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Reflective Type Small-Angle Sensor Based on Multiple Total Internal Reflections in Heterodyne Interferometry

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Abstract: A reflective type small-angle sensor based on the multiple total internal reflections (MTIRs) in heterodyne interferometry is proposed. In the paper, we try to measure the phase difference variation between s- and p-polarizations due to MTIRs. The phase difference variation depends on the incident angle. Therefore, only evaluating the phase difference variation can perform small-angle measurement. The resolution of the method can reach 5.5×10^{-7} radian. The method has some merits, e.g., a simple optical setup, easy operation, high measurement accuracy, high resolution, rapid measurement, and high stability etc. And its feasibility is demonstrated. *Copyright © 2008 IFSA.*

Keywords: Multiple total internal reflections (MTIRs), Heterodyne interferometry (HI), Small-angle measurement

1. Introduction

As is well-known, the measurement of small angles plays an important role in many fields of precision industries. In the past few decades, as far as we know, there has been considerable interest in the development of small-angle sensors based on the total internal reflection theory [1-5]. Chiu and Su proposed some improved techniques [3-4] for measuring small angles some years ago. At that time, they set up an optical system by modifying Huang's method [1-2]. Although these optical methods were based on the total-internal-reflection effect, they measured the phase difference instead of the reflectance difference. And the phase difference can be extracted and measured accurately with heterodyne interferometry despite surrounding light and an unstable light source. But the resolution of the improved technique for measuring small angles isn't high enough. In order to increase the resolution, some small-angle sensors based on the surface plasmon resonance (SPR) technology have been proposed in recent years. Wang *et al*. proposed the novel method of small-angle measurement based on the surface plasmon resonance (SPR) technology and heterodyne interferometry [6-7]. But the measured signal was very weak around the resonant angle owing to the attenuated total reflection (ATR) effect. It is for the reason that we propose a new type small-angle sensor based on the multiple total internal reflections in heterodyne interferometry in this paper. The small-angle sensor is designed as a reflective type elongated prism [8-9] that is made of BK7 glass. As a heterodyne light source enters into the elongated prism, it will undergo multiple total-internal reflections (MTIRs).

In this paper, a common-path heterodyne interferometer is applied to measure the phase differences between s- and p-polarizations at the total-internal reflection effect. Some special equations are derived according to the optical configuration and Fresnel's equations [10]. By substituting the datum of the phase differences into these equations, a small rotation angle can be achieved. The new type small-angle sensor has some merits, e.g., a simple optical setup, high resolution, high sensitivity, rapid measurement, and high stability etc. And its feasibility is demonstrated.

2. Principle

A ray of light in air is incident at θ on one side surface of a right-angle prism with refractive index n, as shown in Fig. 1. The light ray is refracted into the prism and it propagates toward the hypotenuse surface of the prism. At that surface, there is a boundary between the prism and air. If the angle of incidence at the boundary is θ_1, then we have

$$\theta_1 = 45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right) \tag{1}$$

Here the signs of θ_1 and θ are defined as positive if they are measured clockwise from a surface normal. If θ_1 is larger than the critical angle θ_C, the light is totally reflected at the boundary. According to Fresnel's equations, the phase difference between s- and p-polarizations is given as

$$\phi = 2\tan^{-1}\left\{\frac{\sqrt{\sin^2\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right] - 1/n^2}}{\tan\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right]\sin\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right]}\right\} \tag{2}$$

In this paper, a refractive-type elongated prism is used to be the new type small-angle sensor. The elongated prism structure appears in Fig. 2. When a laser light enters into the elongated prism, it will undergo multiple total internal reflections. In the paper, we choose $\theta = \theta_0 = -3^\circ$ as the initial angle of rotation stage. At this moment, the light is incident on the two side surfaces of the reflective elongated prism at $\theta_1 = \theta_{10} \cong 43.02^\circ$. As we rotate the rotation stage at an angle $\Delta\theta$, the incident angle θ_1 is equal to $\theta_{10}+\Delta\theta_1$, we can obtain

$$\theta_1 = \theta_{10} + \Delta\theta_1 = 45^\circ + \theta' \tag{3}$$

$$1\times\sin(\theta_i) = n\times\sin\theta' \tag{4}$$

$$l = h \times \tan(45° + \theta') \tag{5}$$

$$m \le \frac{L}{l} < m + 1, \tag{6}$$

where l is the reflection distances of between two adjacent points along the z axis, n is the refractive index of the elongated prism, L is the length of the smaller side of the elongated prism, m is the TIR times as the light travels from the hypotenuse surface of the prism to the end surface of that. The parameters of the sensor are L=100 mm, and h=5 mm. From Eq. (3) ~ Eq. (6), the heterodyne light will undergo 40 times TIRs for a round trip. It deserves to be mentioned that the light under 39 times TIRs at the two sides of the elongated prism and one time TIR at the end surface of that. Thus, the total phase difference $\Delta\phi_t$ between s- and p-polarizations is given by

$$\Delta\phi_t = 39\phi_1 + \phi_2, \tag{7}$$

where ϕ_1 and $\Delta\phi_2$ are the phase differences between s- and p-polarizations as the light undergoes one time of total internal reflection at the side surface and the end surface of the elongated prism, respectively.

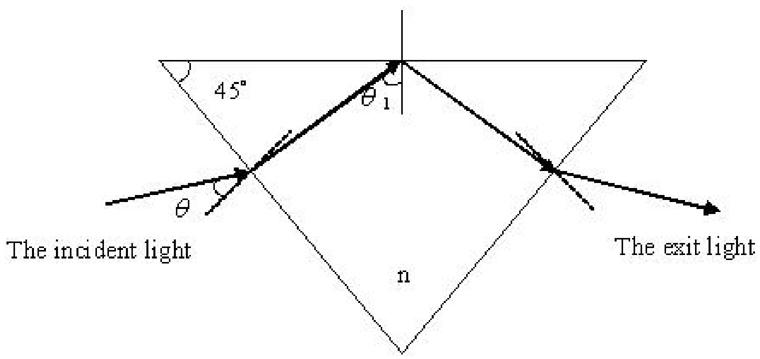

Fig. 1. A ray of light in air is incident at θ on one side surface of a right-angle prism with refractive index n .

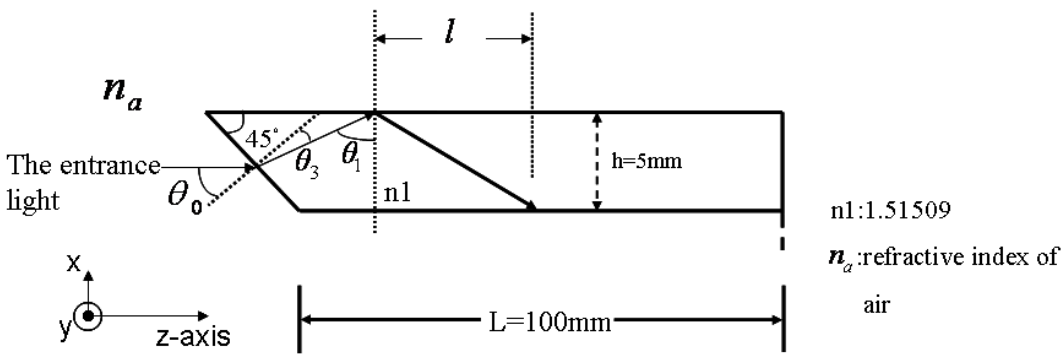

Fig. 2. The Elongated prism.

3. Experimental Setup and Results

The experimental setup of the system is shown in Fig. 3. In the experimental configuration, the new type small-angle sensor is mounted on a precision rotary stage (manufactured by Newport, Model:

URS150PP & EPS300) with the resolution of 0.0002°. A heterodyne light passing through half-wave plate is incident on a beamsplitter BS and is divided into the reflected and transmitted lights. The reflected light passes thorough an analyzer AN_r, and then enters a photo-detector PD_r. The signal measured by PD_r is the reference signal. The transmitted light from the BS enters the new-type small-angle sensor and undergoes ATRs. Finally, the light propagates out of the sensor and passes through an analyzer AN_t. We can obtain the test signal by a photo-detector PD_t. These two signals are sent to a lock-in amplifier (LIA) with the resolution $\Delta\Phi$ of $0.01°$. Thus, we can obtain the phase difference due to the MTIRs effect. After some numerical computations by a computer, a small rotation angle can be measured.

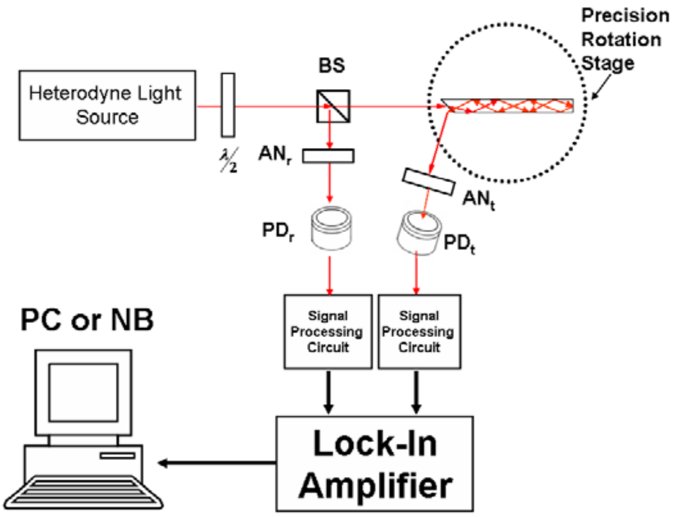

Fig. 3. The experimental setup.

Fig. 4 shows the experimental curve of the phase difference versus the rotating angle $\Delta\theta$ of the rotary stage. It is evident that only evaluating the phase difference variation can perform small-angle measurement.

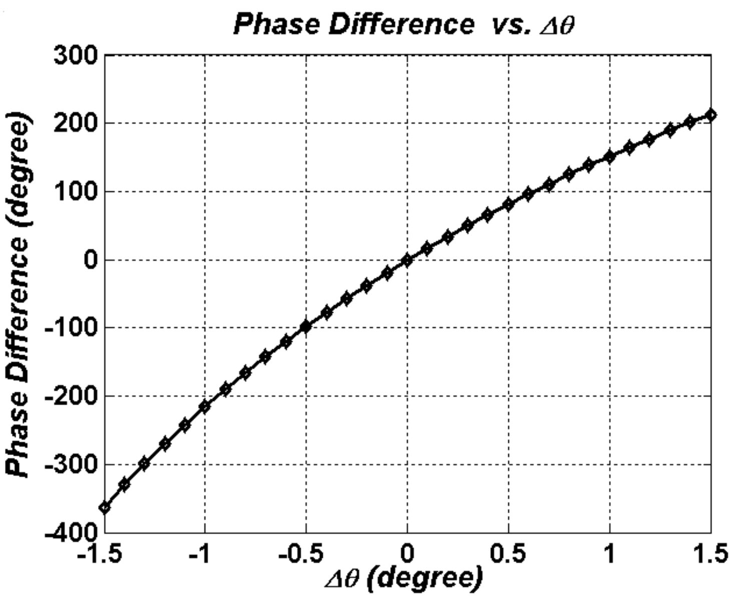

Fig. 4. Total phase difference versus the small rotation angle $\Delta\theta$.

4. Discussion

At this moment, let us discuss the sensitivity of the small-angle sensor. The sensitivity R of the small-angle sensor is defined as

$$S = \frac{d(\Delta\phi_t)}{d(\Delta\theta)}, \tag{8}$$

where $d(\Delta\phi_t)$ is the variation of the phase difference and $d(\Delta\theta)$ is the variation of a small-angle change made by the rotation of the rotary stage. As shown in Fig. 5, we can obtain the curve of sensitivity S versus $\Delta\theta$. It is evident that the sensitivity is better than 120 (degree/degree) over the measurement range $-1.5° \le \theta \le 1.5°$.

Besides, the resolution $\Delta\Phi$ of the lock-in amplifier is equal to $0.01°$, then the angular resolution $A_{resolution}$, defined as

$$A_{resolution} = \frac{d(\Delta\theta)}{d(\Delta\phi_t)}\Delta\Phi \cdot \tag{9}$$

As shown in Fig. 6, we can also obtain the curve of angular resolution $A_{resolution}$ versus $\Delta\theta$. It is clear that the resolution $A_{resolution}$ of the small-angle sensor can reach 5.5×10^{-7} radian over the measurement range of $-1.5° \le \theta \le 1.5°$.

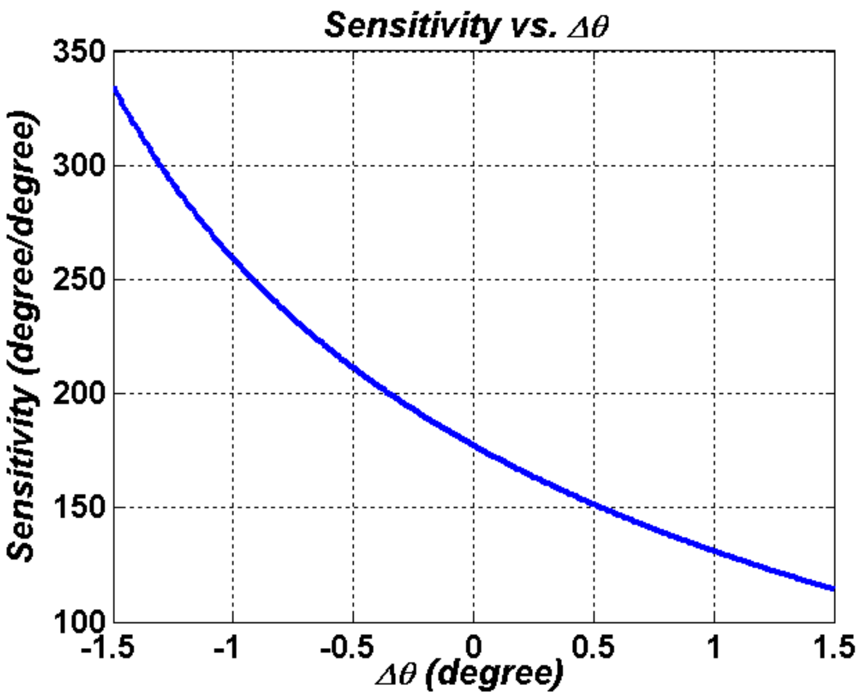

Fig. 5. The angular sensitivity of the small-angle sensor.

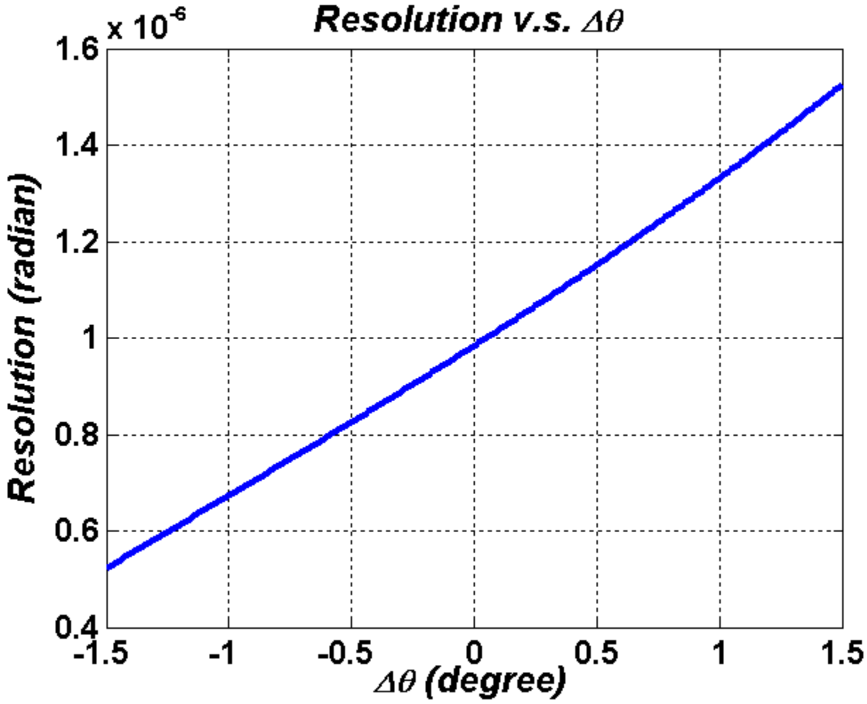

Fig. 6. The angular resolution of the small-angle sensor.

5. Conclusions

Why do we use a reflective type parallelogram prism, i.e., a reflective type elongated prism, as a small-angle sensor in this paper? It is because that we want to increase the resolution and the sensitivity of a small-angle measurement by increasing the number of multiple total-internal reflections inside the prism. Besides, the optical structure is designed as a common-path structure and the principle is based on multiple total-internal reflections and the heterodyne interferometry. Thus, it is stable against the turbulences of the environment such as air turbulences or mechanical vibrations. The new type small-angle sensor has some merits, e.g., a simple optical setup, high resolution, high sensitivity, rapid measurement, and high stability etc. And its feasibility is demonstrated.

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