{x}}_i(t) + K_{i1,\cdots,im}(t)[\bar{\mathbf{y}}_{i1,\cdots,im}(t) \\ &-\gamma_{i1,\cdots,im}(t)C_{i1,\cdots,im}\hat{\mathbf{x}}_i(t)],\end{aligned} \tag{9}$$

$$\begin{aligned}P_i(t+1) &= [A_i - \gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)\times C_{i1,\cdots,im}]P_i(t) \\ &\times[A_{i1,\cdots,im} - \gamma_{il,\cdots,im}(t)K_{i1,\cdots,im}(t)C_{i1,\cdots,im}]^T \\ &+[\mathrm{B}_i - \gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)\mathrm{D}_{i1,\cdots,im}]\textstyle\prod_i(t) \\ &\times[\mathrm{B}_i - \gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t) \\ &\times R_{i1,\cdots,im}K^T_{i1,\cdots,im}(t),\end{aligned} \tag{10}$$

$$\textstyle\prod_i(t+1) = A_i\prod_i(t)A_i^T + B_i\prod_i(t)B_i^T M_i + Q_i, \tag{11}$$

where

$$\bar{\mathbf{y}}_{i1,\cdots,im}(t) \triangleq \begin{bmatrix}\bar{\mathbf{y}}_{i1}(t) \\ \vdots \\ \bar{\mathbf{y}}_{im}(t)\end{bmatrix}, \tag{12}$$

$$\gamma_{i1,\cdots,im}(t) \triangleq \begin{bmatrix} \gamma_{i1}(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_{im}(t) \end{bmatrix}, \tag{13}$$

$$C_{i1,\cdots,im} \triangleq \begin{bmatrix} C_{i1} \\ \cdots \\ C_{im} \end{bmatrix},\ D_{i1,\cdots,im} \triangleq \begin{bmatrix} D_{i1} \\ \cdots \\ D_{im} \end{bmatrix}, \tag{14}$$

$$\mathbf{v}_{i1,\cdots,im}(t) \triangleq \begin{bmatrix} \mathbf{v}_{i1}(t) \\ \cdots \\ \mathbf{v}_{im}(t) \end{bmatrix}, \tag{15}$$

$$R_{i1,\cdots,im} \triangleq \begin{bmatrix} R_{i1}(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{im}(t) \end{bmatrix}, \tag{16}$$

with

$$\begin{aligned} K_{i1,\cdots,im}(t) \triangleq & \left[A_i P_i(t) C^T_{i1,\cdots,im} + B_i M_i \textstyle\prod_i(t) D_{i1,\cdots,im}\right] \\ & \times[C_{i1,\cdots,im} P_i(t) C^T_{i1,\cdots,im} + D_{i1,\cdots,im} \\ & \times \textstyle\prod_i(t) M_i D^T_{i1,\cdots,im} + R_{i1,\cdots,im}]^{-1}. \end{aligned} \tag{17}$$

For $\gamma_{i1}(t) = \cdots = \gamma_{i(m-1)} = 1, \gamma_{im}(t) = 0,$

Lemma 3: Consider state-space description (1)-(3) for fixed-i, if $\gamma_{i1}(t) = \cdots = \gamma_{i(m-1)}(t) = 1, \gamma_{im}(t) = 0,$ i.e., only the m[th] measurement is lost at time t for the i[th] sub-system, Kalman filter can be given as,

$$\begin{aligned} \hat{\mathbf{x}}_i(t+1) = & A_i \hat{\mathbf{x}}_i(t) \\ & + K_{i1,\cdots,i(m-1)}(t)[\bar{\mathbf{y}}_{i1,\cdots,i(m-1)}(t) \\ & - \gamma_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)} \hat{\mathbf{x}}_i(t)], \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{\mathbf{x}}_i(t+1) = & A_i \hat{\mathbf{x}}_i(t) \\ & + K_{i1,\cdots,i(m-1)}(t)[\bar{\mathbf{y}}_{i1,\cdots,i(m-1)}(t) \\ & - \gamma_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)} \hat{\mathbf{x}}_i(t)], \\ P_i(t+1) = & [A_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)}] \\ & \times P_i(t) [A_{i1,\cdots,i(m-1)} - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)}]^T \\ & + [B_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) D_{i1,\cdots,i(m-1)}] \\ & \times \textstyle\prod_i(t) [B_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) D_{i1,\cdots,i(m-1)}]^T M_i \\ & + Q_i + \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) \\ & \times R_{i1,\cdots,i(m-1)} K^T_{i1,\cdots,i(m-1)}(t), \end{aligned} \tag{19}$$

where

$$\bar{\mathbf{y}}_{i1,\cdots,i(m-1)}(t) \triangleq \begin{bmatrix} \bar{\mathbf{y}}_{i1}(t) \\ \vdots \\ \bar{\mathbf{y}}_{i(m-1)}(t) \end{bmatrix}, \tag{20}$$

$$\gamma_{i1,\cdots,i(m-1)}(t) \triangleq \begin{bmatrix} \gamma_{i1}(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_{i(m-1)}(t) \end{bmatrix}, \tag{21}$$

$$C_{i1,\cdots,i(m-1)} \triangleq \begin{bmatrix} C_{i1} \\ \cdots \\ C_{i(m-1)} \end{bmatrix},\ D_{i1,\cdots,i(m-1)} \triangleq \begin{bmatrix} D_{i1} \\ \cdots \\ D_{i(m-1)} \end{bmatrix}, \tag{22}$$

$$\mathbf{v}_{i1,\cdots,i(m-1)}(t) \triangleq \begin{bmatrix} \mathbf{v}_{i1}(t) \\ \cdots \\ \mathbf{v}_{i(m-1)}(t) \end{bmatrix}, \tag{23}$$

$$R_{i1,\cdots,i(m-1)} \triangleq \begin{bmatrix} R_{i1}(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{i(m-1)}(t) \end{bmatrix}, \tag{24}$$

with

$$\begin{aligned} & K_{i1,\cdots,i(m-1)}(t) \\ & \triangleq [A_i P_i(t) C^T_{i1,\cdots,i(m-1)} + B_i M_i \textstyle\prod_i(t) D^T_{i1,\cdots,i(m-1)}] \\ & \times[C_{i1,\cdots,i(m-1)} P_i(t) C^T_{i1,\cdots,i(m-1)} + D_{i1,\cdots,i(m-1)} \\ & \times \textstyle\prod_i(t) M_i D^T_{i1,\cdots,i(m-1)} + R_{i1,\cdots,i(m-1)}]^{-1}. \end{aligned} \tag{25}$$

For the case of $\gamma_{i1}(t) = \cdots = \gamma_{i(m-1)} = 0, \gamma_{im}(t) = 1.$ we have the following.

Lemma 4: Consider the state-space description (1)-(3), if $\gamma_{i1}(t) = \cdots = \gamma_{i(m-1)} = 0, \gamma_{im}(t) = 1,$ i.e., only the m[th] measurement is not lost at time t, thus the Kalman filter can be given as,

$$\begin{aligned} \hat{\mathbf{x}}_i(t+1) = & A_i \hat{\mathbf{x}}_i(t) \\ & + K_{im}(t)[\bar{\mathbf{y}}_{im}(t) - \gamma_{im}(t) C_{im} \hat{\mathbf{x}}_i(t)], \end{aligned} \tag{26}$$

$$\begin{aligned} P_i(t+1) = & [A_i - \gamma_{im}(t) K_{im}(t) C_{im}] P_i(t) \\ & \times [A_i - \gamma_{im}(t) K_{im}(t) C_{im}]^T \\ & + \left[B_i - \gamma_{im}(t) K_{im}(t) D_{im}\right] \textstyle\prod_i(t) \\ & \times \left[B_i - \gamma_{im}(t) K_{im}(t) D_{im}\right]^T M_i \\ & + Q_i + \gamma_{im}(t) K_{im}(t) R_{im} K^T_{im}(t), \end{aligned} \tag{27}$$

where $\prod_i$ is in (11) and

$$\begin{aligned} K_{im}(t) \triangleq & \left[A_i P_i(t) C^T_{im} + B_i M_i \textstyle\prod_i(t) D^T_{im}\right] \\ & \times \left[C_{im} P_i(t) C^T_{im} + D_{im} \textstyle\prod_i(t) M_i D^T_{im} + R_{im}\right]^{-1}. \end{aligned}$$

For the case of $\gamma_{i1}(t) = \cdots = \gamma_{im}(t) = 0,$ we will give the following lemma

Lemma 5: [15] considers the state-space description (1)-(3), if $\gamma_{i1}(t) = \cdots = \gamma_{im}(t) = 0,$ the Kalman filter can be given as follows since the open loop,

$$\hat{\mathbf{x}}_i(t+1) = A_i \hat{\mathbf{x}}_i(t), \quad (28)$$

$$P_i(t+1) = A_i P_i(t) A_i^T + B_i \prod_i(t) B_i^T M_i + Q_i, \quad (29)$$

$$\prod_i(t+1) = A_i \prod_i(t) A_i^T + B_i \prod_i(t) B_i^T M_i + Q_i, \quad (30)$$

For the general case that some measurement are lost, for example, the second measurement and the m^{th} measurement are available, and the other measurement (the first, the third to the $(m-1)^{th}$ measurement) are lost, we have the following.

Lemma 6: Consider the state-space description (1)-(3) for fixed-i, if $\gamma_{i2}(t) = \gamma_{im} = 1, \gamma_{i1}(t) = \gamma_{i3}(t) = \cdots = \gamma_{i(m-1)}(t) = 0$, i.e., only the second measurement and the m^{th} measurement are available at time t for the i^{th} sub-system, and Kalman filter can be given as,

$$\begin{aligned} \hat{\mathbf{x}}_i(t+1) = A_i \hat{\mathbf{x}}_i(t) + K_{i2,im}(t)[\bar{\mathbf{y}}_{i2,im}(t) \\ - \gamma_{i2,im}(t) C_{i2,im} \hat{\mathbf{x}}_i(t)] \end{aligned} \quad (31)$$

$$\begin{aligned} P_i(t+1) = {} & [A_i - \gamma_{i2,im}(t) K_{i2,im}(t) C_{i2,im}] P_i(t) \\ & \times [A_{i2,im} - \gamma_{i2,im}(t) K_{i2,im}(t) C_{i2,im}]^T \\ & + [B_i - \gamma_{i2,im}(t) K_{i2,im}(t) D_{i2,im}] \prod_i(t) \\ & \times [B_i - \gamma_{i2,im}(t) K_{i2,im}(t) D_{i2,im}]^T M_i + Q_i \\ & + \gamma_{i2,im}(t) K_{i2,im}(t) R_{i2,im} K_{i2,im}^T(t), \end{aligned} \quad (32)$$

where

$$\bar{\mathbf{y}}_{i2,im}(t) \triangleq \begin{bmatrix} \bar{\mathbf{y}}_{i2}(t) \\ \bar{\mathbf{y}}_{im}(t) \end{bmatrix}, \quad (33)$$

$$\gamma_{i2,im}(t) \triangleq \begin{bmatrix} \gamma_{i2}(t) & 0 \\ 0 & \gamma_{im}(t) \end{bmatrix}, \quad (34)$$

$$C_{i2,im} \triangleq \begin{bmatrix} C_{i2} \\ C_{im} \end{bmatrix}, D_{i2,im} \triangleq \begin{bmatrix} D_{i2} \\ D_{im} \end{bmatrix}, \quad (35)$$

$$\mathbf{v}_{i2,im}(t) \triangleq \begin{bmatrix} \mathbf{v}_{i2}(t) \\ \mathbf{v}_{im}(t) \end{bmatrix}, \quad (36)$$

$$R_{i2.im} \triangleq \begin{bmatrix} R_{i2}(t) & 0 \\ 0 & R_{im}(t) \end{bmatrix}, \quad (37)$$

with

$$\begin{aligned} K_{i2,im}(t) \triangleq {} & [A_i P_i(t) C_{i2,im}^T + B_i M_i \prod_i(t) D_{i2,im}^T] \\ & \times [C_{i2,im} P_i(t) C_{i2,im}^T + D_{i2,im} \\ & \times \prod_i(t) M_i D_{i2,im}^T + R_{i2,im}]^{-1}. \end{aligned} \quad (38)$$

3.3. Kalman Filter for Sub-system Equations

Five important cases have been given in the above subsection, and we can give the main theorem for WSN,

Theorem 1: Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], one-step predictor for i^{th} sub-system is

$$\begin{aligned} \hat{\mathbf{x}}_i(t+1) = {} & A_i \hat{\mathbf{x}}_i(t) + \gamma_{i1}(t)\gamma_{i2}(t)\cdots\gamma_{im}(t) K_{i1,\cdots,im}(t) \\ & \times [\bar{\mathbf{y}}_{i1,\cdots,im}(t) - \gamma_{i1,\cdots,im}(t) C_{i1,\cdots,im} \hat{\mathbf{x}}_i(t)] \\ & + \gamma_{i1}(t)\gamma_{i2}(t)\cdots\gamma_{i(m-1)}(t)(1-\gamma_{im}(t)) \\ & \times K_{i1,\cdots,i(m-1)}(t)[\bar{\mathbf{y}}_{i1,\cdots,i(m-1)}(t) \\ & - \gamma_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)} \hat{\mathbf{x}}_i(t)] \\ & + \cdots \\ & + (1-\gamma_{i1}(t))\gamma_{i2}(t)(1-\gamma_{i3}(t))\cdots \\ & \times \left(1-\gamma_{i1}(t)\right)\cdots\left(1-\gamma_{i(m-1)}(t)\right)\gamma_{im}(t) \\ & \times K_{im}(t)[\bar{\mathbf{y}}_{im}(t) - \gamma_{im}(t) C_{im} \hat{\mathbf{x}}_i(t)], \end{aligned} \quad (39)$$

$$\begin{aligned} P_i(t+1) = {} & Q_i + \gamma_{i1}(t)\cdots\gamma_{im}(t) \\ & \times \{[A_i - \gamma_{i1,\cdots,im}(t) K_{i1,\cdots,im}(t) C_{i1,\cdots,im}] P_i(t) \\ & \times \left[A_i - \gamma_{i1,\cdots,im}(t) K_{i1,\cdots,im}(t) C_{i1,\cdots,im}\right]^T \\ & + \left[B_i - \gamma_{i1,\cdots,im}(t) K_{i1,\cdots,im}(t) D_{i1,\cdots,im}\right] \prod_i(t) M_i \\ & + \left[B_i - \gamma_{i1,\cdots,im}(t) K_{i1,\cdots,im}(t) D_{i1,\cdots,im}\right]^T \\ & + \gamma_{i1,\cdots,im}(t) K_{i1,\cdots,im}(t) R_{i1,\cdots,im} K_{i1,\cdots,im}^T(t)\} \\ & + \gamma_{i1}(t)\gamma_{i2}(t)\cdots\gamma_{i(m-1)}(t)(1-\gamma_{im}(t)) \\ & \times \{[A_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) \\ & \times C_{i1,\cdots,i(m-1)}] P_i(t) \\ & \times [A_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) C_{i1,\cdots,i(m-1)}]^T \\ & + [B_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) \\ & \times D_{i1,\cdots,i(m-1)}] \prod_i(t) M_i \\ & \times \left[B_i - \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) D_{i1,\cdots,i(m-1)}\right]^T \\ & + \gamma_{i1,\cdots,i(m-1)}(t) K_{i1,\cdots,i(m-1)}(t) \\ & \times R_{i1,\cdots,i(m-1)} K_{i1,\cdots,i(m-1)}^T(t)\} \\ & + \cdots \\ & + (1-\gamma_{i1}(t))\gamma_{i2}(t)(1-\gamma_{i3}(t))\cdots \\ & \times (1-\gamma_{i(m-1)}(t))\gamma_{im}(t) \\ & \times \{\left[A_i - \gamma_{i2,im}(t) K_{i2,im}(t) C_{i2,im}\right] P_i(t) \\ & \times \left[A_i - \gamma_{i2,im}(t) K_{i2,im}(t) C_{i2,im}\right]^T \\ & + \left[B_i - \gamma_{i2,im}(t) K_{i2,im}(t) D_{i2,im}\right] \prod_i(t) M_i \\ & \times \left[B_i - \gamma_{i2,im}(t) K_{i2,im}(t) D_{i2,im}\right]^T \\ & + \gamma_{i2,im}(t) K_{i2,im}(t) R_{i2,im} K_{i2,im}^T(t)\} \\ & + \cdots \\ & + (1-\gamma_{i1}(t))\cdots(1-\gamma_{i(m-1)}(t))\gamma_{im}(t) \\ & \times \{\left[A_i - \gamma_{im}(t) K_{im}(t) C_{im}\right] P_i(t) \\ & \times \left[A_i - \gamma_{im}(t) K_{im}(t) C_{im}\right]^T \\ & + \left[B_i - \gamma_{im}(t) K_{im}(t) D_{im}\right] \prod_i(t) M_i \\ & \times \left[B_i - \gamma_{im}(t) K_{im}(t) D_{im}\right]^T \\ & + \gamma_{im}(t) K_{im}(t) R_{im} K_{im}^T(t)\}, \end{aligned} \quad (40)$$

where $\prod_i(t+1)$ is as in (11).

Remark 1: It can be easily seen from (39) and (40) that, there are 2^m parts in the structures of $\hat{\mathbf{x}}_i(t+1)$ and $P_i(t+1)$, which shows the 2^m cases of Kalman predictors.

Furthermore, we can give the filter formulae as in the following theorem

Theorem 2: Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], the Kalman filter is

$$\begin{aligned}
&\hat{\mathbf{x}}_i(t+1|t+1)\\
&=\hat{\mathbf{x}}_i(t+1)\\
&\quad+\gamma_{i1}(t+1)\cdots\gamma_{im}(t+1)\Im_{i1,\cdots,im}(t+1)\\
&\quad\times[\bar{\mathbf{y}}_{i1,\cdots,im}(t+1)-\gamma_{i1,\cdots,im}(t+1)\\
&\quad\times C_{i1,\cdots,im}\hat{\mathbf{x}}_i(t+1)]\\
&\quad+\gamma_{i1}(t)\gamma_{i2}(t)\cdots\gamma_{i(m-1)}(t)(1-\gamma_{im}(t))\\
&\quad\times\Im_{i1,\cdots,i(m-1)}(t+1)\\
&\quad\times[\hat{\mathbf{y}}_{i1,\cdots,i(m-1)}(t+1)-\gamma_{i1,\cdots,i(m-1)}(t+1)\\
&\quad\times C_{i1,\cdots,i(m-1)}\hat{\mathbf{x}}_i(t+1)]\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\gamma_{i2}(t)(1-\gamma_{i3}(t))\cdots\\
&\quad\times(1-\gamma_{i(m-1)}(t))\gamma_{im}(t)K_{i2,im}(t)\\
&\quad\times[\bar{\mathbf{y}}_{i2,im}(t+1)-\gamma_{i2,im}(t+1)C_{i2,im}\hat{\mathbf{x}}_i(t+1)]\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\cdots(1-\gamma_{i(m-1)}(t))\gamma_{im}(t)\\
&\quad\times\Im_{im}(t+1)[\bar{\mathbf{y}}_{im}(t+1)-\gamma_{im}(t+1)C_{im}\hat{\mathbf{x}}_i(t+1)],
\end{aligned}\tag{41}$$

$$\begin{aligned}
&P_i(t+1|t+1)\\
&=\{1-\gamma_{i1}(t+1)\cdots(1-\gamma_{im}(t+1))\\
&\quad+\gamma_{i1}(t+1)\cdots\gamma_{im}(t+1)\\
&\quad\times[I-\Im_{i1,\cdots,im}(t+1)C_{i1,\cdots,im}]\\
&\quad+\gamma_{i1}(t)\gamma_{i1}(t)\cdots\gamma_{i(m-1)}(t)(1-\gamma_{im}(t))\\
&\quad\times[I-\Im_{i1,\cdots,i(m-1)}(t+1)C_{i1,\cdots,im}]\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\cdots(1-\gamma_{i(m-1)}(t+1))\gamma_{im}(t)\\
&\quad\times[I-\Im_{im}(t+1)C_{im}]\}P_i(t+1),
\end{aligned}\tag{42}$$

and $\Im_{ij}$ can be given similarly as follows:

$$\begin{aligned}
&\Im_{ij}(t)\triangleq P_i(t)C_{ij}^T[C_{ij}P_i(t)C_{ij}^T+D_{ij}\textstyle\prod_i(t)M_iD_{ij}^T+R_{ij}]^{-1},\\
&j=1,\cdots,m,
\end{aligned}\tag{43}$$

or

$$\begin{aligned}
\Im_{i1,\cdots,im}(t)\triangleq{}&P_i(t)C_{i1,\cdots,im}^T[C_{i1,\cdots,im}P_i(t)C_{i1,\cdots,im}^T+D_{i1\cdots,im}\\
&\times\textstyle\prod_i(t)M_iD_{i1,\cdots,im}^T+R_{i1,\cdots,im}]^{-1}
\end{aligned}\tag{44}$$

3.4. Computation of $P_{ij}(t|t)$

The cross covariance matrix $P_{ij}(t|t)(i\neq j)$ will be given here, since $P_{ij}(t+1|t+1)=P_{ij}^T(t+1|t+1)$, so $P_{ij}(t+1|t+1)$ only needs to be given.

Theorem 3: Consider the state-space description (1)-(3) with the similar corresponding assumptions and definitions in [15], the cross covariance matrix $P_{ij}(t+1|t+1)$ can be given as

$$\begin{aligned}
&\mathrm{P}_{ij}(t+1|t+1)\\
&\triangleq F(Q_{ij}(t))+\gamma_{i1}(t)\cdots\gamma_{im}(t)\gamma_{j1}(t)\cdots\gamma_{jm}(t)\\
&\quad\times[I-\Im_{i1,\cdots,im}(t+1)C_{i1,\cdots,im}]\\
&\quad\times[A_i-\gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)C_{i1,\cdots,im}]P_{ij}(t)\\
&\quad\times[A_j-\gamma_{j1,\cdots,jm}(t)K_{j1,\cdots,jm}(t)C_{j1,\cdots,jm}]^T\\
&\quad\times[I-\Im_{j1,\cdots,jm}(t+1)C_{j1,\cdots,jm}]^T\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\cdots(1-\gamma_{im}(t))(1-\gamma_{j1}(t))\cdots\\
&\quad\times(1-\gamma_{im}(t))A_i\mathrm{P}_{ij}(t)A_j^T,
\end{aligned}\tag{45}$$

where $P_{ij}(t+1)$ is

$$\begin{aligned}
&P_{ij}(t+1)\\
&=Q_{ij}(t)+\gamma_{i1}(t)\cdots\gamma_{im}(t)\gamma_{j1}(t)\cdots\gamma_{jm}(t)\\
&\quad\times\left[A_i-\gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)C_{i1,\cdots,im}(t)\right]P_{ij}(t)\\
&\quad\times\left[A_j-\gamma_{j1,\cdots,jm}(t)K_{j1,\cdots,jm}(t)C_{j1,\cdots,jm}(t)\right]^T\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\cdots(1-\gamma_{im}(t))(1-\gamma_{j1}(t))\cdots\\
&\quad\times(1-\gamma_{jm}(t))A_iP_{ij}(t)A_j^T,
\end{aligned}\tag{46}$$

and

$$\begin{aligned}
&F(Q_{ij}(t))\\
&=\gamma_{i1}(t)\cdots\gamma_{im}(t)\gamma_{j1}(t)\cdots\gamma_{jm}(t)\\
&\quad\times\left[I-\Im_{i1,\cdots,im}(t)C_{i1,\cdots,im}\right]Q_{ij}(t)\\
&\quad\times\left[I-\Im_{J1,\cdots,Jm}(t)C_{J1,\cdots,Jm}\right]^T\\
&\quad+\cdots\\
&\quad+(1-\gamma_{i1}(t))\cdots(1-\gamma_{im}(t))(1-\gamma_{j1}(t))\cdots\\
&\quad\times(1-\gamma_{jm}(t))Q_{ij}(t),
\end{aligned}\tag{47}$$

with the other variables can be referred in the above.

Proof: For $P_{ij}(t)$. As is well known, for $\mathbf{e}_i(t+1)$, there are 2^m cases in sum

$$\begin{aligned}
&(i-1)\gamma_{i1}(t)=\cdots=\gamma_{im}(t)=1\\
&\mathbf{e}_i(t+1)=[A_i-\gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)C_{i1,\cdots,im}]\mathbf{e}_i(t)\\
&\qquad+B_i\mathbf{x}_i(t)\mathbf{w}_i(t)+\mathbf{u}_i(t)\\
&\qquad-\gamma_{i1,\cdots,im}(t)K_{i1,\cdots,im}(t)\mathbf{v}_{i1,\cdots,im}(t)
\end{aligned}\tag{48}$$

$$\vdots\quad\vdots$$

$$\begin{aligned}
&(i-2^m)\gamma_{i1}(t)=\cdots=\gamma_{im}(t)=0\\
&\mathbf{e}_i(t+1)=A_i\mathbf{e}_i(t)+B_i\mathbf{x}_i(t)\mathbf{w}_i(t)+\mathbf{u}_i(t)
\end{aligned}\tag{49}$$

For $\mathbf{e}_j(t+1)$ of the j-th sub-system, there are also 2^m cases:

$$
\begin{aligned}
&(j-1)\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=1 \\
&\mathbf{e}_j(t+1)=[A_j-\gamma_{j1,\cdots,jm}(t)K_{j1,\cdots,jm}(t)C_{j1,\cdots,jm}]\mathbf{e}_j(t) \\
&(j-1)\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=1 \\
&\mathbf{e}_j(t+1)=[A_j-\gamma_{j1,\cdots,jm}(t)K_{j1,\cdots,jm}(t)C_{j1,\cdots,jm}]\mathbf{e}_j(t) \\
&\qquad +B_j\mathbf{x}_j(t)\mathbf{w}_j(t)+\mathbf{u}_j(t) \\
&\qquad -\gamma_{j1,\cdots,jm}(t)K_{j1,\cdots,jm}(t)\mathbf{v}_{j1,\cdots,jm}(t) \qquad (50) \\
&\qquad \vdots\ \vdots \\
&(j-2^m)\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=0 \\
&\mathbf{e}_j(t+1)=A_j\mathbf{e}_j(t)+B_j\mathbf{x}_j(t)\mathbf{w}_j(t)+\mathbf{u}_j(t). \qquad (51)
\end{aligned}
$$

Thus, from the definition of $P_{ij}(t)$, there are 2^{2m} cases as

$$
\begin{aligned}
&(ij-1).\gamma_{i1}(t)=\cdots=\gamma_{im}(t)=1,\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=1 \\
&P_{ij}(t+1)=\left[A_j-\gamma_{1,\cdots im}(t)K_{i1,\cdots im}(t)C_{i1,\cdots im}(t)\right]P_{ij}(t) \\
&\qquad \times\left[A_j-\gamma_{j1,\cdots jm}(t)K_{j1,\cdots jm}(t)C_{j1,\cdots jm}\right]^T+Q_{ij}(t), \quad (52) \\
&\qquad \vdots\vdots \\
&(ij-2^{2m}).\quad \gamma_{i1}(t)=\cdots=\gamma_{im}(t)=0,\gamma_{j1}(t)=\cdots= \\
&\gamma_{jm}(t)=0 \\
&\qquad P_{ij}(t+1)=A_jP_{ij}(t)A_j^T+Q_{ij}(t) \qquad (53)
\end{aligned}
$$

So the proof of $P_{ij}(t)$ is over.

For $P_{ij}(t+1|t+1)$, also 2^{2m} cases are dealt with, for $\mathbf{e}_i(t+1|t+1)$ when $\gamma_{i1}(t)=\cdots=\gamma_{im}(t)=1,\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=1$, it can be given as

$$
\begin{aligned}
&P_{ij}(t+1|t+1) \\
&=E\left[\mathbf{e}_i(t+1|t+1)\mathbf{e}_j^T(t+1|t+1)\right] \\
&=\left[I-\Im_{i1,\cdots,im}(t+1)C_{i1,\cdots,im}\right]P_{ij}(t+1) \\
&\quad\times\left[I-\Im_{i1,\cdots,im}(t+1)C_{i1,\cdots,im}\right]^T.
\end{aligned} \tag{54}
$$

For the last case of $\gamma_{i1}(t)=\cdots=\gamma_{im}(t)=0,\gamma_{j1}(t)=\cdots=\gamma_{jm}(t)=0$, and for $\mathbf{e}_i(t+1|t+1)$, it can be given as

$$
\begin{aligned}
&P_{ij}(t+1|t+1) \\
&=E\left[\mathbf{e}_i(t+1|t+1)\mathbf{e}_j^T(t+1|t+1)\right] \\
&=P_{ij}(t+1),
\end{aligned} \tag{55}
$$

The other parts are omitted, thus (45) can be given.

3.5. Kalman Information Fusion Filter

The main result which extends the case of [15] will be given in form of theorem in the paper

Theorem 4: For the state-space description (1)-(3) with $\gamma_{ij}(0),\cdots\gamma_{ij}(t),(i=1,\cdots,l,\ j=1,\cdots,m)$, the similar corresponding assumptions and definitions in [15], the information fusion Kalman filter is given by (5), where $\hat{\mathbf{x}}_i(t|t)$ is calculated in (39), $P_i(t|t)$ is calculated in (40), and $P_{ij}(t|t)$ is calculated in (45).

4. Numerical Example

In the section, we will give an example to show the efficiency of the presented results. For the simplicity of discussion, let $l=m=2$, then the optimal information fusion filter is

$$\hat{\mathbf{x}}_0(t|t)=c_1(t)\hat{\mathbf{x}}_1(t|t)+c_2(t)\hat{\mathbf{x}}_2(t|t) \tag{56}$$

where

$$
\begin{aligned}
&c_1(t) \\
&=\frac{trP_2(t|t)-trP_{21}(t|t)}{trP_1(t|t)-trP_{12}(t|t)-trP_{21}(t|t)+trP_2(t|t)} \\
&c_2(t) \\
&=\frac{trP_1(t|t)-trP_{12}(t|t)}{trP_1(t|t)-trP_{12}(t|t)-trP_{21}(t|t)+trP_2(t|t)}
\end{aligned} \tag{57}
$$

and the corresponding error covariance matrix is

$$
\begin{aligned}
P_0(t|t)&=c_1^2(t)P_1(t|t)+c_1(t)c_2(t)P_{12}(t|t) \\
&\quad+c_1(t)c_2(t)P_{21}(t|t)+c_2^2(t)P_2(t|t)
\end{aligned} \tag{58}
$$

and $trP_0(t|t)\le trP_i(t|t)(i=1,2)$.
Consider the system (1)-(3) with N = 100, and

$$
\begin{aligned}
&A_1=\begin{bmatrix}0.4 & 0.1\\ 0 & 0.3\end{bmatrix}, A_2=\begin{bmatrix}0.4 & 0\\ 0.1 & 0.8\end{bmatrix}, \\
&B_1=\begin{bmatrix}0.3 & 0.1\\ 0 & 0.8\end{bmatrix}, B_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.6\end{bmatrix}, \\
&C_{11}=\begin{bmatrix}0.5 & 0.1\end{bmatrix},\ C_{12}=\begin{bmatrix}1 & 0.2\end{bmatrix}, \\
&C_{21}=\begin{bmatrix}0.5 & 0.1\end{bmatrix}, C_{22}=\begin{bmatrix}2 & 0.3\end{bmatrix}, \\
&D_{11}=\begin{bmatrix}0.5 & 0.2\end{bmatrix}, D_{12}=\begin{bmatrix}2 & 0.1\end{bmatrix}, \\
&D_{21}=\begin{bmatrix}0.2 & 0.1\end{bmatrix}, D_{22}=\begin{bmatrix}2 & 0.3\end{bmatrix}.
\end{aligned} \tag{59}
$$

The initial state value $\mathbf{x}_1(0)=\mathbf{x}_2(0)=\begin{bmatrix}1 & 0.5\end{bmatrix}^T$, and noises $\mathbf{u}_i(t)$ and $\mathbf{w}_i(t)$ are white noises with zero means and unity covariance matrices $Q_i=Q_{ij}=1, M_i=1(i,j=1,2)$. Observation noise $\mathbf{v}_{ij}(t)(i,j=1,2)$ is of zero means and with covariance matrix $R_{ij}=0.01$.

We simulate four cases for fixed i-subsystem of
1) $\gamma_{i1}=\gamma_{i2}=0$,
2) $\gamma_{i1}=1,\gamma_{i2}=0$,
3) $\gamma_{i1}=0,\gamma_{i2}=1$,
4) $\gamma_{i1}=\gamma_{i2}=1$,

Since the simulated example has 16 cases, it's complicate to give all cases, so we only give the filters $\hat{\mathbf{x}}_1(t\,|\,t)$ of the signal $\mathbf{x}_1(t)$ based on observation $\left\{\overline{\mathbf{y}}_{1j}(s)\right\}_{s=0}^{t}$.

The following four figures show the origin and its estimator under the case (4).

The tracking performance of Kalman filter $\begin{bmatrix}\hat{\mathbf{x}}_{11}(t\,|\,t)\\ \hat{\mathbf{x}}_{12}(t\,|\,t)\end{bmatrix}$ is drawn in Fig. 1 and Fig. 2 when $\gamma_{11}=1, \gamma_{12}=0$, and they are also given in Fig. 3 and Fig. 4 when $\gamma_{11}=1, \gamma_{12}=1$. Seen from the above figures, the filter are good, and the estimate performance when $\gamma_{11}=1, \gamma_{12}=1$ is better than them when $\gamma_{11}=1, \gamma_{12}=0$, since the available measurements are more.

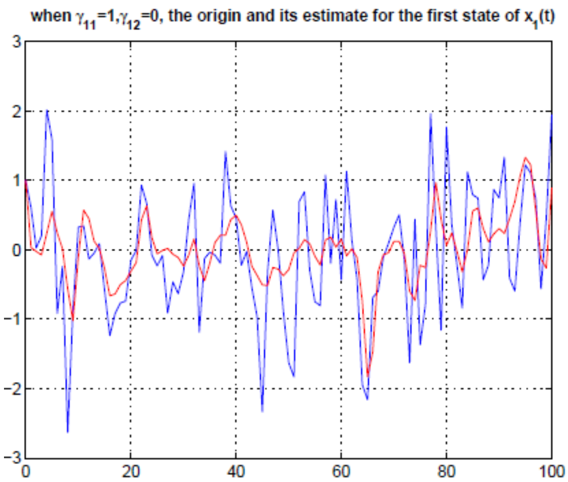

Fig. 1. The origin and its filter for the first state of $\hat{\mathbf{x}}_1(t\,|\,t)$ when $\gamma_{11}=1, \gamma_{12}=0$, where the blue line is the origin signal, the red line is the smoother.

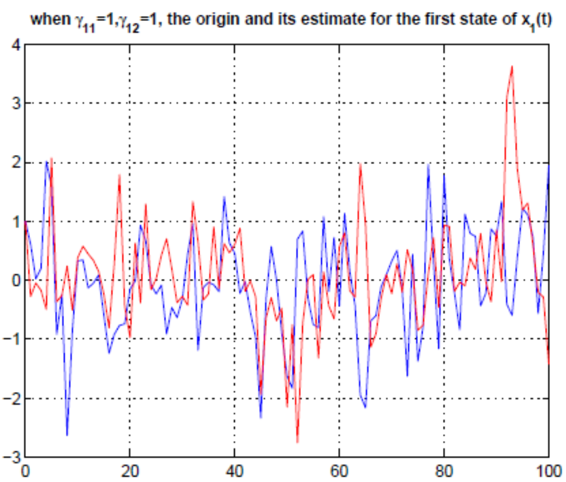

Fig. 2. The origin and its filter for the second state of $\hat{\mathbf{x}}_1(t\,|\,t)$ when $\gamma_{11}=1, \gamma_{12}=0$, where the blue line is the origin signal, the red line is the smoother.

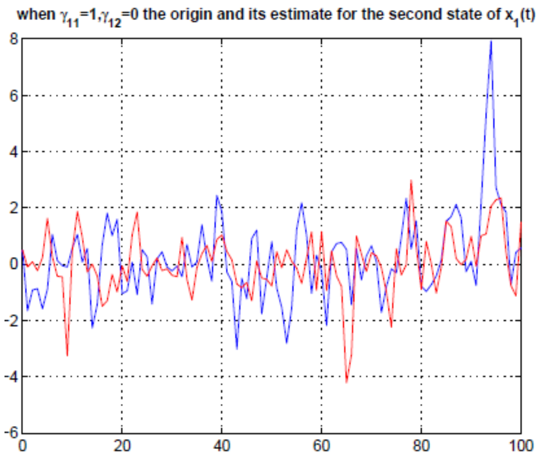

Fig. 3. The origin and its filter for the first state of $\hat{\mathbf{x}}_1(t\,|\,t)$ when $\gamma_{11}=1, \gamma_{12}=1$, where the blue line is the origin signal, the red line is the smoother.

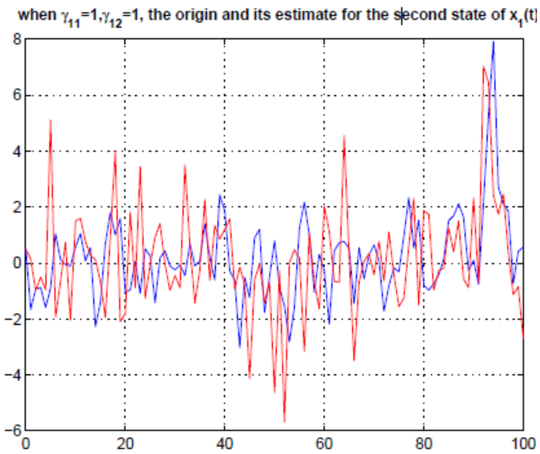

Fig. 4. The origin and its filter for the second state of $\hat{\mathbf{x}}_1(t\,|\,t)$ when $\gamma_{11}=1, \gamma_{12}=1$, where the blue line is the origin signal, the red line is the smoother.

5. Conclusions

Information fusion Kalman filter for multiple WSNs with multiplicative noises has been given. The filter is given by the scalar weighting criterion and the projection theorem. The proposed Kalman filtering formulae for each sub-system can obviously show the structure of results. The given examples show the efficiency of the proposed approach. In addition, the proposed approach can be used to deal with many practice problems, for example Intelligent Body Sensor Network [17].

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